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ABSTRACTS

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On the 100th Anniversary of the Birth of Academician of the National Academy of Sciences of Ukraine V.S. Korolyuk (1925-2020)

Volodymyr Korolyuk was born on August 19, 1925, in Kyiv, on Andriyivskyi Descent. He graduated from secondary school with distinction just a week before the outbreak of hostilities in June 1941. In the same year, he entered Kyiv University. In 1942, at the age of 17, he was conscripted into the army. After his demobilization in 1947, he resumed his studies as a third-year student at the Faculty of Physics and Mathematics of Kyiv State University named after T.G. Shevchenko, from which he graduated with honors.

V. Korolyuk began his scientific career under the supervision of Academician Borys Volodymyrovych Gnedenko, one of the founders of the Kyiv school of probability theory and cybernetics. Already in the 1950s, Korolyuk proposed a number of innovative ideas in limit theorems, and later in the asymptotic analysis of stochastic processes.

From 1961 onward, Volodymyr Semenovych worked at the Institute of Mathematics of the National Academy of Sciences of the Ukraine, with which he remained closely associated throughout his life.

One of the significant directions of Academician Korolyuk's research was the study of random walks and related boundary problems. These investigations laid the foundation for a new mathematical paradigm in the analysis of the limiting behavior of stochastic processes in both discrete and continuous time, particularly in the context of complex Poisson processes.

Korolyuk was among the first to formulate and develop a mathematical theory of the stability of dynamical systems with stochastic Markovian switching, modeling situations with rapid random perturbations. In such models, the system's dynamics depend on a random state process (Markov or semi-Markov) governing the switching between different regimes of evolution.

Volodymyr Semenovych Korolyuk is a figure who embodies the strength of the Ukrainian mathematical tradition and the continuity of generations. His legacy consists not only of numerous theorems and methods but also of a culture of scientific thought that continues to shape the future.

CONTENT[1]

[1] *The abstracts are publishing in authors edition*

COMPLIMENTS IN SOCIAL MEDIA: GENDERED AND CULTURAL DIFFERENCES IN THE GEORGIAN AND GERMAN DIGITAL DISCOURSE

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Compliments represent one of the most socially and pragmatically loaded speech acts, functioning not only as expressions of positive social attitude but also as tools for negotiating identity, power relations, and cultural expectation sininteraction. This presentation aims to examine the linguistic forms, thematic focuses, tones, and recipient responses of compliments within Georgian and German social media contexts, with particular emphasis on gendered communication patterns. The research explores how compliments function across Facebook and Instagram and how their use reflects broader cultural norms and gender roles.

The study is based on a corpus of 200 comments (100 in Georgian and 100 in German), collected from 40 public posts (equally distributed between male and female users). The analysis applies content and discourse-analytical methods and is grounded in the theoretical frameworks of Brown & Levinson's politeness theory and Holmes' gendered pragmatics.

The findings reveal significant cultural and gender-based distinctions in the use of compliments. In the Georgian context, compliments tend to be direct, emotionally expressive, and focused on visual appearance, especially when directed toward women. Typical expressions such as "You are stunningly beautiful" or "You are radiant inside and out" highlight a culturally embedded emotional openness. In contrast, compliments in the German context are often more restrained, indirect, and formally polite, as reflected in expressions like "Du siehst gut aus" or "Sehr sympathisch!", suggesting a more reserved communicative style.

The comparative analysis demonstrates that compliments in digital discourse are not universal speech acts but rather culturally embedded and gender-conditioned practices.

FOREIGN LANGUAGE AND ARTIFICIAL INTELLIGENCE: EMERGING OPPORTUNITIES IN LANGUAGE LEARNING

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One of the most pressing challenges of the 21st century is the growing need for multilingualism in an increasingly globalized world. Proficiency in a foreign language is no longer merely a tool for communication - it has become an essential element of professional growth, cultural integration, and the development of critical thinking skills. In this context, the rapid advancement of artificial intelligence (AI) has significantly transformed traditional approaches to language education.

This study explores how artificial intelligence is transforming the teaching and learning of foreign languages at both individual and institutional levels. A mixed-methods approach was employed, combining a systematic literature review, an analysis of widely used technological platforms (including Duolingo, ChatGPT, Google Translate, Grammarly, and others), and in-depth interviews conducted with teachers and students. The findings indicate that AI enhances personalized learning, supports multimodal instruction (through text, audio, and visual content), and boosts learners' motivation to engage with the target language. However, several challenges were also identified, including a loss of contextual nuance, difficulties in interpreting cultural elements, and a shifting or diminished role of the teacher in the learning process.

In conclusion, artificial intelligence should not be viewed as a substitute for traditional teaching methods, but rather as a complementary tool that, when effectively integrated, can substantially improve the quality of foreign language learning. Moving forward, there is a critical need to strengthen interdisciplinary research to better understand AI's role, not only from a technological perspective but also within pedagogical and ethical frameworks.

CONVERGENCE TO THE CONTINUOUS STATE SPACE BRANCHING PROCESSES

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Let $\vec{\xi}_n(\varepsilon)=\left(\xi_n^{'}(\varepsilon),...,\xi_n^{d}(\varepsilon)\right)$, $n=0,1,2...$ be a sequence of branching processes with d-type of particles and generating function $\vec{F}_\varepsilon(\vec{s})=\left(\vec{F}_\varepsilon^{1}(\vec{s}),...,\vec{F}_\varepsilon^{d}(\vec{s})\right)$, where ε – series parameter, $\vec{s}=(s_1,...,s_d)$, $F_\varepsilon^{j}(s)=E_j s_1^{\xi_n^{'}(\varepsilon)},...,s_d^{\xi_n^{d}(\varepsilon)}$, ξ_n^{j} is interpreted as a number of particles of j-type in n-th generation, E_j – conditional expectation as condition that at the beginning there was only one partical of j –type. The average number of j-type descendants from one partical of i-type denote by $a_{ij}(\varepsilon)$, i.e.

$$a_{ij}(\varepsilon)=E\left[\xi_1^{j}(\varepsilon)\,|\,\vec{\xi}_0(\varepsilon)=\vec{e}_i\right],\quad \vec{e}_i=(\underset{i-1}{0,..,0}1,\underset{d-i}{0,...,0})$$

Denote

$$A(\varepsilon)=\left\|d_{ij}(\varepsilon)\right\|_{i,j=1}^{d}.$$

In this work, according the behavior of matrix $A(\varepsilon)$ the limit theorems on the convergence of suitably normalized of sequence of branching processes with many type of particles to the one or multidimensional continuous state space branching processes are obtained.

HYBRID TWO-POINT METHODS FOR SOLVING NONLINEAR EQUATIONS

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There are numerous problems that using mathematical modeling can be expressed in the form of an equation $F(x) = 0$, where $F: D \subseteq X \to Y$ is a nonlinear operator, defined on a convex domain D, X and Y are Banach spaces. Solving nonlinear equations typically relies on iterative methods that involve multiple inversions of linear operators, which can be computationally intensive. One approach to solving large-scale problems involves approximating inverse operators [1].

In this work, we introduce hybrid two-point methods designed to reduce this cost by requiring only a single inversion of a fixed linear operator throughout the entire iteration process.

Local and semilocal convergence analyses are conducted in this work. Numerical experiments show that the outlined method performs comparably to traditional techniques, with significantly lower computational cost, offering an efficient alternative to Newton's method and other inversion-based schemas.

The proposed method is:

$$\Delta(x) = \Delta = G^{-1}(G - A(x_{n-1}, x_n)),$$
$$B_k(x) = B(x) = I + \Delta + ... + \Delta^k,$$
$$s_0 \in D, s_{n+1} = s_n - BG^{-1}F(s_n),$$

where $A(\cdot,\cdot): D \times D \to \mathcal{L}(X,Y)$ from X into Y and k is a fixed number. Several popular choices for the operator A include the secant operator, defined as $A(x,y) = F(x,y)$, and the Kurchatov operator, given by $A(x,y) = F(2y - x, x)$. The operator $G \in \mathcal{L}(X,Y)$ is such that both G and $(I - G^{-1}(G - F'(x)))$ are invertible.

This method requires only a single inversion of the fixed operator G, making it a computationally efficient alternative to Newton's method.

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EXTENDING THE APPLICABILITY OF NEWTON-JARRATT-LIKE METHODS WITH CONVERGENCE ORDER 2m + 1 FOR SOLVING NONLINEAR EQUATIONS

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Let B_0 and B be Banach spaces and D be a convex and nonempty subset of B_0. We consider a nonlinear equation $F(x) = 0$, where $F: D \subset B_0 \to B$ is a Fréchet-differentiable operator. For solving such equations, a class of computationally efficient (m+1)-step methods was introduced in [1]:

$$y_n^{(0)} = x_n - \frac{2}{3}F'(x_n)^{-1}F(x_n),$$
$$A_n = F'(x_n)^{-1}F'\left(y_n^{(0)}\right),$$
$$y_n^{(1)} = y_n^{(0)} - \frac{1}{12}(13I - 9A_n)F'(x_n)^{-1}F(x_n),$$
$$E_n = E\left(x_n, y_n^{(0)}\right) = \frac{1}{2}(5I - 3A_n),$$
$$y_n^{(j)} = y_n^{(j-1)} - E_nF'(x_n)^{-1}F\left(y_n^{(j-1)}\right),$$
$$x_{n+1} = y_n^{(m)} = y_n^{(m-1)} - E_nF'(x_n)^{-1}F\left(y_n^{(m-1)}\right),$$

where $x_0 \in D;\ m \geq 2;\ j = 2, \dots, m;\ n = 0,1,2, \dots$ and the sequence of approximations $\{y_n{}^{(j)}\}$ is generated starting from the Newton-Jarrat-type prediction step. The original analysis in [1] was limited by its reliance on high-order derivatives. We extend the method's applicability by proving convergence under weaker assumptions, using only generalized Lipschitz conditions on the first derivative. Our work provides a comprehensive analysis of local and semi-local convergence. The semi-local theory, based on majorizing scalar sequences, guarantees convergence without prior knowledge of a solution, and the domain of uniqueness of a solution is found. Numerical experiments validate our theoretical findings.

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INTEGRATING XGBOOST FOR INTELLIGENT SECRET MINING IN SOURCE CODE

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The inclusion of secrets (e.g. passwords) in application source code often results in unauthorized access to sensitive resources and leads to quick infrastructure compromise. Tools like Yelp's **detect-secrets** use regular expressions, keywords, and searching for strings with high entropy to detect secrets. However, such tools are limited by fixed heuristics which results in high false positive rates for obfuscated or novel secret formats.

In this work, I propose using the XGBoost classifier [1], trained on a hand-curated dataset of source code lines that contains both secret and non-secret examples. The 8449-line long dataset was constructed by combining over 60 types of real-world leaked secrets from GitHub and StackOverflow [2], 200 synthetically generated and manually reviewed code samples, and non-secret code samples from GitHub. The XGBoost classifier was trained on the following features: line entropy, token count, line length, file extension, the reliability score of a known secret regex (from 0 to 10 depending on the regex' robustness), and the reliability score of a common secret keyword (from -10 to 0 if the keywords are not interesting e.g. HTML tags, or from 0 to 10 if the keyword is interesting e.g. "aws_secret_access_key").

The proposed model identified multiple novel secrets when compared to the **detect-secrets** tool. However, the increase in sensitivity resulted in higher false positive rates for long strings and languages other than English. To address these issues, I intend to add the average token entropy feature, and also to split long strings into 300 character-long chunks in a round-robin manner and analyze each chunk.

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WORKING MEMORY IMPAIRMENT IN ALCOHOL DEPENDENCE

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Patients with alcohol dependence have disorders of multiple cognitive functions. This leads to inability to control alcohol consumption, relapse or continued use of alcohol despite negative physical, psychological, and social consequences. Executive control refers to a group of cognitive functions that allow people to regulate independently their behavior and to choose appropriate actions depending on their long-term goals.

The aim of the study was to evaluate indexes of spatial working memory in patients with alcohol dependence syndrome using the quantitative EEG method and the Corsi computer test.

Methods. 50 patients diagnosed with alcohol dependence (F10.2 on MKB-10) were evaluated after detoxification. The control group consisted of 30 mentally and somatically healthy, age-matched subjects. The study of spatial working memory was performed by a Corsi computer test (Corsi Block-Tapping). 16-channel EEGs were recorded at background with open eyes and during the Corsi test. Spectral power absolute values of θ –, α – and β – rhythms were analyzed.

Results. According to the Corsi test in alcohol dependence, the WM volume was markedly decreased compared to the healthy control group. Changes in EEG during the Corsi test in patients with alcohol dependence syndrome were associated with alpharhythm desynchronization in the prefrontal cortex and left temporal locus, which was not observed in the healthy control group. Decreased beta rhythm was observed in the central and parietal cortex of patients with alcohol dependence syndrome in response to a cognitive stimulus.

Conclusion. The results of this study may serve as additional diagnostic criteria for cognitive disorders in patients with alcohol dependence syndrome.

MANAGING ETHICAL DECISIONS WITHIN A SPIRITUAL CULTURE: PHILOSOPHY OF EDUCATION

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The present scientific report deals with the main issues of managing ethical decisions within a spiritual culture, especially Philosophy of Education. As we know, Spiritual culture encompasses the values, beliefs, practices, and expressions that shape a society's understanding and engagement with the realm of the matters of soul. It's a broad concept that includes religion, morality, art, and other aspects of social life, often intertwined with cultural traditions and beliefs. Spiritual culture "can be a source of meaning, purpose, and connection for individuals and communities, influencing their worldview and interactions with the world"[6]. It is interesting some opinion considering the moral decisions problems connected with protection of digital data in the educational culture, there is an actual question on research conducts to better understand the research role and responsibility of the Internet in Philosophy of education generally involves five overlapping areas: linguistic features of language teaching, interaction between practical and theoretical problems, attitude, and context and language impact.

The report tells us, that ethical maintenance of digital intellectual information safety is rather new practical contemporary area of professional work of the classical pragmatic educational experts. The basic and fundamental purposes of such traditional activity are:

- Maintenance of security from external and internal threats in pragmatic sphere of the social-cultural and institutional formation, business learning management and use of the digital information resources [5];
- Prevention of infringements of the worldview ethical legal rights of educational institutions and the academic organizations on moral preservation of confidentiality and privacy of the digital educational information;
- Association and consolidation of the distance online conditions interfering deliberate distortion or concealment of the theoretical digital information at absence for it of the instructional and academic worldview bases.

Thus, we can assume that the selection of alternatives and criteria for assessing the quality and safety of the individual functional unit of a

vehicle was carried out using an analytical hierarchical procedure method"; So it "is one of the promising trends for the improvement of the range of cosmetics in order to provide higher quality and safety of products on the world market. However, it should be noted that the scientific community is presently mainly researching the use of new technologies. Accordingly, the report is interested in the point of view of scientific work "Ethics of Information Technology in the Context of a Global Worldview" by the way of spiritual culture categories: "The virtual symbolic environment is made our reality. Virtual culture reality is a world of images, interpreted signs and symbols. You can say information society culture filled with simulacra. The civilization world of simulacra owes much of its existence to technical information technology. So, for example, on the internet human communicates with model another, and not with the other. Media Machine Absorbing Products computer revolution blurs the boundaries between simulated and real, between cultural spectacle and politics. Information technology that enriches virtual worlds has great success and is developing intensively" [2].

As is considered in spiritual culture, in the process of managing ethical decisions, especially in the direction of educational philosophy, it is important to consider that "an individual image of culture takes and uses material to shape itself and represent the archetype through the sensory-emotional characteristics of the modern era and the qualitative elements of its spiritual architecture" At the same time, attention is focused on the main elements of the usual, typical characteristics of the cultures under consideration, forming painful and exemplary instances of spiritual culture"[4] Here it needs to be noticed, that the types of educational research fall along the following main actual continua, among others: online experimental versus natural, quantitative versus qualitative, etic product versus process. Experimental research is based on the goal of trying to find generalized answers to specific questions. To achieve that, as many variables as possible are held constant so that one or two specific questions can be addressed in tightly controlled experimental situations.

In the axiological worldview context, the research "wants to convince us that in the case of the above two general types of world culture, both purely mental-discursive and purely figurative-ideological and creative progress, aspiration and feat are a direct result of the spiritual abilities of the morals and customs of the historical-cornerstone and national-social generation and, thus, this is a phenomenological manifestation unconscious feelings and subconscious knowledge"[1]. We may say that

true experiments must involve two groups, an experiment and control group, into which subjects have been randomly assigned. A "quasi-experiment" may involve one group that takes a pretest, has some kind of "treatment" and then takes a posttest. In contrast, naturalistic research seeks to understand phenomena in their natural, holistic context. In conducting naturalistic research, instead of controlling outside variables, the researcher attempts to observe and understand how a wide range of contextual factors may be affecting a situation. However, "they do not give a qualitative change in the situation in education and, in principle, cannot give, since the same results, sometimes even with less time, human, moral and intellectual resources, can be provided by traditional forms, methods and means of teaching" [3]

From the above-mentioned we may conclude that: We hope that the references cited in this paper provide a useful starting point for readers interested in conducting research on on-line learning and theoretical worldview in the internet space. Cultural and technological industry espionage prospers. But are frequent also a case of inadvertent presentation of the information concerning an institutional secret of the educational organization; as a specific traditional classical gold rule, here plays a delicate role a negligence of employees, misunderstanding of conditions by them, otherwise, «the moral human factor».

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ALGORITHMS FOR SEARCHING FOR PATTERNS AND RELATIONS IN BIG DATA USED BY ARTIFICIAL INTELLIGENCE SYSTEMS

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Over the last few years, the use of artificial intelligence systems has grown rapidly. It involves both specialized and more general tools. Among the latter, the most prominent are free publicly available Large Language Model based AI systems. In this work it was studied what algorithms AI systems use for solving the problems of finding patterns and relations in big data. Both literature data and own responses of Grok4, ChatGPT 5.0, Gemini 2,5 Flash Deep research AI systems were used for analysis. The main results may be summarized so.

In the initial stages, various data collection, integration and pre-processing algorithms are used. This may involve: Web Crawling & Scraping algorithms of importing data from online resources; Schema Matching (SM) and Record Matching algorithms required to integrate multiple relational tables of different schemas; Extract-Transform-Load (ETL) algorithms for integrating data into a data warehouse; Federated Query Processing algorithms for joining datasets without centralizing them;

In the following stages, various distributed and scalable processing frameworks are used. Among them there are: MapReduce algorithms for data scalable sorting, filtering, counting and aggregation; Graph Processing systems (Pregel, GraphX, Giraph) for link analysis and network relationship detection; Stream processing frameworks (Flink, Kafka Streams, Storm) for near-real-time patterns.

For searching patterns and relations such groups of algorithms are directly used: association and correlation discovery; clustering; classification; prediction; graph-based relationship mining; sequential pattern mining; anomaly and novelty detection; multi-modal and cross-source pattern learning, etc.

APPLYING THE MAXIMAX APPROACH TO MILITARY DECISION-MAKING

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The Maximax approach involves making a decision that provides the best results under the most favorable scenario. It is usually used in situations where: other options are guaranteed to be unsuccessful; there is an opportunity to ensure a high probability of a favorable scenario; it is necessary to obtain estimates of results for different scenarios, in particular for an optimistic one, etc. Some recent examples of Maximax decisions during the russian invasion of Ukraine and military operations in the Middle East are analyzed. The results confirm the high riskiness of maximax decisions. At the same time, they prove potentially high effectiveness of Maximax decisions supported by measures that ensure a high probability of a scenario that is the best or close to the best for achieving the set goals.

In this contest the following examples were analyzed using information available in open sources.

1. russia's full-scale invasion of Ukraine in 2022. This operation was based on fundamentally erroneous assessments of the potential of Ukraine and its allies, as well as its own resources. It didn't take into account any alternative scenarios or action plans. Because of this, it turned out to be a failure and did not achieve any of the set goals.

2. The Ukrainian operation "Pavutyna" ("Web") in 2025 is the opposite example of careful planning of the main scenario and measures aimed at supporting its implementation. This ensured the achievement of its goals and significant losses of russian strategic aviation at great distances from the border.

3. The MOSSAD attack on Hezbollah using pagers and other electronic devices is another example of the successful application of the Maximax approach. The success is largely due to the same factors. In addition, in both cases, the non-standard and unexpectedness of the solutions used was important.

4, 5. The Israel and American operations to eliminate military leaders and key facilities of Iran's nuclear program were expected, relatively standard, and judging by the available information, only partially achieved their goals.

COMPLETE SEMIGROUPS OF BINARY RELATIONS, DEFINED BY *X*-SEMILATTICES OF $\Sigma_2(X,5)$ CLASS

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Let $\Sigma_2(X,5)$ be such class between themselves isomorphic X-semilattice of unions, each element of which is isomorphic to X-semilattice of unions $D_\varsigma = \{\bar{D}_\varsigma, Z_1, Z_2, Z_3, Z_4\}$, satisfying the following conditions: $Z_4 \subset Z_3 \subset Z_1 \subset \bar{D}_\varsigma$, $Z_4 \subset Z_2 \subset \bar{D}_\varsigma$, $Z_3 \setminus Z_2 \neq \varnothing$, $Z_2 \setminus Z_3 \neq \varnothing$, $Z_1 \setminus Z_2 \neq \varnothing$, $Z_2 \setminus Z_1 \neq \varnothing$.

Theorem 1. Let $D = D_\varsigma \in \Sigma_2(X,5)$. A binary relation of a semigroup $B_X(D)$ is idempotent if and only if it satisfies one of the following conditions:

a) $\alpha = X \times Z$ for any $Z \in D$;

b) $\alpha = (Y_1^\alpha \times Z) \cup ((X \setminus Y_1^\alpha) \times Z')$, for some $Z, Z' \in D$ and $Y_1^\alpha \subset X$, satisfies the following conditions: $Z \subset Z'$, $Y_1^\alpha \neq \varnothing$, $Y_1^\alpha \supseteq Z$ and $Z' \setminus Y_1^\alpha \neq \varnothing$;

c) $\alpha = (Y_1^\alpha \times Z) \cup (Y_2^\alpha \times Z') \cup \left((X \setminus (Y_1^\alpha \cup Y_2^\alpha)) \times Z''\right)$ for some $Z, Z', Z'' \in D$ and disjoint subsets Y_1^α and Y_2^α of the set X, satisfying the following conditions: $Z \subset Z' \subset Z''$, $Y_1^\alpha \neq \varnothing$, $X \setminus (Y_1^\alpha \cup Y_2^\alpha) \neq \varnothing$, $Y_1^\alpha \supseteq Z$, $Y_1^\alpha \cup Y_2^\alpha \supseteq Z'$, $Y_2^\alpha \cap Z' \neq \varnothing$;

d) $\alpha = (Y_1^\alpha \times Z_3) \cup (Y_2^\alpha \times Z_2) \cup \left((X \setminus (Y_1^\alpha \cup Y_2^\alpha)) \times \breve{D}\right)$ for some disjoint subsets Y_1^α and Y_2^α of the set X, satisfying the following conditions: $Z_3 \cap Z_2 = \varnothing$, $Z_3 \subseteq Y_1^\alpha \subseteq X \setminus Z_2$, $Z_2 \subseteq Y_2^\alpha \subseteq X \setminus Z_3$;

e) $\alpha = (Y_1^\alpha \times Z_4) \cup (Y_2^\alpha \times Z_3) \cup (Y_3^\alpha \times Z_1) \cup \left((X \setminus (Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha)) \times \breve{D}\right)$ for some disjoint subsets Y_1^α, Y_2^α and Y_3^α of the set X, satisfying the following conditions: $Y_1^\alpha \supseteq Z_4$, $Y_1^\alpha \cup Y_2^\alpha \supseteq Z_3$, $Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha \supseteq Z_1$, $Y_2^\alpha \cap Z_3 \neq \varnothing$, $Y_3^\alpha \cap Z_1 \neq \varnothing$;

f) Let' $Z = Z_3, Z' = Z_2$ or $Z = Z_1, Z' = Z_2$, when $\alpha = (Y_1^\alpha \times Z_4) \cup (Y_2^\alpha \times Z) \cup (Y_3^\alpha \times Z') \cup \left((X \setminus (Y_1^\alpha \cup Y_2^\alpha \cup Y_3^\alpha)) \times \breve{D}\right)$ for some disjoint subsets Y_1^α, Y_2^α and Y_3^α of the set X, satisfying the

following conditions: $Y_1^{\alpha} \supseteq Z_4$, $Y_1^{\alpha} \cup Y_2^{\alpha} \supseteq Z$, $Y_1^{\alpha} \cup Y_2^{\alpha} \cup Y_3^{\alpha} \supseteq Z'$, $Y_2^{\alpha} \cap Z \neq \varnothing$, $Y_3^{\alpha} \cap Z' \neq \varnothing$.

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ON CONJUGATE TRIGONOMETRIC SERIES

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Some approximate properties of Cesaro means of negative order of Fourier trigonometric series are established.

Below we suppose that $T=[-\pi,\pi]$ and functions $f: R \to R$ are 2π - periodic. For $f \in L(T)$, by $\sigma[f]$ and $\bar{\sigma}[f]$ we denote Fourier trigonometric series of f and its conjugate series respectively, i. e.

$$\sigma[f,x]=\frac{a_0}{2}+\sum_{K=1}^{\infty} a_K \cos kx + b_K \sin kx, \quad k=0,1,2,...$$

$$\bar{\sigma}[f,x]=\sum_{K=1}^{\infty} -b_K \cos kx + a_K \sin kx, \qquad k \in N.$$

Below $A(\alpha)$, $A(p)$, $A(\alpha, p)$ they indicate absolute or positive values, depending only indicated parameters.

Theorem 1. Let $\alpha \in]0,1]$, $\alpha p > 1$ and $f \in C(T)$. Then

$$\| t_n^{-\alpha}(f) - \bar{f}_n \|_c \leq A(\alpha,p)\left\{\omega\left(\frac{1}{n^{1-\alpha}}, f\right)_c + n^{\alpha}\left(\omega\frac{1}{n}, f\right)_c\right\}, \; n \geq 4.$$

Theorem 2. Let $p \in]1,+\infty[$ and $f \in C(T)$. Then

$$\| t_n^{-\frac{1}{p}}(f) - \bar{f}_n \|_c \leq A(p)\left\{\omega\left(\frac{1}{n^{1-\frac{1}{p}}}, f\right)_c + n^{\frac{1}{p}}\omega\left(\frac{1}{n}, f\right)_p (\ln n)^{1-\frac{1}{p}}\right\}.$$

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ON SOME NEWTON METHOD MODIFICATIONS FOR SOLVING SYSTEM OF NON-LINEAR EQUATIONS

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Let's consider the problem

$$P(x) = 0 \; where \; P: R^n \to R^m \qquad (1)$$

There are a number of algorithms for solving problem (1). We will consider a particular new algorithm, namely:

$$u_k = \Phi(x_k);$$
$$x_{k+1} = x_k - [P'(\tfrac{x_k+u_k}{2})]^{-1}P(x_k),\ k = 0,1,\ldots \qquad (2)$$

Theorem. Let's follow the conditions that are satisfied.

1. $\Phi(x)$ operator that satisfies the condition $\|\Phi(x) - x_*\| \le C\|x_k - x_*\|^\tau$, where C>0, $1 \le \tau \le 2$, x_* - solution of the problem (1).
2. For $x \in X = \{x \in R^n: \|P(x)\| \le \|P(x_0)\|\}$,
   $\|P'(x)^{-1}\| \le B$, where $B > 0$.
3. For $x, x+h, x-h \in X$, $h \in R^n$
   $||P'(x) - P(x+h, x-h)|| \le k_1||h||^\tau$.
4. For $x, y, z \in X$:
   $\|P(x,y) - P(x,z)\| \le k_2\|y - z\|$, where $k_2 > 0$.
5. The initial approximation x_0 is chosen to satisfy the condition
   $q = B(k_1 + k_2)C\|x_0 - x\ \|^\tau < 1$,

then the sequence x_k is defined correctly by (2), and the next evaluation can take place

$$\|x_k - x_*\| \le q^{(1+\tau)^k - 1}\|x_0 - x_*\|.$$

Propose considering some cases of choosing formulas.

$$u_k = x_k - [P'(\tfrac{x_{k-1}+u_{k-1}}{2})]^{-1}P(x_k);$$

The start of the algorithm is $x_{k+1} = x_k - [P'(\tfrac{x_k+u_k}{2})]^{-1}P(x_k)$; $u_0 = x_0$; $k = 0,1,\ldots$.

REINFORCEMENT LEARNING FOR BANDWIDTH ESTIMATION IN REAL TIME COMMUNICATIONS

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Accurate bandwidth estimation is crucial for real-time communication systems such as WebRTC. There is little room for further advancement using classical approaches, such as delay gradient estimation [1].

We propose to use offline reinforcement learning for creating agent that can estimate available bandwidth during P2P calls. For learning we used modern offline RL method called Implicit Q-Learning (IQL). It is a recent offline RL algorithm that avoids querying any actions not present in the dataset, yet still enables significant policy improvement beyond the behavior policy. Instead of explicitly constraining or penalizing the policy, IQL changes how the value function is learned. The key insight is to replace the standard max operator in Q-learning with an implicit policy improvement step using expectile regression [2]:

$$\arg\min_{m_\tau} \mathbb{E}_{x\sim X}[L_2^\tau(x-m_\tau)],$$

Where $L_2^\tau(u) = |\tau - 1(u<0)|u^2$.

IQL use expectile regression in minimizing loss on Value function:

$$L_V(\psi) = \mathbb{E}_{(s,a)\sim D}[L_2^\tau(Q_{\hat{\theta}}(s,a) - V_\psi(s))] .$$

And next loss for updating Q-function:

$$L_Q(\theta) = \mathbb{E}_{(s,a,s')\sim D}[(r(s,a) + \gamma V_\psi(s') - Q_\theta(s,a))^2]. .$$

Our trained model showed better on 10% result on overestimate errors than previous efforts in this fields. It shows potential in such kind of methods and large place for improvements.

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THE LEAST-SQUARES METHOD IN THE THEORY OF NONLINEAR PERIODIC BOUNDARY VALUE PROBLEMS WITH DELAY

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Among numerous studies of functional-differential equations, research of periodic boundary value problems for differential equations with delay holds a special place [1]. This is primarily due to the wide application of periodic boundary value problems for differential equations with delay in physics, economics, biology, and mechanics. When constructing solutions of the nonlinear periodic boundary value problem for the equation using the least squares method, the problem of impossibility of finding solutions in elementary functions arises. This, in turn, leads to significant errors in the solutions of nonlinear boundary value problems.

By applying the Adomian decomposition method [2] and the least squares method scheme [3], we have derived the necessary and sufficient conditions for the existence of solutions to the weakly nonlinear periodic boundary value problem for a system of differential equations with delay in the critical case. The efficiency of the iterative schemes we have developed is demonstrated using an example of solving the problem of approximating periodic solutions to an equation with delay, which models a non-isothermal chemical reaction.

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AUTONOMOUS NONLINEAR PERIODIC BOUNDARY VALUE PROBLEMS WITH SWITCHING AT NON-FIXED MOMENTS OF TIME

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We analyze the relationship between boundary value problems with impulsive perturbation at fixed moments of time and boundary value problems with switching at fixed [1,2] and non-fixed moments of time [3]. We are found the constructive conditions of solvability and the scheme for constructing solutions of a nonlinear periodic boundary value problem with switching's at non-fixed moments of time. Using the Adomian decomposition method, solvability conditions are obtained and a new iterative technique for finding solutions to a weakly nonlinear periodic boundary value problem with switching's at non-fixed times is constructed. In addition, we obtained the constructive conditions for the convergence of the iterative scheme to the solution of the weakly nonlinear boundary value problem, as well as the switching's moments. The obtained iterative scheme is applied to find approximations to the periodic solution of the equation with switching's at non-fixed moments of time, which models a nonisothermal chemical reaction [3].

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SECURITY OF DATA TRANSMISSION IN THE MODERN DIGITAL SPACE

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When analyzing issues related to information security, it is necessary to take into account the specifics of the aspect of security, that security is an integral part of information technology - a field that is developing at an unprecedented pace. In the modern digital landscape, the security of data transmission over networks, especially Wide Area Networks (WANs), is of paramount importance. As WANs span large geographical areas and often traverse multiple administrative domains, they are particularly vulnerable to various security threats. One critical aspect of ensuring the security of WANs is the development of secure routing algorithms.

Developing secure routing algorithms for WANs presents several challenges due to the unique characteristics of these networks:

- Scalability. WANs often consist of numerous interconnected networks and devices, making scalability a crucial consideration in designing routing algorithms. Traditional routing protocols may struggle to scale efficiently while maintaining security measures.
- Dynamicity. WANs are subject to dynamic changes in network topology, traffic patterns, and security threats. Routing algorithms must adapt to these changes in real-time to ensure continuous and secure data transmission.

To address the aforementioned challenges, researchers have proposed innovative approaches to developing secure routing algorithms in WANs:

- Trust-Based Routing. Trust-based routing algorithms establish trust relationships among network entities and use trust metrics to make routing decisions. By considering the trustworthiness of neighboring nodes and network paths, these algorithms can mitigate routing attacks and ensure secure data transmission.
- Cryptographic Techniques. Cryptographic techniques, such as digital signatures, cryptographic hashing, and secure key exchange protocols, can enhance the security of routing protocols in WANs.

TO THE ISSUES OF FINDING IDENTIFIERS OF TENNIS PLAYERS' SPORTSMANSHIP AND STRATEGIES FOR ITS IMPROVEMENT

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Tennis players know that to get into the top ten in the world, it is necessary to achieve the physical data of the top ten (serve power, hit power, jump height, etc.). But this is not enough, and therefore the search continues for key indicators of a tennis player's individual skill that can become "sufficient" to get into the top ten. The identifiers for improving the sportsmanship of tennis players, training strategies, and the tasks of building game strategies to maximize the probability of winning points and matches are considered. Finding optimal strategies is carried out using mathematical models of cause-and-effect relationships between identifiers of individual sportsmanship of tennis players, the results of previously played matches involving these tennis players, and the probabilities of victories. The initial set of sportsmanship identifiers and important variables includes: physical indicators (strength, endurance, speed, agility, mobility); performance indicators (strength and direction of shots, strength and direction of serves, number of aces, unforced errors, percentage of first serves, second serves and doubles); physical, psychological, environmental (clay, grass, hard court), etc. These and many other important variables that determine the causal and consequential factors of adequate models are available in large databases with match results from many Grand Slam tournaments, ATP/WTA mandatory and auxiliary tournaments and many other lower-ranked tournaments. Statistical data on these and many other important tennis model variables are available in many databases with results from Grand Slam matches, ATP/WTA mandatory and auxiliary tournaments, and lower-ranked tournaments. The availability of large volumes of statistical data contribute to the successful game strategies optimization and in-depth research into the development of tennis mathematical modelling using data mining methods, hypothesis testing, machine learning, and artificial intelligence tools.

AN EMPIRICAL EVALUATION OF REASONING LANGUAGE MODELS FOR PROPAGANDA DETECTION

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The automated detection of propaganda presents a challenging decision-making problem under uncertainty, where systems must classify nuanced text that is often deliberately ambiguous. This work investigates the use of Reasoning Language Models (RLMs) for identifying propaganda techniques and manipulative narratives in text. In particular, we evaluate advanced RLMs against non-reasoning models and previously established benchmarks for two well-known and highly cited challenges: (1) propaganda technique classification (SemEval-2020 Task 11, Subtask TC), and (2) propaganda narrative detection (DIPROMATS 2024 Task 2).

Results show that reasoning-capable models can improve detection performance and provide interpretable outputs (e.g. step-by-step justifications), at the cost of increased computation. For technique classification, the top reasoning model achieved an F1 score of up to 0.569, outperforming the best non-reasoning model by nearly five percentage points, with some techniques like “Loaded Language” achieving F1 of up to 0.693. In narrative detection, the best RLM registered an average F1 score of 0.670, significantly outperforming both the official baseline (0.416) and the highest performing system reported in the competition overview paper.

The effectiveness of RLMs for propaganda detection is demonstrated by benchmarking our system against the official competition leaderboards, where it achieves a score corresponding to the top rank for DIPROMATS 2024 Task 2 and a performance level equivalent to a top-three placement for SemEval-2020 Task 11, underscoring their outstanding performance. We conclude that future progress might involve developing hybrid multi-agent systems that leverage the strengths of different model tiers to balance accuracy, cost-efficiency, and explainability.

GENERALİZİNG THE ANTİVİRAL İMMUNE RESPONSE MODEL TO ACCOUNT FOR ADSORPTİON, DİFFUSİON PERTURBATİONS AND TEMPERATURE

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The ability of adsorbents to bind, retain and naturally remove from the organism not only various types of toxins, metabolic products and heavy metals, but also pathogenic microorganisms and products of their vital activity ensures an increase in the therapeutic effect with the complex application of traditional therapeutic procedures and adsorption drugs. Considering the specifics of adsorption therapy application is important for forecasting the infectious disease dynamics in decision-making systems in the development of effective individual treatment programs.

The antiviral immune response model has been generalized to take into account the effect of adsorption therapy on the development of disease process under conditions of small diffuse scattering of acting factors based on the synthesis of perturbation theory and modeling ideas of the adsorption mass transfer process. A computational technology of step-by-step asymptotic approximation of the solution to the corresponding model problem with time delay as a perturbation of solutions to degenerate problems without delay has been developed. The numerical experiment results demonstrate a predictive decrease in the concentration of viral elements in the target organ through their neutralization into the organism by adsorbents introduced. It is noted that the efficiency of adsorption drugs depends, in particular, on the moment of time for their introduction, which must be considered when making decisions on the formation of complex treatment programs using appropriate therapy.

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THE MAXIMUM PRINCIPLE FOR A MODIFIED MODEL OF VIRAL INFECTION

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The problems of funding optimal controls for various systems, particularly those governed by partial differential equations, continue to retain their significance. The foundations of the theory of optimal control for systems described by partial differential equations were established by J. L. Lions [1] as early as 1971. Nevertheless, this theory continues to be advanced, with numerical algorithms being developed for the solution of specific classes of problems. In this paper, we consider the optimal control problem for a modified model of viral infection, governed by a nonlinear parabolic system of differential equations involving a singularity in the state description:

$$\frac{\partial V}{\partial t}+\upsilon_x^V\frac{\partial V}{\partial x}+\upsilon_y^V\frac{\partial V}{\partial y}-\varepsilon D_V\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)=w^V+(\beta(\theta)-\gamma F)V,$$

$$\frac{\partial C}{\partial t}+\varepsilon\upsilon_x^C\frac{\partial C}{\partial x}+\varepsilon\upsilon_y^C\frac{\partial C}{\partial y}-\varepsilon^2 D_C\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)=\xi(m)\cdot\alpha(\theta)\cdot F(t)\cdot V(t)\text{-}\mu_C(C-C^*),$$

$$\frac{\partial F}{\partial t}+\upsilon_x^F\frac{\partial F}{\partial x}+\upsilon_y^F\frac{\partial F}{\partial y}-\varepsilon D_F\left(\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}\right)=u^F+\rho C-(\mu_f+\eta\gamma V)\cdot F,$$

$$\frac{\partial m}{\partial t}-\varepsilon^2 D_m\left(\frac{\partial^2 m}{\partial x^2}+\frac{\partial^2 m}{\partial y^2}\right)=\sigma\cdot V-\mu_m(\theta)\cdot m,$$

$$\frac{\partial \theta}{\partial t}+\varepsilon\upsilon_x^\theta\frac{\partial \theta}{\partial x}+\varepsilon\upsilon_y^\theta\frac{\partial \theta}{\partial y}-\varepsilon D_\theta\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)=\alpha_\theta VF-\mu_\theta(\theta-\theta^*)-\lambda_\theta\cdot u^\theta,$$

where $V(x,y,t)$ – concentration of antigens), $C(.)$ – concentration of specific immune agents, antibodies, $F(.)$ – concentration of immunological cells $m(.)$ – relative characteristic of the level of damage to the target organ, θ – temperature distribution in the target organ, u^F, u^θ – control functions.

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OPTIMAL CONTROL FOR THE MATHEMATICAL MODEL OF COVID-19 SPREAD SVEAIHR

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In this paper we investigate the mathematical model of COVID-19 spread described by the initial value problem for a system of nonlinear ODEs:

$$
\begin{aligned}
&s'(t) = \Lambda - \left(\beta_1(1-u(t))i(t) + \beta_2(1-u(t))a(t)\right)s(t)/N - \left(\omega+\mu\right)s(t) + \\
&\qquad \kappa r(t) + \theta v(t) \\
&v'(t) = \omega s(t) - \left(1-\psi\right)\left(\beta_1(1-u(t))i(t) + \beta_2(1-u(t))a(t)\right)v(t)/N - \\
&\qquad \left(\theta+\mu\right)v(t) \\
&e'(t) = \left(\beta_1(1-u(t))i(t) + \beta_2(1-u(t))a(t)\right)s(t)/N + \\
&\qquad \left(1-\psi\right)\left(\beta_1(1-u(t))i(t) + \beta_2(1-u(t))a(t)\right)v(t)/N - \left(\delta+\mu\right)e(t) \\
&a'(t) = q\delta e(t) - \left(\mu+\rho_2\right)a(t) \qquad (1) \\
&i'(t) = \left(1-q\right)\delta e(t) - \left(\mu+d+\rho_1+\delta_1\right)i(t) \\
&h'(t) = \delta_1 i(t) - \left(\sigma_0/\left(1+bh(t)\right) + d_h + \mu + \xi\right)h(t) \\
&r'(t) = \rho_1 i(t) + \rho_2 a(t) + \sigma_0/\left(1+bh(t)\right)h(t) - \left(\kappa+\mu\right)r(t) \\
&s(0) = s_0 > 0, \quad v(0) = v_0 \geq 0, \quad e(0) = e_0 \geq 0, \quad a(0) = a_0 \geq 0, \quad i(0) = i_0 \geq 0, \\
&h(0) = h_0 \geq 0, \quad r(0) = r_0 \geq 0 \qquad (2) \\
&s(t) + v(t) + e(t) + a(t) + i(t) + h(t) + r(t) = N \quad \text{for all } t \in [0,T].
\end{aligned}
$$

In model: s – susceptible individuals, v – vaccinated individuals, e – exposed (latent) individuals, a – asymptomatic infected, i – symptomatic infected, h – hospitalized, r – recovered individuals; $u(t)$ - control function, which represents the level of social distancing, social restrictions, and public awareness campaigns; Λ, $\beta_1, \beta_2, N, \omega$, μ, κ, θ, ψ, δ, q, $\rho_2, d, \rho_1, \delta_1, \sigma_0, b, d_h, \xi$, b, d_h, ξ – fixed parameters.

The optimal control problem consists in minimizing the cost functional: $\min J(u) = \int_0^T \left[c_1 i(t) + c_2 a(t) + c_3 \omega s(t) + c_4 u(t)^2\right]dt$ subject to the state equations (1)-(2) and subject to the control constraint $0 \leq u(t) \leq u_{\max}$. We apply the Pontryagin Maximum Principle to derive

the necessary conditions for optimality. Results of numerical experiments for both the direct and optimal control problems are presented.

RETRIAL QUEUES WITH PROPORTIONAL SERVICE RATE STRATEGY

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We are dealing with a retrial queue of $M/M/1/\infty$ type (see. [1]) with intensity λ of the input Poisson flow of the primary calls, and ν is a parameter of the repeated calls. It is supposed the number of places on the orbit to be equal to $m \leq \infty$. The parameter of the service time is submitted to the proportional strategy: if at the moment, when a customer (repeated or primary) hits the swerver we have j customers in the system, then the service time of that customer follows an exponential distribution with parameter $\mu j = aj + b$. So, in contrast to the classic systems, we are dealing with the case when service time depends on the number of the customers in the system.

Let $\xi(t)$ stand for the number of the customers on the orbit at the instant t. If at that moment the server is busy with the service and its time distribution has parameter μi, $i \geq 1$, we say that the server is in the phase i. If at the moment t the server is free, we say that it is in the phase $i = 0$. Let $\eta(t) \in \{0, 1, 2, ..., m + 1\}$ denote the phase of the server at the time t . The process $(\eta(t), \xi(t))$ is a homogeneous Markov process with $E = \{(i, j), j \geq 0, 0 \leq i \leq j + 1\}$ as a state space.

In the report we obtain the condition when the steady-state regime of the system exists and the formulae for that distribution are obtained. Using these formulae we considered some optimization problems for such a system.

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DEVELOPMENT OF DISCRETE SPATIAL MODELS OF A NEW SPECIES MIGRATION IN A SPATIALLY HETEROGENEOUS ENVIRONMENT

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The mechanism of migration and dispersal of living organisms on the surface and in three-dimensional space is an important issue in theoretical ecology [1].

We have developed discrete mathematical models of the dispersal of individuals of one species of living organisms across a spatially heterogeneous environment.

For one territorial cell, all pairs of individuals (living and not sated) are listed for which the Boolean function $F(i, j)$ is defined: $F(i, j) = 1$, if individuals of the i-th species can eat individuals of the j-th species, and $F(i, j) = 0$ – otherwise. Here i is the species of the first individual of the pair, and j is the species of the second individual of the pair that can feed on the first individual of the pair, and the satiety $S(n, j)$ of the second individual in the pair (the predator individual) does not reach the boundary value $S^*(j)$ (that is, $S(n, j) < S^*(j)$, where j is the species number of the second individual of the pair, and n is the individual number).

Then, the set of all those territorial cells is found in which either no individuals live at all, or only individuals of an age equal to or greater than the age T_V live, and such cells are located no more than distance L_P away from the mother individual. From this set of cells, cells with a total number of K_D are selected in such a way that the probability of choosing the m-th cell is proportional to the level V_r of hostility of the relocated young to the individual living in this cell from the previous iteration. Environmental heterogeneity is modeled by the impossibility of settlement or low probability of survival of individuals of certain species in certain cells.

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CONVERGENCE OF THE CONTROL PROBLEM FOR A DIFFUSION PROCESS IN A LÉVY SCHEME WITHIN A SEMI-MARKOV ENVIRONMENT

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The control problem for a random transport process $y(t) \in R^d$ with control $u(t) \in C([0,\infty))$ [1], and semi-markov environment $x \in (X, \boldsymbol{X})$ [1] is defined by the equation:

$$dy(t) = C\big(y(t), x(t), u(t)\big)dt + d\eta\big(t, u(t)\big), \tag{1}$$

where $\eta(t,u), t \geq 0$, is an impulse disturbance process, which in the Lévy approximation scheme is defined by the relation [1]

$$\eta\big(t, u(t)\big) = \int_0^t \eta\big(ds, x(s), u(s)\big)ds. \tag{2}$$

Let the quality criterion of control $u(t)$ for the process (1) be defined by the function $G(y,x,u)$ with a unique point u^* where $G(y,x,u^*) = \min G(y,x,u)$.

Thus, for the control $u(t)$ let us consider the stochastic optimization procedure [1]

$$\frac{du(t)}{dt} = \alpha(t)\, \nabla_{\beta(t)} G(y,x,u), \tag{3}$$

where $\nabla_{\beta(t)} G(y,x,u) = \frac{\big(G(y,x,u+\beta(t)) - G(y,x,u-\beta(t))\big)}{2\beta(t)}$.

The conditions for the convergence of problem (1), (3) to the diffusion process are established.

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CONVERGENCE OF THE DIFFUSION PROCESS UNDER MARKOV-MODULATED POISSON PERTURBATION IN THE DIFFUSION APPROXIMATION SCHEME

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A stochastic evolutionary system [1] in an ergodic Markov environment within the diffusion approximation scheme is defined by the following evolutionary equation:

$$du^{\varepsilon}(t) = C\left(u^{\varepsilon}(t), x\left(\frac{t}{\varepsilon^2}\right)\right)dt + \varepsilon^{-1}a(t)\widetilde{N}_x(t)dt,$$

where compensated Poisson process $\widetilde{N}_x(t)$ defined as

$$\widetilde{N}_x(t) = N^{\varepsilon}(t) - \Lambda(t) = N^{\varepsilon}(t) - \int_0^t \lambda(x(s/\varepsilon^2))ds.$$

A Poisson process $N^{\varepsilon}(t)$ is a counting process that tracks the number of events occurring up to time t. The event arrival rate at time *t*, is defined as $\lambda(x(t))$, i.e., the intensity function is modulated by the state of a Markov process $x \in X$ [1]

The limit process $\hat{u}(t)$ can be obtained as the solution to the stochastic differential equation

$$d\hat{u}(t) = C(\hat{u}(t))dt + B(t)dW(t),$$

where $W(t)$ is the increment of a Wiener process.

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OPTIMAL CONTROL PROBLEM FOR THE STOCHASTIC EVOLUTION UNDER MARKOV-MODULATED POISSON PERTURBATIONS

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We consider a stochastic evolution system perturbed by a Markov-modulated Poisson process (MMPP) and aim to determine an optimal control strategy via a stochastic optimization procedure.

Let $x(t) \in (X, \boldsymbol{X})$ be an ergodic Markov process, and let the evolution system dynamics be given by the equation:

$$dy^{\varepsilon}(t) = C\big(y^{\varepsilon}(t), x(t/\varepsilon^2)\big)dt + \varepsilon^{-1}a\big(t, u^{\varepsilon}(t)\big)N_x(t),$$

where $N_x(t)$ is a compensated Poisson process with intensity modulated by $x(t/\varepsilon^2)$ [1].

The control $u^{\varepsilon}(t)$ is chosen to minimize a quality criterion $J(y, x, u)$ using a stochastic optimization procedure:

$$\frac{du(t)}{dt} = \alpha(t)\,\nabla_{\beta(t)}J(y, x, u),$$

where $\nabla_{\beta(t)}J(y, x, u) = \frac{\big(J(y,x,u+\beta(t))-J(y,x,u-\beta(t))\big)}{2\beta(t)}$.

The limit generator for the joint process $(y^{\varepsilon}(t), u^{\varepsilon}(t), N_x(t))$ is derived using the singular perturbation analysis. This allows us to show the convergence to a diffusion process $(\hat{y}(t), \hat{u}(t))$.

Under mild assumptions on the Lyapunov stability of the averaged system and boundedness of the gradients, we prove that $\mathrm{P}\{\lim_{\varepsilon\to 0} u^{\varepsilon}(t) = \hat{u}(\mathrm{t})\} = 1$.

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ASYMPTOTIC STABILITY OF THE SEIR MODEL WITH SEMI-MARKOVIAN SWITCHING

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In the presented material, the procedure of stochastic approximation (PSA) is constructed by

$$\frac{dS}{dt} = a(t)\left[-\beta_X \frac{SI}{P} + \sigma_1(S - S^*)w_1'\right],$$
$$\frac{dE}{dt} = a(t)[\beta_X \frac{SI}{P} - \alpha_X E + \sigma_2(E - E^*)w_2'],$$
$$\frac{dI}{dt} = a(t)[\alpha_X E - \gamma_X I + \sigma_2(I - I^*)w_3'],$$
$$\frac{dR}{dt} = a(t)[\gamma_X I + \sigma_2(R - R^*)w_4']$$

for the trends in SEIR model. There β − transmission rate that describes the spreading speed of the epidemic, γ − recovery rate, α − exposed rate, P − population, E − exposed population, R − recovered population, I – infected population, S− susceptible population, t − time. Here w_1, w_2, w_3, w_4 denote mutually independent standard Wiener processes, σ_1, σ_2, σ_3, σ_4 indicate noises for susceptible, infected and recovered populations, respectively [1].

The variations observed in the specified coefficients are indicative of the semi-Markov switching dynamics inherent to the model [2]. Moreover, the implementation of PSA establishes convergence to the equilibrium point of the original model, which is rigorously derived through analytical procedure.

Comparable to those applied in model formulation parameters $\beta_X = \beta + X_1$, $\gamma_X = \gamma + X_2$, $\alpha_X = \alpha + X_3$, where X_i , i = [1, 2, 3], is a semi-Markov process with arbitrarily chosen states

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ESTIMATION OF MATRIX EQUATIONS SOLUTIONS UNDER UNCERTAINTY FOR NONLINEAR OBSERVATIONS

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We studied the problem of estimating a matrix X of given dimension $n \times n$ that is a solution to the equation

$$AX + XB = F,$$

where A, B, F - are known matrices, and the set $D = \{X : AX + XB = 0\}$ of solutions is non-empty.

To determine estimates of the matrix X , values were considered

$$y_k = F_k(X) + \eta_k, \; k = \overline{1, N},$$

where $F_k(x)$ is a known function, and the unknown values η_k satisfy the inequalities

$$|\eta_k| \le \gamma_k, \; k = \overline{1, N}$$

with a known value γ_k .

We investigated the boundedness of the set of matrices that are solutions to the inequalities

$$|y_k - F_k(x)| \le \gamma_k, \quad k = \overline{1, N},$$

and the problem of approximate estimates of the matrix X .

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EXTENDING GRAPH RETRIEVAL-AUGMENTED GENERATION FOR SEMI-STRUCTURED SCIENTIFIC DATA

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Large Language Models (LLM) based on AI are becoming more and more popular all over the world. However, they can make mistakes in data analysis, which can lead to incorrect conclusions and decisions.

We present a graph-centric extension of Graph Retrieval-Augmented Generation (Graph RAG) [1] that injects explicit statistical aggregation into the knowledge graph, closing the gap between free-form LLM summarization and evidence-based reasoning:

1. Each paper is parsed by an LLM into structured *claims* linking a canonical entity to a dynamically inferred population segment, storing effect direction, *p*-value, sample size, citation count, and publication year as edge attributes.
2. Claims are clustered via embedding similarity and ontology-aware rules, unifying semantically equivalent statements and creating dense evidence bundles for every entity-population pair.
3. For each pair, we compute a conditional evidence profile — proportions of supporting, neutral, and adverse findings, citation-weighted confidence, median *p*-value, and temporal trend — and attach these statistics to a summary edge in the graph.
4. At query time, the engine retrieves both raw findings and their evidence profiles, enabling the LLM to answer with quantitatively grounded statements such as "… 75 % of studies (median $p = 0.02$) support …".

This hybrid design produces balanced, transparent, and scalable answers, illustrating how statistical synthesis can be embedded directly into graph-based RAG systems.

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OPTIMAL CONTROL OF THE TWO-LINK MANIPULATOR

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We investigate the optimal control problem for the motion of a two-link manipulator performing a transport operation in the horizontal plane. The solution is constructed based on the Pontryagin's maximum principle, with a combination of the shooting method and the corresponding residual minimization procedure.

The two-link manipulator, under the action of controls $u_1(t)$, $u_2(t)$ (torque moments at the joints), performs a transport operation: moves the load from a specified initial position (α_0, β_0) to a specified final position (α_T, β_T), where (α, β) – generalized coordinates of the manipulator. The velocities of the manipulator's links are assumed to be zero at both the start and the end of the operation. The time duration of the transport operation is a specified value, while no explicit constraints are imposed on either the control or the angular characteristics of the manipulator. We formulate the following optimal control problem: find the controls $u_1^*(t)$, $u_2^*(t)$, $t \in [0, T]$, which perform specified transport operation with the minimum value of the functional $\Phi = \int_0^T [u_1^2(t) + u_2^2(t)]dt$.

The motion of the two-link manipulator is described by a system of four highly nonlinear first-order differential equations. The solution to the formulated problem is built using the Pontryagin's maximum principle, according to which the optimal controls $u_1^*(t)$, $u_2^*(t)$ are expressed explicitly and depend on the four adjoint variables. Therefore, the original optimal control problem is reduced to the corresponding two-point boundary value problem for a eight first-order differential equations (four equations for the basic variables and four equations for the conjugate variables). To solve the resulting boundary value problem, we use the shooting method, which leads to the solution of a sequence of Cauchy problems. We define the initial values of the conjugate variables, which are determined by the minimization of the corresponding quadratic function (residual) that quantifies the deviation of the calculated terminal values of the basic variables from their specified values (according to the transportation operation). The residual is minimized through iterative methods of nonlinear programming.

PARAMETRIC OPTIMIZATION OF MOTION OF A PORTAL ROBOT WITH A TWO-LINK MANIPULATOR

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We consider the controlled motion (in the vertical plane) of a portal robot consisting of a carriage G and a two-link manipulator (fig. 1). The carriage under the action of the force $F(t)$ moves along a horizontal beam *1* mounted on vertical columns *2* and *3*. A two-link manipulator is attached to the carriage using a cylindrical joint O_1. The rotation of the manipulator links occurs under the action of torque moments $u_1(t)$, $u_2(t)$ at the joints O_1, O_2.

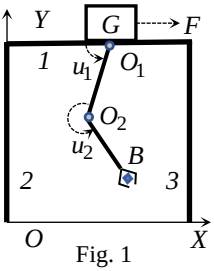

Fig. 1

Let $\mathbf{q} = (x, \alpha, \beta)$ be the vector of generalised coordinates of the robot, and $\mathbf{u} = (F, u_1, u_2)$ be the vector of controls. Let the manipulator perform a transport operation at the time interval $[0, T]$ (it moves the load from a given initial position to a given final position). Let us formulate the optimal control problem.

Problem 1. Determine the optimal control $\mathbf{u}^*(t)$, $t \in [0, T]$, which will ensure that the robot performs the given transport operation with the minimum value of the functional $E = \int_0^T \mathbf{u}(t)\mathbf{u}^{\mathrm{tr}}(t)dt$.

To construct a suboptimal solution to *Problem 1*, we use the parametric optimisation method, according to which we present the generalised coordinates

$$q_i = \sum_{k=0}^{3} p_{ik} t^k + \sum_{k=1}^{n} (a_{ik} \cos 2\pi k t/T + b_{ik} \sin 2\pi k t/T),$$

where $\{p_{ik}\}_{k=0}^{3}$ are the coefficients determined from the initial and final conditions of the transport operation, n is the specified parameter, $\{a_{ik}\}_{k=1}^{n}$ are the coefficients found as a solution to the auxiliary nonlinear programming problem $\tilde{E}(\mathbf{z}) \underset{\mathbf{z}}{\to} \min$, where $\tilde{E}(\mathbf{z})$ is the function of parameterization coefficients $\mathbf{z} = (a_{ik};\ k = \overline{1, n},\ i = \overline{1, 3})$, which is obtained after substituting the controls $\mathbf{u}(t, \mathbf{z})$ into the objective function E.

CREATING SECURE ROUTING ALGORITHMS

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The current methods of routing message packets exhibit several drawbacks, such as insufficient adaptation to changes in the network structure, low communication security, and various other limitations. Selecting a transmission route involves establishing the sequence of transit network nodes through which messages should be forwarded to the intended recipient. This selection process typically occurs within the network nodes of the provider operators. The presence of nodes with low security levels creates conditions for network intrusions, consequently diminishing communication security. The approaches outlined in the paper and the algorithm presented are geared towards enhancing communication security by effectively managing information exchange routes between network subscribers.

To facilitate the exchange of information between subscribers within a communication network, a secure communication route must be chosen from the set of available routes connecting the network's subscribers. Routing a message entails identifying the sequence of nodes within the transit network through which the message should traverse. Determining the route poses a challenging task, particularly when numerous potential routes exist between a pair of subscribers. Route determination involves selecting one or more routes from the set of possible options based on specific criteria. In existing methods of route selection, typical criteria include nominal bandwidth, congestion of communication channels, delays introduced by channels, number of intermediate network transit nodes, reliability of channels, and network transit nodes. However, in many cases, a contradiction arises between the need to ensure communication security and these existing methods.

In the initial version of the method, secure routes are selected based on the communication network's structure, initial data about network nodes and subscribers, and the calculation of complex security indicators for network nodes.

DECISION MAKING UNDER LEGAL UNCERTAINTY: A FUZZY LOGIC APPROACH TO PROTECTING CHILDREN'S RIGHTS

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Legal decisions concerning the protection of children's rights—such as those involving custody, adoption, child protection, or refugee status—are frequently made under conditions of significant uncertainty. These uncertainties arise from incomplete or ambiguous evidence, emotionally charged contexts, and the often vague nature of legal standards such as the "best interests of the child." Traditional legal reasoning, which typically relies on binary classifications and rigid frameworks, may not adequately capture the complexity of such cases, leading to inconsistent or unjust outcomes.

The paper explores how fuzzy logic—an approach that allows for reasoning with degrees of truth rather than fixed categories—can serve as a valuable tool for improving decision-making in child-related legal contexts. Fuzzy logic models can formalize subjective criteria, weigh conflicting factors, and provide a structured yet flexible framework for evaluating nuanced legal scenarios. The paper proposes a model for applying fuzzy decision-making in key areas such as child custody determination, child protection risk assessments, and legal evaluation of vulnerable refugee minors. By integrating fuzzy logic with legal reasoning, decision-makers can reduce arbitrariness, improve consistency, and better align outcomes with the principles of justice and child welfare.

The study also addresses potential challenges in applying such interdisciplinary methods, including ethical concerns, practical implementation, and the importance of maintaining judicial discretion. Ultimately, the paper advocates for a more adaptive and transparent approach to legal decision-making under uncertainty—one that not only respects legal traditions but also embraces innovation in the service of protecting children's rights.

PROBLEMS UNEMPLOYMENT IN AN ECONOMICALLY LESS ACTIVE REGION

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Unemployment, and especially chronic unemployment, has quite negative socio-economic consequences for society, which is adequately reflected in the standard of living of the population and the process of economic development of the country. Therefore, in international practice, serious attention is paid to the assessment of these negative socio-economic consequences. It is relatively easy to assess the economic consequences of unemployment, while the social consequences, which are difficult to distinguish and are complex and cannot be fully evaluated. The negative consequences of unemployment and the degree of its impact on socio-economic development depend on the situation in the country, on the specific parameters of the social situation.

To fight unemployment, we must first analyze its causes. There are many reasons for the emergence of unemployment: the transformation of the economy, the economic downturn, the mistakes made during economic reforms, the introduction of innovations, the high level of corruption, the presence of imperfect institutions, the difficult political and criminogenic situation, etc.

We focus on one of the problems in the field of employment, namely the shadow part of the employment market. People employed in the shadow part of the labor market avoid mandatory taxes for employees in the country, whose income can be much higher than the income of many taxpayers, however, in most cases, the income of people employed in the shadow part of the labor market is below average, and they might be using various types of social assistance and benefits. Most, but not all, people in the informal labor market are low-paid, low-skilled workers.

The main indicators of unemployment are: the number of unemployed; duration of unemployment; unemployment rate; the sex-age structure of unemployed; professional-qualified structure of unemployed; sectoral and territorial structure of unemployed.

DECISION SUPPORT MODEL BASED ON TOPSIS AND AI

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This paper presents an intelligent decision support model designed to improve strategic decision-making by integrating multi-criteria analysis (TOPSIS) with elements of explainable artificial intelligence [1]. A prototype implementation was developed using Python and a graphical interface, demonstrating usability and adaptability in real-world scenarios.

This paper proposes a novel model that addresses these limitations by combining classical multi-criteria techniques with modern explainability approaches.

The model consists of three modules: predictive-analytical, multi-criteria evaluation [2], and an interface acting as a chatbot, which provides textual explanations. Users can input alternatives, criteria, and weights; select one of four analysis modes; and receive transparent rankings with natural-language explanations. All processing is done offline, without external dependencies.

Main advantages include full transparency of calculations, deployment simplicity for SMEs, enhanced user-interaction, and applicability across domains (finance, logistics, energy) [3]. Future work may include database integration and automated weight generation. Such a system can support strategic planning, investment evaluation, or risk-sensitive project selection.

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THE ANALYTICAL VIEW OF SOLUTION OF FRACTIONAL KINETIC EQUATIONS

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Fractional diffusion equations were introduced to describe physical phenomena such as diffusion in porous media with fractal geometry, kinematics in viscoelastic media, glassy materials, synthetic polymers, biopolymers, anomalous diffusion and turbulence et al.

The fractional operators are natural mathematical objects to describe the long-range dependence and intermittency phenomena. In this article we present a fractal heat equation in which the *n*-dimensional Laplacian Δ is replaced by a fractional Laplacian of the form $-(I-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}$, $\alpha > 0$, $\gamma \geq 0$, where the operators $-(I-\Delta)^{\gamma/2}$, $\gamma \geq 0$, and $(-\Delta)^{\alpha/2}$, $\alpha > 0$, are interpreted as inverses of the Bessel and Riesz potentials respectively. Both Bessel and Riesz potentials are considered to be defined in a weak sense, in frequency domain, in terms of fractional Sobolev spaces.

We consider the following fractional kinetic equation

$$\frac{\partial^{\beta} u(t,x)}{\partial t^{\beta}} = -\mu(I-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}u(t,x) + f(x,t,u(x,t-h)), t > h, \quad (1)$$

subject to the initial condition

$$u(t,x)\big|_{0\leq t\leq h} = \varphi(x,t), \quad (2)$$

$$u(0,t) = \psi(t), t > h, \quad (3)$$

where $u(t,x)$, $0<t\leq T$, $x\in \mathbb{R}^n$ is the kinetic field and $\beta\in(0,1]$, $\gamma \geq 0$, $\alpha > 0$, are fractional parameters.

The time derivative of order $\beta\in(0,1]$ is defined as follows

$$\frac{\partial^{\beta} u(t,x)}{\partial t^{\beta}} = \begin{cases} \dfrac{\partial u(t,x)}{\partial t}, \beta = 1, \\ \left(D_t^{\beta} u\right)(t,x), \beta \in (0,1), \end{cases}$$

where

$$\left(D_t^{\beta}u\right)(t,x)=\frac{1}{\Gamma(1-\beta)}\left[\frac{\partial}{\partial t}\int_0^t (t-\tau)^{-\beta}\,u(\tau,x)\,d\tau-\frac{u(0,x)}{t^{\beta}}\right],\ 0<t<T,$$

is the regularized fractional derivative in the Caputo-Djrbashian sense. Note that for $\gamma = 0$, $\beta = 1$, $\alpha = 2$. Eq. (1) is the classical linear diffusion equation or heat equation with initial condition (2), (3) [1] and for $n = 1$ operator $\Delta \equiv \partial/\partial x_1$.

Cauchy problem (1) – (3) we study of the steps method. Let $x \in \mathbb{R}^n$, $h \le t \le 2h$, then $u(x,t-h) = \varphi(x,t)$ and in (1) – (3) we get the problem:

$$\frac{\partial^{\beta}u(t,x)}{\partial t^{\beta}}=-\mu(I-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}u(t,x)+f(x,t,\varphi(x,t)),$$

$$u(t,x)\big|_{t=h}=\varphi(x,h),\quad u(0,t)=\psi(t).$$

with the conditions of agreed $\varphi(0, h) = \psi(h)$.

By the Fourier transform with respect to x we obtain the Cauchy problem and we shall extensively use the Mittag–Leffler function $E_{\beta}(z)$ (see [1]).

In the article [2,3] we provide a formula for solving the nonclassical boundary value problem and in [4] the first boundary value problem for the equation of thermal conductivity with a variable diffusion coefficient and with a nonlinear term, which depends on the sought function with the deviation of the argument, is solved.

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LINEAR CONTRACTS: SIMPLICITY AND OPTIMALITY

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A linear contract can be fully described by a single real number $\alpha \in [0,1]$, that is it, the principal pays the agent a fraction α of the realized reward. Accordingly, the expected utilities of the agent and the principal for an agent's action i are given by $\alpha R_i - c_i$ and $(1-\alpha)R_i$. To determine the action motivated by a linear contract with parameter α, it suffices to identify which of the lines $\alpha R_i - c_i$ has the highest value under this particular value of α [1]. These points are named as indifference points. For an action $1 \le i \le n'$ on the upper envelope, intersection point with the preceding action α_i, is defined by formula 1 as:

$$\alpha_i := \begin{cases} 0 & for \quad i = 1 \\ (c_i - c_{i-1}) / (R_i - R_{i-1}) & for \quad 2 \le i \le [n'] \\ 1 & for \quad i = n' + 1 \end{cases} \qquad (1)$$

At point α_i the agent is indifferent between actions i and $i-1$. The choice of action at such points is governed by a tie-breaking rule. Under the canonical tie-breaking rule, action i is preferred over action $i-1$. Thus, the interval $[0,1]$ of α values divides onto n' subintervals, each corresponding to motivated action. For each $i \in [n']$ the interval of i-th action $[\alpha_i, \alpha_{i+1})$ includes all values α, given which agent selects this action. Worth noting that the gap between the revenue from the optimal contract and the best linear contract does not exceed n [1].

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APPLICATION OF CONVOLUTIONAL NEURAL NETWORKS IN THE DEVELOPMENT OF COMPUTATIONAL BIOLOGY TOOLS

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The rapid growth of biological data in computational biology requires the development of efficient analytical tools. Convolutional neural networks (CNNs), originally created for computer vision, have proven effective in analyzing DNA, RNA, and protein sequences. Their ability to automatically extract features and detect local dependencies makes them useful for tasks such as classification, functional region prediction, protein structure modeling, and motif recognition.

In bioinformatics, sequences are represented as numerical matrices (e.g., via one-hot encoding), enabling CNNs to apply convolutions with filters of different sizes to capture relevant patterns without manual feature engineering. Applications include protein function prediction, transcription factor binding site identification, and variant analysis in personalized medicine.

The integration of CNNs with language models enables hybrid architectures that model both local and global contexts, improving the performance of biological sequence analysis. Overall, CNNs support the creation of accurate, scalable, and automated tools in computational biology and bioinformatics [1].

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STOCHASTIC DIFFERENTIAL EQUATIONS AND COGNITIVE MODELLING OF DEFENDER SELF-DECEPTION IN CYBERSPACE

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In this article, we explore a novel direction in modeling decision-making in cybersecurity by integrating stochastic differential equations (SDEs) with cognitive models that describe defender behavior under conditions of self-deception. Drawing on the Instance-Based Learning Theory (IBLT), we analyze human decisions in cyber deception scenarios, particularly in the Insider Attack Game (IAG). Our focus lies in the defender's response to deceptive signals, highlighting the inherent cognitive biases and bounded rationality that influence behavior. We propose that recent advances in the theory of SDEs can provide a powerful mathematical foundation for modeling such dynamic and uncertain decision-making environments. The integration of adaptive cognitive models and stochastic methods allows for more accurate simulation and prediction of human behavior in cyber defense, especially when the attacker or defender employs deceptive tactics.

Cyber defense often involves asymmetric decision-making, where defenders must act on incomplete or deceptive information. Detecting deception is particularly difficult due to the lack of physical cues and the high-dimensional nature of cyberspace. Traditional models of rational choice fall short in capturing the nuances of human cognition under uncertainty. To address this gap, we adopt a cognitive modeling approach based on IBLT and propose its integration with stochastic methods.

Experimental studies [1,2] show that decisions in the IAG are largely driven by confirmation bias, memory recency, and frequency of past experiences. This behavior deviates from purely rational strategies and highlights the need to incorporate cognitive dynamics into cyber defense models. Self-deception—when defenders misinterpret or selectively attend to signals—further complicates the decision process.

Stochastic Differential Equations in Modeling Decision Dynamics

Stochastic differential equations offer a formalism to represent decision-making as a continuous-time process under uncertainty. By modeling belief updates, confidence thresholds, and memory decay as stochastic

processes, we can better capture how decisions evolve. For example, the decision variable x(t) may follow an SDE of the form:

$$dx(t) = f(x,t)dt + g(x,t)dW(t),$$

where $W(t)$ is a Wiener process, and f and g encode cognitive features like attention to deception or memory strength.

Adaptive Signaling and Defender Behavior: Recent research in signaling schemes [3] suggests that defenders can use deception not only to mislead attackers but also to influence their own decisions. Adaptive signal design, guided by individual cognitive profiles, can enhance defensive strategies. A key insight is that humans do not always process available information rationally, especially under repeated interactions.

Towards Realistic Cognitive-Driven Defense Models: To increase ecological validity, ongoing work integrates these models with environments like CyberVAN [4] where domain experts evaluate the feasibility of implementing adaptive deception. Combining cognitive architectures with SDE-based dynamics enables a flexible and robust framework that can scale to more complex cyber scenarios.

Conclusion. Understanding and modeling self-deception in cyber defense decision-making require a transdisciplinary approach. By merging cognitive modeling with stochastic dynamics, we aim to predict and influence defender behavior more effectively. Future work should explore the parameterization of these models using real-world data and extend the methodology to collaborative defense systems.

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ON THE DEVELOPMENT OF ADAPTATION AND SELF-LEARNING ALGORITHMS FOR DIGITAL TWINS FOR SEMI-NATURAL MODELING OF COMPLEX OBJECTS

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In the process of development and semi-naturalistic modeling of modern complex objects, there is a need to provide hardware and information support for the construction of various components of these complex objects. The creation of accurate complex models, virtualization of components, and the use of digital twin technologies will ensure the joint uninterrupted operation of these components in real time, and the optimal conduct of various studies and tests.

In semi-naturalistic modeling of complex objects, digital twins of components must behave realistically. For this purpose, adaptation and self-learning algorithms can be used. In semi-naturalistic simulation systems with the implementation of the Industrial Internet of Things (IIoT), measured data from a real object helps to increase the accuracy, flexibility and efficiency of such digital twins. Adaptation algorithms are used to update the parameters of the model or simulator in real time using data from various sensors and IIoT tools. Self-learning algorithms are used to adjust the digital twin during the simulation using streaming data from IIoT tools. They are divided into online learning algorithms and self-learning algorithms with time series. Such algorithms can help to identify degradations or anomalies in telemetry during semi-naturalistic simulation. Models in which a machine learning algorithm complements the physical model are also suitable for semi-naturalistic simulation. This can be used to adapt the model to changes in operating conditions.

In the future, it is planned to use self-learning digital twins that will automatically update based on sensor data in real time and will initiate changes in architecture or parameters. It is necessary to provide for the semi-naturalistic modeling system its own adaptation and learning logic for configuring modules and for adapting the system topology when changing parameters. This approach will provide the ability to adjust the modeling process and reproduce the behavior of a real complex system (object) without interfering with the operation of the object itself. In addition, it will be possible to influence the accuracy of forecasting and system management using digital twins.

AUTOMATION OF AN EXPERT CHATBOT BASED ON GROK 3 (XAI) FOR EVALUATING DECISION-MAKING ALTERNATIVES

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1. Research Objective: To develop an algorithm for integrating Grok 3 (xAI) via REST API for automated evaluation of alternatives, ensuring adherence to criteria of objectivity, processing speed (real-time / batch), and user-friendliness.

2. Solution Components: User Input: A JSON containing alternatives, evaluation criteria, and associated weights. **API Integration**: Integration with Grok 3 / Grok 3 mini REST API, utilizing reasoning modes ("Think", "Big Brain") and DeepSearch. [1, 2, 4]. **Results**: Structured evaluations, recommendations, and justified reasoning (chain-of-thought).

3. Technical Implementation: Backend: Python (requests, pandas); integration via frameworks such as Flask/Django or automation platforms like Airflow/Zapier. **Database**: PostgreSQL / MongoDB for storing alternatives, criteria, and evaluation results. **Modes of Use**: Real-time (online) or batch processing modes.

4. Advantages: Enhanced reasoning using "Think" / "Big Brain" modes [2, 5]. High performance powered by the Colossus supercomputer [1]. Multimodal capabilities: DeepSearch provides access to current data [2, 4].

5. Limitations & Risks: "Black Box" nature: Limited transparency in model logic, potentially reducing trust. Limited adaptability in non-standard contexts — requires manual tuning. Controversial behavior: Criticized for bias in responses [3]. Dependence on input data quality.

6. Recommendations: Use Grok 3 as an auxiliary tool, not the sole decision-maker. Implement Explainable AI (XAI) to make decisions transparent to users. Ensure GDPR compliance, content moderation, and provide feedback channels. Regularly update criteria and validate input data.

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CLUSTERING EXCHANGE-TRADED FUND (ETF) TIME SERIES ACROSS COMMODITY SECTORS

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Time series clustering plays a crucial role in financial analytics, enabling the discovery of structural relationships between assets without relying on predefined labels. In commodity markets, such analysis is particularly relevant due to the complex interplay of macroeconomic, geopolitical, and seasonal factors that drive asset prices. These time series are typically non-linear, non-stationary, and exhibit sector-specific volatility patterns, making the choice of similarity measures and clustering algorithms critical for obtaining meaningful results.

This study investigates the clustering of monthly historical price time series for a set of commodity-related Exchange-Traded Funds (ETFs) from January 1, 2022, to August 1, 2025. The dataset covers precious metals, industrial metals, energy resource, and agricultural commodities. After standardizing prices five clustering approaches were applied: K-means, K-means + DTW, agglomerative clustering, FFT-based K-means, and K-shape.

The best-performing method in terms of producing semantically coherent clusters was K-means + DTW.

The results largely align with known commodity sectors, yet some consistent cross-sector relationships emerged. Notably, natural gas clustered with wheat, while crude oil repeatedly grouped with sugar. Such patterns may stem from policy-driven and geopolitical factors.

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THE A PRIORI ESTIMATES AND REGULARITY OF SOLUTIONS THE NONLINEAR PARABOLIC EQUATIONS IN GENERALIZED MORREY SPACES

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This study establishes generalized Morrey estimates for the gradient of weak solutions pertaining to a class of nonlinear parabolic equations within a highly irregular domain. The nonlinearity is assumed to be merely measurable with respect to the time variable t and belongs to the small BMO class concerning the spatial variable x. In addition, we present a numerical application that models the time-dependent propagation of heat or pressure in a subsurface environment using a parabolic PDE and the Crank–Nicolson method, supporting our theoretical findings with visual simulations.

Keywords: Nonlinear parabolic equation; regularity; Reifenberg flat domain; fractional maximal operator; generalized Morrey spaces; Calderón–Zygmund estimates; Crank–Nicolson method.

Mathematics Subject Classification: 35J25, 35K20, 46E30.

GENERATING SETS OF THE COMPLETE SEMIGROUPS OF BINARY RELATIONS DEFINED BY FIVE-ELEMENT DOUBLE CHAIN

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Abstract. In this article, we study generating sets of the complete semigroups of binary relations defined by five-element double chain.

Key words: Semigroup, semilattice, binary relation, double chain.

Let us call the complete D semilattice of unions a five-element double chain, if it has the following form: $D=\{Z_1, Z_2, Z_3, Z_4, Z_5\}$, where the elements Z_1, Z_2, Z_3, Z_4, Z_5 are nonempty subsets of a nonempty set

X that satisfy the following conditions:

$$Z_1 \subset Z_3 \subset Z_4 \subset Z_5,$$
$$Z_2 \subset Z_3 \subset Z_4 \subset Z_5,$$
$$Z_1 \cup Z_2 = Z_3.$$

We will consider only the case when $Z_1 \cap Z_2 = \varnothing$ and $X \neq Z_5$.

We always assume that every element of the complete semigroup of binary relations defined by a five-element double chain is written in normal form.

Let us identify the following subsets in the semigroup $B_X(D)$:

$$B_2 = \left\{\beta \mid \beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2)\right\};$$
$$B_3 = \left\{\beta \mid \beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2) \cup (Y_5^\beta \times Z_5); \left|Y_5^\beta\right| = 1\right\};$$
$$B_4 = \left\{\beta \mid \beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2) \cup (Y_4^\beta \times Z_4) \cup (Y_5^\beta \times Z_5)\right\};$$
$$B_5 = \left\{\beta \mid \beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2) \cup (Y_3^\beta \times Z_3) \cup (Y_4^\beta \times Z_4) \cup (Y_5^\beta \times Z_5)\right\}.$$

Let us define the binary relations $\varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$ on the subsets B_2, B_3, B_4, B_5 respectively as follows:

- If

$$\alpha = (Y_1^\alpha \times Z_1) \cup (Y_2^\alpha \times Z_2) \in B_2$$

and

$$\beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2) \in B_2,$$

then $(\alpha, \beta) \in \varepsilon_2$ iff $\alpha = \beta$ or $Y_1^\alpha = Y_2^\beta$ and $Y_2^\alpha = Y_1^\beta$;

- If

$$\alpha = (Y_1^\alpha \times Z_1) \cup (Y_2^\alpha \times Z_2) \cup (Y_5^\alpha \times Z_5) \in B_3$$

and

$$\beta = (Y_1^\beta \times Z_1) \cup (Y_2^\beta \times Z_2) \cup (Y_5^\beta \times Z_5) \in B_3,$$

(obviously $\left|Y_5^\alpha\right| = \left|Y_5^\beta\right| = 1$),

then $(\alpha, \beta) \in \varepsilon_3$ iff $\alpha = \beta$ or $Y_1^\alpha = Y_2^\beta$, $Y_2^\alpha = Y_1^\beta$ and $Y_5^\alpha = Y_5^\beta$;

- If

$$\alpha = (Y_1^{\alpha} \times Z_1) \cup (Y_2^{\alpha} \times Z_2) \cup (Y_4^{\alpha} \times Z_4) \cup (Y_5^{\alpha} \times Z_5) \in B_4$$

and

$$\beta = (Y_1^{\beta} \times Z_1) \cup (Y_2^{\beta} \times Z_2) \cup (Y_4^{\beta} \times Z_4) \cup (Y_5^{\beta} \times Z_5) \in B_4,$$

then $(\alpha, \beta) \in \varepsilon_4$ iff $\alpha = \beta$ or $Y_1^{\alpha} = Y_2^{\beta}$, $Y_2^{\alpha} = Y_1^{\beta}$, $Y_4^{\alpha} = Y_4^{\beta}$ and $Y_5^{\alpha} = Y_5^{\beta}$;

And finally,

- If

$$\alpha = (Y_1^{\alpha} \times Z_1) \cup (Y_2^{\alpha} \times Z_2) \cup (Y_3^{\alpha} \times Z_3) \cup (Y_4^{\alpha} \times Z_4) \cup (Y_5^{\alpha} \times Z_5) \in B_5$$

and

$$\beta = (Y_1^{\beta} \times Z_1) \cup (Y_2^{\beta} \times Z_2) \cup (Y_3^{\beta} \times Z_3) \cup (Y_4^{\beta} \times Z_4) \cup (Y_5^{\beta} \times Z_5) \in B_4,$$

then $(\alpha, \beta) \in \varepsilon_5$ iff $\alpha = \beta$ or $Y_1^{\alpha} = Y_2^{\beta}$, $Y_2^{\alpha} = Y_1^{\beta}$, $Y_3^{\alpha} = Y_3^{\beta}$, $Y_4^{\alpha} = Y_4^{\beta}$ and $Y_5^{\alpha} = Y_5^{\beta}$.

It is easy to check that the binary relations $\varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$ are equivalence relations on the subsets B_2, B_3, B_4, B_5 respectively, so these subsets will be divided by the mentioned equivalences into disjoint classes of mutually equivalent elements. It is easy to see that each class will contain two elements.

Let S_2 (accordingly S_3, S_4, S_5) denote the subset of B_2 (accordingly B_3, B_4, B_5) that contains exactly one member of each of the ε_2 (accordingly $\varepsilon_3, \varepsilon_4, \varepsilon_5$) equivalence classes.

Theorem. The set $S = S_2 \bigcup S_3 \bigcup S_4 \bigcup S_5$ is a ireducible system of generators of the complete semigroup $B_X(D)$ of binary relations defined by a five-element double chain.

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TO OPTIMAL CONTRACTS UNDER UNCERTAINTY

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In the principal-agent model, risk aversion is more important than the limited liability of the agent [1]. Let the agent have a set A of actions of size n, and the costs of these actions for the agent are $0 \le c_i \le c_{i+1}$, $i = 1,...,n-1$. Let the principal have a set of m outcomes with rewards $0 \le r_j \le r_{j+1}$, $j = 1,...,m-1$. The agent's action stochastically leads to an outcome based on a probability matrix Q (the agent's technological matrix) consisting of elements q_{ij}, $i = 1,...,n$, $j = 1,...,m$, where q_{ij} is the probability of obtaining the principal's outcome j as a result of the agent's action i. Therefore, the i-th row $(q_{i1}, q_{i2},..., q_{im})$ of the matrix Q is the distribution (probability mass function) of the gains as a result of the agent's action i. The principal's expected gain as a result of the agent's action i is denoted

$$R_i \doteq \sum_{j=1}^{m} q_{ij} r_j ,$$

and the expected (social) welfare as a result of the agent's action i is denoted $W_i \doteq R_i - c_i$. It is important here that the agent's action i is hidden from the principal, who observes only the stochastic outcome j and his/her payoff r_j.

A contract is a payment rule $\vec{t} \doteq (t_1, t_2,..., t_m)$ from the principal to the agent, where $0 \le t_j$ is a transfer corresponding to the outcome $j = 1,..., m.$ The solution to the principal-agent problem consists in developing a contract $\vec{t}$ [2]. The expected payment from the principal to the agent as a result of the agent's action i is denoted

$$T_i \doteq \sum_{j=1}^{m} q_{ij} t_j .$$

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INTELLIGENT ANOMALY DETECTION FOR CYBERSECURITY: REAL-TIME SIMULATION AND UNSUPERVISED DEEP LEARNING

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In this paper, we tackle the challenge of detecting increasingly sophisticated cyber attacks that evade traditional signature-based intrusion detection systems. We propose an unsupervised deep learning framework for real-time anomaly detection, eliminating the need for labeled attack data. The method employs a deep autoencoder trained solely on benign network traffic from the CIC-IDS2017 dataset, modeling normal behavior and detecting anomalies via reconstruction error. A real-time Streamlit interface visualizes detection outcomes and enables user interaction with both benign and malicious data streams. Experimental results show high accuracy in identifying abnormal patterns without prior attack knowledge, demonstrating the framework's scalability, adaptability, and effectiveness for autonomous intrusion detection in dynamic environments.

Keywords: Unsupervised Anomaly Detection; Autoencoder; Cybersecurity; Intrusion Detection System; Real-Time Simulation

Subject Classification: 68T05, 68T07, 62H30

SOLVING A DEGENERATE ELLIPTIC VARIATIONAL INEQUALITY WITH DIFFERENT WEIGHTS

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Let us consider the following degenerate elliptic variational inequality:

$$\sum_{i,j=1}^{N} \int_{\Omega} a_{ij}(x) \frac{\partial y}{\partial x_j} \frac{\partial (v-y)}{\partial x_i} \rho_2 \, dx + \int_{\Omega} y(v-y)\rho_1 \, dx \geq \int_{\Omega} f(v-y) \, dx, \forall v \in K \quad (1)$$

where $f \in L^2(\Omega)$ is the right-hand side; Ω is a bounded open subset of $\mathbb{R}^N (N \geq 1)$ with Lipschitz boundary; and $\rho_{i=1,2}$ are degenerate weight functions of $\mathbb{R}^N$; $\rho_i(x) > 0$ a.e. of $\mathbb{R}^N$ and $\rho_i + \rho_i^{-1} \in L^1_{loc}(\mathbb{R}^N)$, $\rho + \rho^{-1} \notin l^{\infty}(\Omega)$ in the general case; K is a non-empty, closed, and convex subset of the weighted Sobolev space $H(\Omega, \rho, dx)$, $\rho = (\rho_1, \rho_2, \cdots, \rho_2)$, $0 \in K$; $H(\Omega, \rho, dx)$ - closure $C_0^{\infty}(\Omega)$ of $W(\Omega^N, \rho \, dx)$ with respect to the norm

$$\|y\|_{\rho} = \left(\int_{\Omega} |y|^2 \rho_1 \, dx + \sum_{i=1}^{N} \int_{\Omega} \left| \frac{\partial y}{\partial x_i} \right|^2 \rho_2 \, dx \right)^{\frac{1}{2}} \quad (2)$$

$W(\Omega, \rho \, dx)$ - the set of functions $y \in W_0^{1,1}(\Omega)$ for which the norm (2) is finite.

Consider the set of all symmetric matrices $u(x) = \{a_{ij}(x)\}_{1 \leq i,j \leq N}$ в $L^{\infty}(\Omega; \mathbb{R}^N \times \mathbb{R}^N)$

Suppose that the following conditions are satisfied:

$$|a_{ij}(x)| \leq \rho \; a.e. \Omega, \forall_{i,j} \in \{1, \dots, N\};$$

$$(u(x)(\xi - \eta), \xi - \eta)_{\mathbb{R}^N} \geq 0, \text{ a.e. in } \Omega, \forall_{\xi,\eta} \in \mathbb{R}^N;$$

$$(u(x)(\xi), \xi)_{\mathbb{R}^N} = \sum_{i,j=1}^{N} a_{ij}(x) \, \xi_i a \xi_j \geq \alpha |\xi|^2 \; a.e. \Omega \text{ where } \alpha > 0.$$

Under natural assumptions, the following result holds for the parameters of the problem.

Theorem. For every $f \in L^2(\Omega)$, there exists a unique solution to the degenerate elliptic variational inequality (1).

HIGH FREQUENCY ACOUSTIC WAVE OF COMPRESSION AND RAREFACTION

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High-frequency acoustic wave of compressions and rarefactions, collisions, fluctuations of concentration and temperature, quantum intermolecular interactions excite the atoms of PHF by transferring a share of energy to them. In the excited state, neutral atoms and molecules of PHF create an electric dipole moment of quantum transitions.

A common property of the particles participating in the reaction is the ability to form instantaneous dipole moments and, as a consequence, participation in dispersed interaction. The state of equilibrium of the PGF molecules depends on the intensity of the ultrasonic wave, its length, the PGF flow rate, the laminarity of the gas flow, the relaxation time constant determined by the mechanism of thermal relaxation, and the properties of the vibrational and rotational scheme of the terms of the molecules. The strict requirement of laminarity is a consequence of the close action of dispersed forces. Therefore, the kinetics of processes in the PGF and the adsorption layer is also determined by dispersed interaction.

Since the mass of electrons is small, the acoustic wave in the PGF deforms the outer electron shell of arsenic (phosphorus) atoms. They lose their tetrahedral symmetry, which contributes to an increase in the number of arsenic (phosphorus) atoms captured by the GC.

The process of arsenic (phosphorus) capture by excited GCs of the substrate surface determines the growth of islands on the GC face (111)A. Since the concentration of Fe atoms is high and they are located predominantly in the plane (111)A, the density of growth islands on this plane is maximum.

The density of localized energy in the spots of the main maxima, as well as the density of nonequilibrium electrons of the surface layer, carried away by the flow of elastic energy to the surface, is maximum at the center of the ellipse. For this reason, the excited complexes of the ellipse center are in states with the largest quantum numbers. They attract the particles of the PGF most strongly, as a result of which the concentration of particles PGF is maximum in the center of the ellipse.

CONVERGENCE OF SOLUTION FOR AVERAGED LINEAR QUADRATIC PARABOLIC OPTIMAL CONTROL PROBLEM

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For unknown functions $y = y(t,x), u = u(t,x) \in L^2(Q_t), t \in [0,T]$, $x \in \Omega \subset \mathbb{R}^d$, $Q_t = (0,T) \times \Omega$, the following linear quadratic optimal control problem is considered:

$$J_\pi(u) = \int_{\mathcal{A}} \left[\int_0^T \left(\int_\Omega y^2(t,x)dx + \gamma_1 \int_\Omega u^2(t,x)dx \right) dt + \right.$$
$$\left. + \gamma_2 \int_\Omega y^2(T,x)dx \right] d\pi(A) \to \inf,$$
$$\begin{cases} \dfrac{\partial y(t,x)}{\partial t} - L_A y(t,x) = u(t,x), \\ y\big|_{x \in \partial\Omega} = 0, \\ y\big|_{t=0} = y_0(x), \end{cases}$$

where

$$L_A = \sum_{i,j=1}^{d} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j}, \quad a_{ij} = a_{ji}.$$

Matrix $A = \{a_{ij}\}$ satisfies the uniform ellipticity condition: $A \geq \gamma_3 I$; $\gamma_1, \gamma_2, \gamma_3$ are positive constants, $y_0 \in L^2(\Omega)$. Here $\mathcal{A}$ is a set of symmetric matrices A satisfying the uniform ellipticity condition with the same constant γ_3. Let's denote π as a probability measure on $\mathcal{A}$.

We prove [1] the convergence of optimal controls as the probability measure representing system uncertainty becomes more concentrated, namely if $\pi^n \to \pi^\infty$ weakly, then

$$\forall A \in \mathcal{A}, \ \overline{y_\pi^n} \to \overline{y_\pi^\infty} \text{ in } C([0,T]; L^2(\Omega)),$$
$$\overline{u_\pi^n} \to \overline{u_\pi^\infty} \text{ in } L^2(\Omega).$$

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GLOBAL BEHAVIOR OF THE TRAJECTORIES OF IMPULSIVE SET-VALUED SEMIGROUP FOR PARABOLIC INCLUSION

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When modeling processes whose trajectories undergo instantaneous impulsive perturbations at the moments when some integral functional reaches a certain threshold value, the theory of impulsive dynamical systems is generally accepted [1]. In conditions of incomplete information about the characteristics of the system, an adequate mathematical tool is set-valued dynamical systems, when at each moment of time we analyze a certain set of system states [2].

In this paper, we investigate the qualitative behavior of mild solutions of a parabolic inclusion with jumps on a sphere in the phase space. More precisely, in a bounded domain $\Omega \subset R^d, d \geq 1$ for unknown function $u = u(t,x),\ t > 0, x \in \Omega$ we consider the following initial boundary value problem

$$\begin{cases} \frac{\partial u}{\partial t} - Au \in \varepsilon F(u), \\ u|_{\partial\Omega} = 0, \\ u|_{t=0} = u_0 \end{cases} \tag{1}$$

where A is a uniformly elliptic symmetric coercive differential operator with bounded coefficients, $\varepsilon \geq 0$ is a small parameter, $F: \mathbb{R} \to C_v(\mathbb{R})$ is a given upper-semicontinuous bounded set-valued map. We consider (1) in the phase space $X = L^2(\Omega)$. It is known [2] that $\forall u_0 \in X$ there exists at least one weak solution of (1) on (0, +∞). The impulsive problem is the following:

a phase point $u(t), t \geq 0$, when reaching the set $M = \{u \in X | \ ||u|| = R\}$, instantly jumps into a new position $u^+ \in u + \varphi(u)$, where $\varphi: X \to C_v(X)$ is a given set-valued map.

After that, the phase point $u(t)$ continues to move along the tra jectory (1) until the next meeting with M, and so on. Assuming each $u(\cdot)$ to be right-continuous at the moments of the jump, we denote by $G_\varepsilon(t, u_0)$ the set of all points of impulsive trajectories at the moment of time t that started from u_0 at $t = 0$. We prove that for sufficiently small $\varepsilon \geq 0$ the set-valued mapping $G_\varepsilon$generates an impulsive set-valued dynamical

system [2] which possesses a compact uniform attractor in X , i.e., there exists a compact $set\ \theta_\varepsilon \subset X$ such that $\forall r > 0$

$$\sup_{||u_0||\leq r} dist(G_\varepsilon(t, u_o), \theta_\varepsilon) \to 0\ as\ t \to \infty,$$

and θ_ε is the minimal among all closed sets with this property.

Under additional assumptions on parameters we also prove that

$$dist(\theta_\varepsilon, \theta) \to 0\ as\ \varepsilon \to 0,$$

where θ is the uniform attractor of the impulsive problem with $\varepsilon = 0$.

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FAST POLYNOMIAL RECONSTRUCTION

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Deep learning dominates modern ML, but low-degree polynomial approximations remain valuable in scenarios that require: computational efficiency; interpretability; modeling based on physically based data (known polynomial structure). This paper proposes a hybrid approach that combines:

1. Polynomial interpolation for 3rd-degree bivariate polynomials[1],[2]:

$$f(x,y) = \sum_{i+j\leq 3} a_{ij} x^i y^j.$$

2. Integration of the constructed approximations into neural networks as interpretable layers.

Application Examples. 1. Polynomial Layers in Neural Networks: A hybrid architecture where the first layer reconstructs a polynomial from the input (x,y), followed by a traditional MLP[3]. 2. Hyperparameter Optimization: If the loss function is locally polynomial, approximating it using a few points enables a quick search for the minimum.[4]

Conclusion: Hybrid approaches combine the advantages of numerical

methods with neural networks. If the dependency is known to be close to cubic, the reconstruction accuracy is higher. Avoiding Overfitting: Third-degree polynomials are less prone to overfitting than higher-degree ones. For further research, it is worth comparing Chebyshev Networks with classical polynomial regressions on real-world data.

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CLUSTERING DYNAMIC CITATION TRAJECTORIES FOR EVALUATING RESEARCH RELEVANCE

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Evaluating the relevance of scientific research is critical for informed decision-making, particularly in environments characterized by uncertainty. Traditional metrics like the Citation Index (CI) and h-index often fall short in reflecting the topicality of recent contributions, as they aggregate impact over extended periods without distinguishing between past and current influence. This limitation motivates a time-sensitive, clustering-based approach to research evaluation.

We model each researcher or topic as a time series of dynamic citation metrics, with the core indicator defined at the publication-year level [1]. Instead of relying on a single static score, we apply unsupervised clustering to these trajectories to reveal typical regimes of influence. The study remains method-agnostic: similarity can be captured via time-aware distances tolerant to shifts and stretching, or via compact window-based representations of local trend and variability, followed by standard clustering algorithms. Internal validity and robustness to parameter and

window choices are assessed.

By focusing on clustered temporal patterns rather than cumulative totals, the approach highlights evolving trends in research influence and supports better allocation of funding and collaboration strategies under uncertainty, offering a clearer view of current momentum than static measures.

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IMPROVING THE RESILIENCE OF DIGITAL SIGNAL SYNCHRONIZATION SYSTEMS IN WAR CONDITIONS

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Intelligent electrical grids and mobile communication systems, which, in war conditions, cannot function properly without resilient synchronization of digital signals, require the use of additional methods and technical means.

To generate synchronization signals with increased resilience in conditions of unintentional failures and targeted attacks, it is recommended to use special Global Navigation Satellite System (GNSS) radio receivers, as well as multi-channel monitoring technologies, hybrid synthesis of synchronization signals, and additional methods of synchronization signal sources’ reservation.

Based on the example of mobile communications, the principles of synchronization network redundancy, optimization of GM and BC clock locations, and the number of retransmissions in MPLS loops are

presented. The main stability parameters that must be measured and monitored in case of emergency situations have been identified. An intelligent computer-integrated system has been developed that provides continuous multi-channel monitoring of synchronization signal parameters in real time, digital processing of monitoring data, and generation of synchronization signals with increased stability and resilience.

The proposed organizational and technical measures to improve resilience ensure the formation of synchronization signals with improved quality and can be recommended for use at the facilities in various sectors of the country's economy both in war conditions and in the post-war period.

A LAW OF THE ITERATED LOGARITHM FOR ITERATED RANDOM WALKS, WITH APPLICATION TO RANDOM RECURSIVE TREES

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Consider a general branching process (a.k.a. Crump-Mode-Jagers process) generated by an increasing random walk whose increments have finite second moment. Let $Y_k(t)$ be the number of individuals in generation $k \in \mathbb{N}$ born in the time interval $[0,t]$. We discuss a law of the iterated logarithm for $Y_k(t)$ with fixed k, as $t \to +\infty$. As a consequence, we present a law of the iterated logarithm for the number of vertices at a fixed level k in a random recursive tree, as the number of vertices goes to ∞.

The talk is based on the joint article [1] with Zakhar Kabluchko (Munster).

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DIGITAL TWINS IN SEMI-NATURALISTIC MODELING SYSTEMS FOR CREATING COMPLEX PRODUCTS

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The current stage of development of information technologies requires new approaches to the design, testing and improvement of complex technical products. One such approach is the use of digital twins in combination with semi-naturalistic modeling. A digital twin is a virtual representation of a physical object, process or system that is supported by updated real-time sensor data, software, and analytical models. This approach allows for the implementation of the object's functioning in real time, analysis of its condition, prediction of failures and optimization of management processes. A digital twin in semi-naturalistic modeling synchronizes the digital model with bench tests in real time; increases the accuracy of predicting the behavior of complex systems due to iterative calibration of the digital twin based on sensor data. Its use will reduce the cost of full-scale testing; identify errors at the early stages of design; provide an opportunity for safe testing in extreme conditions; increase the adaptability of the product to changes in the external environment. When developing complex products, such as tracked machinery, digital twins allow simulating the dynamics of movement on complex terrain, checking the operation of the suspension, power units, and electronics. In the future, digital twins will become the main components of cyber-physical systems in industry, defense, automotive and energy. The introduction of IoT technologies will significantly expand the functionality of semi-naturalistic modeling, ensure continuous data acquisition from physical objects, adaptation of models in real time and automatic updating of the behavioral characteristics of the system. Promising areas include: creating self-adaptive digital twins capable of independently adjusting the model for quick access to real systems; integration with cloud technologies, which makes it possible to expand capabilities and ensure scalability; implementation of artificial intelligence for forecasting, autonomous diagnostics and testing optimization; use of virtual environments for visualization, which will facilitate personnel training and quick decision-making in critical situations. Thus, the integration of digital twins into semi-naturalistic modeling systems allows combining the accuracy of modeling with the reality of physical testing, while ensuring high efficiency of engineering solutions in modern conditions.

DECISION SUPPORT AND MAKING SYSTEM ON UNCERTAINT AND FUZZY CONTRACTS

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Decision support in conditions of uncertainty and unclear definition of the subject and subjects of processing require a special approach to an automated software decision support system (DSS). A description of some approaches to contracts - subjects involved in the automation of the DSS management process is given.

VCG-contracts (principal–agent problem)

In complex dynamic systems, participants interact in a multi-period or continuous-time manner, making decisions based on current information and future expectations.

The principal–agent problem can be described as a conflict of interest between one party (the principal), who delegates authority to another party (the agent) to carry out certain actions. This conflict may arise due to information asymmetry (the agent is more knowledgeable in their area of activity), opportunism (the agent acts in their own interest), and other factors.

One modern approach to this problem involves the use of VCG-contracts [1], named after the VCG-auction, used in digital marketing [2] and potential applicable in the renewable energy sources market [3].

The core idea of VCG-contracts is that the agent's compensation depends not only on their own actions but also on the proposals of other principals. This structure eliminates the incentive to manipulate resource allocation in the agent's favor. The compensation can be represented as [1]:

$$t_{m,n}^{VCG}(s_n, b) = \max_{x \in S_n} \left\{ \sum_{m' \in M \setminus m} b_n^{m'}(x) \right\} - \max_{y \in S_n} \left\{ \sum_{m' \in M} b_n^{m'}(y) \right\} + b_n^m(S_n).$$

The VCG-contract mechanism incentivizes agents in complex dynamic systems, offering applications in management, finance (particularly in digital transactions and fintech solutions) and energy branch.

Ambigous contracts

Since it's impossible to account for all circumstances, contracts are often vague, incomplete, or ambiguous. Ambiguous contract models are

of particular interest. Unlike classical models, here the principal is not required to commit to a single specific contract. Instead, they offer the agent a set of possible contracts $\tau = \boldsymbol{t}^1, ..., \boldsymbol{t}^k$ and commit to selecting one of them after the agent chooses an action. The agent adopts a cautious strategy, aiming to maximize their minimum possible payoff. This worst-case approach gives the principal additional leverage for benefit through uncertainty.

Thus, the chosen action $i^\star$ under an ambiguous contract τ is defined as:

$$i^*(\tau) \in \arg\max_{i \in [n]} \min_{t \in \tau} (T_i(\boldsymbol{t}) - c_i), \text{ where } T_i(t) = \sum\nolimits_{j \in [m]} q_{ij} t_j.$$

Here, the expected payoff $T_i(\boldsymbol{t})$ is determined as the sum of the probabilities q_{ij} of outcomes and the payments for them t_j, while c_i represents the agent's cost for performing the chosen action [4].

An optimal ambiguous contract must be consistent: all contracts must yield the same benefit to the principal for the chosen action, eliminating the agent's uncertainty. Such a contract can be represented as a set of simple contracts that pay for only one outcome. A polynomial algorithm was proposed to compute the optimal solution, and the potential advantage over classical contracts was estimated. However, if the agent uses mixed strategies, the benefit of ambiguity disappears, limiting the effectiveness of ambiguous contracts.

In general, despite the substantive difference of the described DSS subjects, it can be emphasized that the use of significantly different approaches to vague or incompletely defined contracts in a purely business process is only an element of the implementation of some applied decision-making tasks, organically included in the automated DSS system.

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STABILITY KERNEL WITH RESPECT TO PERTURBATIONS OF ALL INPUT DATA OF A PARTIALLY INTEGER VECTOR OPTIMIZATION PROBLEM

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$Z(F^u, X^u)$: $\max\{F(x) \mid x \in X\}$, where $X = G \cap (R^{n_1} \times Z^{n_2}) \subset R^n$, $X \neq \varnothing$, $n_1 + n_2 = n$, $F(x) = (f_1(x), ..., f_\ell(x))$, $f_i(x) = \langle x, D_i x \rangle + \langle c_i, x \rangle$, $c_i \in R^n$, $D_i \in R^{n \times n}$, $i \in N_\ell = \{1, ..., \ell\}$, $G = \{x \in R^n \mid Ax \leq b\}$, $A \in R^{m \times n}$, $b \in R^m$, $x = (y, z) \in R^{n_1} \times Z^{n_2}$. $S\ell(F, X)$, $Sm(F, X)$, $P(F, X)$ are sets of optimal solutions of the problem $Z(F^u, X^u)$. For a set of input data $u \in U$ and $\forall \delta > 0$ define a set of perturbed input data $O_\delta(u) = \{u(\delta) = (u_1(\delta), u_2(\delta)) \in U \mid \|u(\delta) - u\| < \delta\}$. Problem $Z(F^{u_1(\delta)}, X^{u_2(\delta)})$ is $\max\left\{F^{u_1(\delta)}(x) \mid x \in X^{u_2(\delta)}\right\}$. Let the set X be closed.

Definition. The **stability kernel** of a set $\mathrm{M}(F, X)$ is the subset of all its solutions that belong stably to it, i.e. $\mathrm{Ker}(\mathrm{M}(F, X)) =$

$$= \left\{ x \in \mathrm{M}(F, X) \middle| \forall \varepsilon > 0 \; \exists \delta > 0 \; \forall u(\delta) \in O_\delta(u) \; (\mathrm{M}(F^{u_1(\delta)}, X^{u_2(\delta)}) \cap O_\varepsilon(x) \neq \varnothing) \right\}.$$

Statement 1. Any point that stably belongs to the Slater set $S\ell(F, X)$ or the Pareto set $P(F, X)$ also belongs to the Smale set $Sm(F, X)$: $\mathrm{Ker}(Sl(F, X)) \subset Sm(F, X)$, $\mathrm{Ker}(P(F, X)) \subset Sm(F, X)$.

Statement 2. If the problem $Z_{S\ell}(F, X)$ ($Z_P(F, X)$) is stable to input data perturbations, then the equality $S\ell(F, X) = Sm(F, X)$ ($P(F, X) = Sm(F, X)$) holds.

Based on the concepts of stable membership of solutions [1] in separate subsets of the feasible set and the kernel of stability of the set of optimal solutions, results were obtained that allowed us to expand the known class of vector optimization problems that are stable with respect to perturbations of input data.

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DEEP GALERKIN METHOD FOR MULTIVALUED NONLINEAR PROCESS MODELING WITH HYBRID NEURAL NETWORKS

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The optimization problem for the approximation of solutions to classes of reaction-diffusion equations with multivalued interaction functions is not suitable for numerical computation. However, with deep learning algorithms, in particular the Deep Galerkin Method, it is possible to transform an infinite-dimensional stochastic optimization problem into a finite-dimensional one by incorporating artificial neural networks (ANNs) [1]. To achieve this, we denote by $N^{\theta,l_0}_{\varphi_1,\varphi_2,...,\varphi_L}:\mathbb{R}^{l_0}\to\mathbb{R}^{l_L}$ the realization function of the fully-connected feedforward artificial neural network associated with θ, which has $L+1$ layers with dimensions $(l_0,l_1,...,l_L)$ and activation functions $(\varphi_1,\varphi_2,...,\varphi_L)$ for all $x\in\mathbb{R}^{l_0}$ and for each $d,m,n\in\mathbb{N}, s\in\mathbb{N}_0:=\mathbb{N}\cup\{0\}, \theta=(\theta_1,\theta_2,...,\theta_d)\in\mathbb{R}^d$, where $d\geq s+mn+m,$ the affine function $A^{\theta}_{s,m,n}:\mathbb{R}^n\to\mathbb{R}^m$ associated with (θ,s), is defined as follows:

$$A^{\theta}_{s,m,n}(x)=\begin{pmatrix}\theta_{s+1} & \theta_{s+2} & \cdots\theta_{s+n}\\ \theta_{s+n+1} & \theta_{s+n+2} & \cdots\theta_{s+2n}\\ \vdots & \vdots & \ddots\ \vdots\\ \theta_{s+(m-1)n+1} & \theta_{s+(m-1)n+2} & \cdots\theta_{s+mn}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{pmatrix}+\begin{pmatrix}\theta_{s+mn+1}\\ \theta_{s+mn+2}\\ \vdots\\ \theta_{s+mn+m}\end{pmatrix}$$

for all $x=(x_1,x_2,...,x_n)\in\mathbb{R}^n$ [2,3].

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INTEGRATING AGENTIC ARTIFICIAL INTELLIGENCE INTO HUMAN RESOURCE MANAGEMENT: FROM PROCESS AUTOMATION TO ETHICAL, ROBUST AUTONOMY UNDER UNCERTAINTY

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HR teams face mounting pressure to eliminate inefficiencies, fix data inaccuracies, and reduce time spent on repetitive administrative tasks that delay talent decisions and drive up costs. Traditional workflow engines address parts of the problem, but often suffer from slow speeds, inconsistent results, unchecked human bias, and limited support for continuous training or compliance.

This paper presents agentic AI—self-directed, cooperative learning agents—as a unified solution that turns eight chronic pain points into levers of savings and strategic value. We introduce a modular architecture of specialized agents (e.g., Process-Optimizer, Bias-Monitor, Learning-Coach, Cost-Controller) that share a knowledge graph, resolve trade-offs via multi-objective reinforcement learning, and escalate edge cases for human input.

At the core are LLM-powered reasoning engines that process résumés, policies, and surveys across languages, converting unstructured data into actionable knowledge. Transformer encoders detect evolving skill taxonomies, while probabilistic meta-learners quantify uncertainty and trigger human review when confidence is low. Generative AI creates personalized onboarding and training content, while causal-inference agents simulate the impact of policy changes on diversity, retention, and cost—turning raw data into transparent, forward-looking insights.

We close with governance principles — sandbox testing, counterfactual audits, and human-in-the-loop controls — to ensure accountability. By directly tackling inefficiencies, bias, and training gaps, agentic AI repositions HR from a cost center to a data-driven strategic partner.

ARTIFICIAL INTELLIGENCE FOR FORECASTING YIELD AND QUALITY OF CORN CROPS IN UKRAINE

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The industrial production and export of grain in Ukraine is an important component of the country's functioning. This is especially true for its Forest-Steppe Zone. The process of such production is being automated as much as possible using modern technical means, computer technologies, and software environments. The basis for such automation is reliable forecasting of yield and quality of the crop, as the finished goods market (including the global market) is highly competitive and expensive service support for grain production needs to be planned, including, first and foremost, its appropriate financial support.

The purpose of the work: to create and test a mathematical model for predicting grain crop yield and harvest quality in Ukraine.

Crop yields and crop quality are significantly affected by natural factors, the number of pests and their harmfulness, and cultivation technologies. In the process of growing grain, plants use a variety of chemicals - insecticides - to minimize the impact of pests. These services are expensive and often have a negative impact on the ecological state of the environment. It is not uncommon to see a situation where the norms of chemical use have been exceeded, including in previous years, which probably affected both the yield of these crops and the quality of the crop itself.

Proposes to add insecticide effectiveness to the factors of influence in addition to natural factors, processing technologies, and the number of pests to assess the systemic impact on the yield and quality of grain crops in the Forest-Steppe Zone of Ukraine.

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FUZZY LOGIC SYSTEM FOR MEDICAL DIAGNOSIS UNDER UNCERTAINTY CONDITIONS

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We propose a fuzzy inference system for medical diagnosis under uncertain symptom descriptions. The method addresses differential diagnosis between flu, cold, COVID-19, and allergic reactions using linguistic symptom evaluation.

Let $S = \{temperature\ T, pain\ P, cough\ C\}$ be input symptoms and $D = \{flu, cold, COVID-19, allergy\}$ be diagnoses. For each symptom $s \in S$, fuzzy membership functions $\mu(s)$ map linguistic variables to $[0,1]$.

Temperature membership functions:

- $\mu\_low(T) = \{1, T \le 36.5; (37 - T)/0.5, 36.5 < T < 37; 0, T \ge 37\}$,
- $\mu\_high(T) = \{0, T \le 37; (T - 37)/1, 37 < T < 38; 1, T \ge 38\}$.

Fuzzy rules: IF temperature is μ_i AND pain is μ_j AND cough is μ_k THEN diagnosis is D_n.

Inference algorithm: For rule i, firing strength $\alpha\_i = min(\mu\_T(...), \mu\_P(...), \mu\_C(...))$. Final confidence via defuzzification:

$$\textbf{Confidence} = (\Sigma \boldsymbol{\alpha}\_\mathbf{i} \cdot \mathbf{w}\_\mathbf{i} \cdot \mathbf{y}\_\mathbf{i}) / (\Sigma \boldsymbol{\alpha}\_\mathbf{i} \cdot \mathbf{w}\_\mathbf{i}),$$

where w_i is rule weight, y_i is diagnosis value.

The approach introduces **adaptive membership functions** adjusting to patient demographics [1], unlike static systems [2]. **Temporal dynamics** incorporate symptom evolution, improving accuracy for evolving conditions.

System validation: The proposed algorithm was implemented and tested on simulated scenarios. The uncertainty quantification mechanism recommends additional diagnostic procedures when confidence falls below predetermined thresholds.

Example: Patient with T=37.5°C, moderate pain, light cough:

$\mu\_high(37.5) = 0.5$; Rule activation $\alpha = min(0.5, 1.0) = 0.5 \rightarrow 65\%$ flu probability.

Conclusion. The fuzzy system effectively handles diagnostic uncertainty, providing transparent decision-making for medical practitioners. Adaptive functions and temporal analysis offer robust solutions for respiratory disease diagnosis under uncertainty.

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AN ARCHITECTURAL APPROACH TO SCALABLE BOT DETECTION IN SOCIAL MEDIA

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The paper outlines one architectural approach to building modern information systems capable of handling large volumes of data. Emphasis is placed on balancing performance, scalability, and reliability through the integration of contemporary technologies and design principles.

As data volumes and system demands continue to rise, revisiting and refining software architecture has become essential. This area of research remains critical for those involved in designing robust and high-performing systems.

This study proposes an architectural concept tailored to current demands for performance, scalability, and security. It focuses on reviewing existing solutions, evaluating their strengths and limitations, and formulating a more resource-efficient and high-speed alternative.

The methodology combined architectural analysis, system modeling, performance benchmarking, and comparative evaluation. The proposed architecture was implemented using contemporary technologies, including microservices, containerization, and distributed computing.

System efficiency was improved through optimized processing and workload distribution. The container-based design enables scalable and stable operation, with benchmarks demonstrating latency reduction and improved utilization of computational resources.

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ABOUT OPTIMIZATION PROBLEMS FOR A SINGLE-CHANNEL SYSTEM WITH QUEUE AND REPEATED CALLS

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Currently, the investigation of queueing systems with retrial calls represents one of the most relevant and dynamically evolving areas in modern queueing theory. These systems have been extensively studied and systematically presented in the monograph [1]. The growing interest in such systems stems from their wide applicability as mathematical models for real-world processes. In particular, retrial queueing systems are used in the design and analysis of computer networks, in the study of information processing systems, and in the modeling of modern telecommunication systems. Furthermore, they are applied to describe operational processes such as the landing of aircraft, among other complex service scenarios [2].

The study focuses on a queueing system with retrial calls. In such systems, when all service devices are occupied, incoming requests are placed into a waiting queue of limited capacity. If the queue is also full, these requests are directed to a virtual orbit, from which they attempt to access the service again after a random time interval. Such requests become sources of retrial calls and generate a secondary input flow. One of the important applications of queueing models with retrials is that they allow for the formulation and solution of optimization problems related to the control of system parameters [3]. The main difficulty in analyzing such systems lies in their infinite state space. This work adopts the approach of approximating the infinite system with a finite one. However, this raises the question of how accurately the characteristics of the original infinite system can be approximated using the stationary distribution of the truncated model. Solving this problem is facilitated by deriving explicit scalar-type formulas for the stationary probabilities of the truncated system.

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THE LEARNING SCENARIO ON DEVELOPING AI AND MACHINE LEARNING MODELS FOR BUSINESS OPTIMIZATION

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The implementation of this work has been possible thanks to the support and within the framework of the ERASMUS+ project: The transferable training model - the best choice for training IT business leaders (2023-2-PL01-KA220-HED-000179445) [1]

The scenario immerses learners in the full lifecycle of AI projects—from problem definition and data preparation to model training, evaluation, deployment, and ethical governance [2]. Using real-world datasets, such as the Telco Customer Churn dataset, participants build predictive models to address critical challenges like customer attrition, inventory optimization, and dynamic pricing. Tasks emphasize both technical proficiency (e.g., data preprocessing, algorithm selection, evaluation metrics) and strategic leadership (e.g., decision-making, stakeholder communication, and change management). Learners also engage in simulations of crisis management, tackling issues such as algorithmic bias and data privacy risks, ensuring alignment with regulatory and ethical standards.

The scenario targets multiple professional roles—data scientists, IT leaders, business executives, and compliance officers—highlighting the need for interdisciplinary collaboration. Learning outcomes encompass three dimensions: (1) technical skills in ML development and deployment, (2) leadership competencies in decision-making and change management, and (3) professional dispositions emphasizing accountability, adaptability, and ethical responsibility.

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THE METHODOLOGY OF THE DESIGN OF TRANSLEADER LEARNING SCENARIOS

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The dynamic development of digital technologies and the increasing complexity of IT business leadership demand innovative educational approaches that integrate technical, strategic, and ethical dimensions of learning. Within the Erasmus+ project "The transferable training model – the best choice for training IT business leaders" (2023-2-PL01-KA220-HED-000179445), the TransLeader learning scenarios methodology has been developed as a systematic framework to design, implement, and evaluate competency-based training modules.

The methodology is grounded in a standardized scenario template that ensures consistency, adaptability, and relevance across multiple institutional and industrial contexts. Its design philosophy emphasizes the balance between academic rigor and real-world applicability, enabling learners to acquire both technical expertise (e.g., Artificial Intelligence, IoT, data management) and leadership competencies (e.g., decision-making, change management, ethical governance).

Key elements of the methodology include: Background and motivation, linking industry needs with educational practices; Problem and use case definition, situating learning in authentic, data-driven business challenges; Scenario-based tasks, encouraging active learning, collaboration, and problem-solving; Practical application and reflection, ensuring knowledge transfer from theory to practice; Evaluation and feedback mechanisms, supporting continuous improvement and adaptability.

The benefits of adopting a standardized template lie in its transferability, allowing partner institutions to replicate and customize learning scenarios while preserving methodological coherence. This approach fosters a common European framework for IT business leadership education, bridging academic knowledge with industrial expectations.

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THE TRANSFERABLE TRAINING MODEL - THE BEST CHOICE FOR TRAINING IT BUSINESS LEADERS

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In the era of digital transformation and Industry 5.0, IT business leaders are expected to combine advanced technical knowledge with strategic, ethical, and managerial competencies. Traditional academic programs often struggle to adapt to the rapidly evolving demands of global markets, leaving a gap between the skills taught in universities and those required in practice. To address this challenge, the Erasmus+ project "The transferable training model – the best choice for training IT business leaders" (2023-2-PL01-KA220-HED-000179445) proposes an innovative and competency-driven educational framework.

The transferable training model is designed to provide flexibility, scalability, and cross-sectoral applicability, ensuring that learners acquire both hard technical skills (e.g., Artificial Intelligence, Machine Learning, IoT, data management, software engineering, cybersecurity) and soft leadership competencies (e.g., strategic decision-making, ethical governance, communication, and change management). By adopting scenario-based learning, hands-on tasks, and problem-driven assignments, the model reflects real-world challenges faced by IT executives and project leaders.

A key feature of the model is its transferability: the curriculum, learning scenarios, and assessment methods can be adapted to different educational institutions, industries, and cultural contexts. This ensures that the training approach remains relevant not only to IT but also to other business and technology-driven sectors. Furthermore, the integration of adaptive evaluation methods, expert insights, and reflective practices supports continuous learner development and long-term employability.

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FUZZY SET OF FUZZY INEQUALITIES IN LINEAR PROGRAMMING PROBLEMS

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A linear programming problem (LPP) with fuzzy inequalities has the form $\max\{c^T x : A_i x \lesssim b_i,\ i \in M;\ x \in X\}$, where $x \in \mathbb{R}^n$ is a vector of non-negative variables; $c \in \mathbb{R}^n$ is a vector of coefficients of the objective function; $X = \{x \in \mathbb{R}^n : Bx \le d,\ x \ge 0\}$ is a crisp set of alternatives; $d \in \mathbb{R}^s$ is a vector of right-hand sides of crisp constraints, and $B \in \mathbb{R}^{s \times n}$ is a matrix of left-hand sides of constraints; $M = \{1, 2, ..., m\}$ is a set of indices of constraints, A_i is the i$^{\text{th}}$ row of the matrix $A \in \mathbb{R}^{m \times n}$, and b_i is the i$^{\text{th}}$ element of the vector b, $i \in M$.

In some practical situations, the decision maker does not crisply say which constraints should be taken into account when making a decision, and only knows their degrees of membership $\mu_{\tilde{M}}(i)$, $i \in M$ to the fuzzy set (FS) $\tilde{M} = \{(i, \mu_{\tilde{M}}(i)) : i \in M\}$ of constraints. Then, the fuzzy LPP

$$\max\{c^T x : A_i x \lesssim b_i,\ (i, \mu_{\tilde{M}}(i)) \in \tilde{M},\ x \in X\} \qquad (1)$$

with the fuzzy set $\tilde{M}$ of fuzzy constraints. The meaning of the notation $A_i x \lesssim b_i,\ (i, \mu_{\tilde{M}}(i)) \in \tilde{M}$ is that the fuzzy inequalities $A_i x \lesssim b_i,\ i \in M$ are present in the system of constraints of problem (1) with the degrees of membership $\mu_{\tilde{M}}(i)$, $i \in M$, respectively.

This report discusses a LPP with a FS of constraints in the form of fuzzy inequalities. It is shown that a solution to this problem forms a type-2 FS. We prove that a solution type-2 FS can be decomposed by secondary membership grades into a finite collection of FSs. Each of these is a solution to the corresponding fuzzy LPP with a crisp set of constraints. This set is the corresponding cut of the initial FS of constraints. The illustrative example is also included for clarity.

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ON THE PROBLEM OF DECISION MAKING UNDER UNCERTAINTIES FOR MEDICAL AND BIOLOGICAL PROCESSES

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This report investigates decision-making problems under uncertainty in medical and biological processes, with particular emphasis on modeling dynamic systems such as immunological responses, pharmacokinetics, tumor growth, and excitable neuronal dynamics. Approaches based on population dynamics, generalized Gompertzian models, compartmental pharmacokinetics, and immune protection systems are discussed. Special attention is given to the stability of solutions, conditions for parameter identification in Hilbert spaces, and methods for handling incomplete or noisy data.

The study explores qualitative and quantitative methods of analysis, including the Adomian polynomial decomposition technique and data-driven approaches such as decision tree induction, which enable classification of complex biological trajectories (e.g., disease progression types, neuronal excitability forms). By integrating system-theoretic models with data mining and uncertainty-handling techniques, the research highlights new opportunities for building robust decision support systems (DSS) in healthcare.

The findings demonstrate that combining mathematical rigor with computational intelligence improves prediction accuracy, supports optimal therapy planning (e.g., in oncology and pharmacology), and enhances the interpretability of medical decisions. Importantly, the approach provides a framework for managing uncertainties inherent in biological processes while remaining clinically applicable.

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PREDICTIVE ESTIMATES OF SOLUTIONS OF DIFFERENTIAL EQUATIONS UNDER UNCERTAINTY

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Let the vector function $\varphi(t) = (\varphi_1(t), ..., \varphi_n(t))^T$ be a solution to the differential equation

$$\frac{d\varphi}{dt} = Af_1(\varphi, t) + f_2(\varphi, t), \quad \varphi(t_0) = \varphi_0,$$

where A is unknown matrix with elements $a_{ij}, i, j = \overline{1,n}$, functions $f_k(\varphi, t)$ are continuous in the variable $t \in (t_0, T)$ and satisfy the Lipschitz condition in φ.

To determine the elements of the matrix A we specify the exact values $\varphi(t_k)$ and derivatives $\frac{d\varphi(t_k)}{dt}$ with errors $\eta_k, k = \overline{1,N}$. The problem of determining solutions $\varphi(t^*)$, where $t^* > t_N$, is to find pessimistic and optimistic values $\varphi_k(t^*), k = \overline{1,n}$. under certain constraints on unknown values η_k and elements of matrix A.

In this paper, we investigate the cases when the matrix A and the vector $\eta = (\eta_1, ..., \eta_n)^T$ belong to known domains, as well as when η_k are fuzzy interval numbers. We show that under certain conditions such a problem reduces to solving linear programming problems.

The obtained results are illustrated in the case where the functions $\varphi(t)$ are solutions of equations describing the dynamics of diseases.

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LINEAR GUARANTEED ESTIMATES OF LINEAR OPERATORS FROM SOLUTIONS OF LINEARIZED EQUATIONS OF A WEAKLY COMPRESSIVE FLUID IN CONDITIONS OF UNDETERMINATION

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Let a given vector from the Hilbert space H be on the form $y = C\varphi + v$, where $\varphi(x)$ is the solution of the equation

$$-\gamma\Delta\varphi - \beta\, grad\, div\,\varphi = f(x),\ x \in \Omega,$$
$$\varphi\big|_{\Gamma} = 0,$$

where Ω is a bounded region in the space R^n with piecewise-smooth boundary Γ, γ and β are known positive numbers, $f(x)$ is an unknown vector-function from $L_2(\Omega)$, C is a linear bounded operator, acting from the space $L_2(\Omega)$ to H, v is an unknown vector from H. Let also A be a linear bounded operator acting from the space $L_2(\Omega)$ to the finite-dimensional space R^m.

If a bounded set G is given to which a function $f(x)$ and a vector v in the space belong $L_2(\Omega)\times H$, then the linear guaranteed estimate of the vector $A\varphi$ is called the vector $\hat{U}y + \hat{c} = A\varphi$, which is from the condition

$$\min_{U,b}\sup_{G}\left\|A\varphi - Uy - b\right\|_{R^m} = \sup_{G}\left\|A\varphi - A\varphi\right\|,$$

where U is linear bounded operator acting from the space H in R^m, b is vector from R^m.

It is shown that under certain conditions the problem of finding the estimate $A\varphi$ reduces to the problem of minimizing the functional $\Phi(\psi_1,...\psi_m,u_1,...,u_m)$, where ψ_k are the solutions of the problem

$$-\gamma\Delta\psi_k - \beta\, grad\, div\,\psi_k = l_k - Cu_k,$$
$$\psi_k\big|_{\Gamma} = 0,$$

where $A\varphi = ((l_1,\varphi),...(l_m,\varphi))^T$, $Uy = ((u_1,y),...,(u_m,y))^T$.

The convergence of guaranteed linear estimates is investigated under $\beta \to \infty$.

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GUARANTEED ESTIMATES AND THEIR ERRORS OF SOLUTIONS OF INDEFINITE LINEAR OPERATOR EQUATIONS IN SPACE OF RECTANGULAR MATRICES

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Let $H_{p,q}$ denote the space of rectangular matrices of dimension $p \times q$ with a scalar product: $\langle X_1, X_2 \rangle \triangleq spX_1X_2^T$, $X_1, X_2 \in H_{p,q}$. Let A be a linear operator that acts from the space $H_{m,n}$ of matrices to the space H_{m_1,n_1}. Consider the equation:

$$AX = B, \tag{1}$$

where $X \in H_{m,n}, B \in H_{m_1,n_1}$. It is known that this equation has a solution if the matrix $B \perp \ker A^*$, where $\ker A^* \triangleq \{B_1 : A^*B_1 = 0\}$ (here A^* is the operator conjugate to the operator A, and the matrix $B_1 \in H_{m_1,n_1}$).

We assume that the kernel of the operator A is not empty (the condition $\{X : AX = 0\} \neq \varnothing$ is fulfilled) and the dimension $r, r \geq 1$. At the same time, the assumed arbitrary solution of equation (1) can be written in the form:

$$X = X_0 + D, \tag{2}$$

where X_0 is some solution of equation (1) and D is an arbitrary matrix from the kernel of the operator A.

Let $\Psi_i, i \in \overline{1,r}$ denote the basis matrices of the set $\ker A$. Then we can write the matrix D by formula (2) in the form:

$$D = \sum_{i=1}^{r} f_i \Psi_i \triangleq \rho f, \tag{3}$$

where $f_i, i \in \overline{1,r}$ are some real numbers, $f = (f_1, f_2, ..., f_r)^T$. It is clear that if the matrix X_0, basis $\Psi_i, i \in \overline{1,r}$ and vector f are known, then formulas (2), (3) determine the solution of equation (1). Note also that we

can choose the matrix X_0 as the minimum norm matrix that satisfies the equation $AX_0 = B$. We find such a matrix from the system of equations:

$$\begin{cases} AX = B, \\ \langle X, \Psi_i \rangle = 0, i \in \overline{1,r}. \end{cases}$$

Further, we assume that the solution X is unknown, which means that the real numbers $f_i, i \in \overline{1,r}$ are unknown. To determine the estimates of the numbers $f_i, i \in \overline{1,r}$ we set the matrices $Y_k, k \in \overline{1,N}$: $Y_k = C_k X + V_k, k \in \overline{1,N}$, where $Y_k \in H_{p(k),q(k)}, k \in \overline{1,N}$; $C_k, k \in \overline{1,N}$ are known linear operators that act from the space $H_{m,n}$ to the spaces $H_{p(k),q(k)}, k \in \overline{1,N}$; $V_k, k \in \overline{1,N}$ are unknown matrices from the spaces $H_{p(k),q(k)}, k \in \overline{1,N}$.

We present the known restrictions on the matrix $V_1, V_2, ..., V_N$ in the form $(V_1, V_2, ..., V_N) \in G$, where $G \subset \prod_{k=1}^{N} H_{p(k),q(k)}$. Therefore, *the purpose of this work is to find estimates of the vector* Lf, where L is a linear operator that acts from the space R^r to the space $R^s, s \le r$.

Definition 1. Vector Lf is called a linear estimate of vector Lf, if it has the form: $Lf = \sum_{k=1}^{N} U_k Y_k + c$, where $U_k, k \in \overline{1,N}$ are operators that act from the spaces $H_{p(k),q(k)}, k \in \overline{1,N}$ to the space R^s, the vector c belongs to the space R^s.

Definition 2. Vector Lf is called a guaranteed linear estimate of vector Lf, if it has the form: $Lf = \sum_{k=1}^{N} U_k Y_k + \hat{c}$, where $\left(\hat{U}_k, \hat{c}\right), k \in \overline{1,N}$ are found from the condition: $\left(U_k, \hat{c}\right), k \in \overline{1,N} \in Arg \min_{(U,c)} \sigma_g (U,c)$, where $U = (U_1, U_2, ..., U_N)$ and the error of estimation of the vector Lf is given by the formula: $\sigma_g (U,c) \triangleq \sup_f \sup_G \left\| Lf - \sum_{k=1}^{N} U_k Y_k - c \right\|$.

Definition 3. The value $\sigma_g \left(U, \hat{c}\right)$ is called the guaranteed error of the linear guaranteed estimate.

Definition 4. The vector $\varphi \in R^s$ is called a guaranteed posterior estimate

if it is from the condition: $\varphi \in Arg \inf_{\varphi} \sigma_a(\varphi)$, $\sigma_a(\varphi) \triangleq \sup_{\varphi_1 \in G_y} \|\varphi - \varphi_1\|$, where G_y is the posterior set of vectors Lf with known data (4) and has the form: $G_y = \left\{ Lf : f \in G_y^{(1)} \right\}$, $G_y^{(1)} = \left\{ f : \left(\tilde{Y}_1 - C_1 \rho f, ..., \tilde{Y}_N - C_N \rho f \right) \in G \right\}$, where $\tilde{Y}_k = Y_k - C_k X_0, k \in \overline{1, N}$.

Definition 5. The value $\sigma_a(\varphi)$ is called the guaranteed posterior error of the estimate of $\hat{\varphi}$.

Further on we assume that the set G has the form:

$$G = \left\{ (V_1, ..., V_N) : \|V_k\| \le \gamma_k, k \in \overline{1, N} \right\}, \tag{4}$$

where $\|V_k\| = \left(sp V_k V_k^T \right)^{1/2}$, and $\gamma_k, k \in \overline{1, N}$ are known additional numbers.

Theorem 1. There is an equality:

$$\sigma_g(U, c) = \begin{cases} \sigma_g^{(1)}(U, c), \text{ при } W_N \ne \varnothing, \\ \infty, \text{ при } W_N = \varnothing, \end{cases}$$

where function $\sigma_g^{(1)}(U, c)$ is determined by the formula $\sigma_g^{(1)}(U, c) = \max_{\|l\|=1} \left\{ \sum_{k=1}^N \|U_k^* l\| \gamma_k + \left| (c, l) + (\psi(X_0), l) \right| \right\}$, here $\psi(X_0) = \sum_{k=1}^N U_k C_k X_0$, $W_N = \left\{ (U_1, ..., U_N) : \sum_{k=1}^N \rho^* C_k^* U_k^* = L^* \right\}$.

Theorem 2. At the value of the parameter $N = 1$, the condition is fulfilled $\min_{U_1, c} \sigma(U_1, c) = \lambda_{\max}^{1/2}(Q) \gamma_1$, for the estimation error, where $Q = L \left(\rho^* C_1^* C_1 \rho \right)^+ L^*$.

Theorem 3. When $N \ge 1$, the inequality $\inf_{U, c} \sigma_g(U, c) \ge \lambda_{\max}^{1/2}(Q_1)$ are holds, where the operator $Q_1 = L \left(\sum_{k=1}^N \gamma_k^{-2} \rho^* C_k^* C_k \rho \right)^+ L^*$, and the equality $\min_{U, c} \max_{f, G} \left\| Lf - Lf \right\| = \lambda_{\max}^{1/2}(Q_1)$ are holds for the error.

Theorem 4. Let the set G given by formula (4). Then the set

$$G_y^{(1)} = \left\{ f : \left(\tilde{Y}_1 - C_1 \rho f, ..., \tilde{Y}_N - C_N \rho f \right) \in G \right\} =$$
$$= \left\{ f : \left(\left\| \tilde{Y}_1 - C_1 \rho f \right\| \le \gamma_1, ..., \left\| \tilde{Y}_N - C_N \rho f \right\| \le \gamma_N \right) \right\}$$

can be presented in the form: $G_y^{(1)} = \bigcap_{k=1}^{N} \bar{G}_y^{(k)}$, where

$$\bar{G}_y^{(k)} = \left\{ f : \left(R_k \left(f - \hat{f}_k \right), f - \hat{f}_k \right) \le \gamma_k^2 - J_k \left(\hat{f}_k \right) \right\},\ k \in \overline{1,N},$$

$$R_k = \rho^* C_k^* C_k \rho,\ \hat{f}_k = \left(\rho^* C_k^* C_k \rho \right)^+ \rho^* C_k^* \tilde{Y}_k,\ J_k(f) = \left\| \tilde{Y}_k - C_k \rho f \right\|^2.$$

Theorem 5. The set $G_y^{(1)}$ is a convex closed set and if the matrix $\rho^* \sum_{k=1}^{N} \gamma_k^{-2} C_k^* C_k \rho \triangleq S_N$ is not degenerate, then the set $G_y^{(1)}$ is also bounded.

Definition 6. Let inclusions $G^- \subseteq G \subseteq G^+$ be fulfilled. Then the posterior sets: $G_y^{\mp} = \left\{ f : \left(\tilde{Y}_1 - C_1 \rho f, ..., \tilde{Y}_N - C_N \rho f \right) \in G^{\mp} \right\}$ are called the lower and upper posterior sets, respectively, and the corresponding guaranteed a posteriori estimates and guaranteed a posteriori errors are lower and upper estimates and errors.

For the case when G is given by formula (4), let's put:

$$G^- = \left\{ (V_1, ..., V_N) : \sum_{k=1}^{N} \|V_k\|^2 \gamma_k^{-2} \le 1 \right\},$$

$$G^+ = \left\{ (V_1, ..., V_N) : \sum_{k=1}^{N} \|V_k\|^2 \gamma_k^{-2} \le N \right\}.$$

Theorem 6. Let condition: $\det S_N \ne 0$ be fulfilled. Then the lower and upper guaranteed posterior estimates coincide with each other and have an inequality: $\sigma_y^- \le \min_f \sigma_y(f) \le \sigma_y^+$, where

$$\sigma_y(f) \triangleq \max_{f_1 \in G} \|f - f_1\|,\ \hat{f} = S_N^{-1} \sum_{k=1}^{N} \rho^* C_k^* \tilde{Y}_k, 1 - \Phi_N \left(\hat{f} \right) \ge 0,$$

$$\sigma_y^- = \lambda_{\max}^{1/2} \left(S_N^{-1} \right) \left(1 - \Phi_N \left(\hat{f} \right) \right)^{1/2} = \lambda_{\min}^{-1/2} \left(S_N \right) \left(1 - \Phi_N \left(\hat{f} \right) \right)^{1/2},$$

$$\sigma_y^+ = \lambda_{\max}^{1/2} \left(S_N^{-1} \right) \left(N - \Phi_N \left(\hat{f} \right) \right)^{1/2}.$$

Further, we find estimates of the scalar product of vectors (l, f) with data (2). Let the matrices $V_k, k \in \overline{1,N}$ belong to the set G of the form (4).

Theorem 7. The guaranteed linear estimate of the scalar product of vectors (l, f) has the form: $\left(l, f \right) = \sum_{k=1}^{N} \left\langle \hat{U}_k, Y_k \right\rangle + \hat{c}$, where

$$\hat{c} = -\sum_{k=1}^{N} \left\langle \hat{U}_k, C_k X_0 \right\rangle,\ \left(\hat{U}_k, k \in \overline{1,N} \right) \in Arg \min_{(U_1, ..., U_N) \in W_N} \Phi_1 (U_1, ..., U_N),$$

$$\Phi_1 (U_1, ..., U_N) \triangleq \sum_{k=1}^{N} \|U_k\| \gamma_k,\ W_N = \left\{ (U_1, ..., U_N) : \sum_{k=1}^{N} \rho^* C_k^* U_k = l \right\}.$$

At the same time, the error satisfies the condition

$$\min_{U,c}\max_{f,G}\left|(l,f)-\sum_{k=1}^{N}\langle U_k,Y_k\rangle-c\right|=\Phi_1\left(\hat{U}_1,...,\hat{U}_N\right).$$

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STATISTICAL MODELING OF THE NUMBER OF TRAFFIC ACCIDENTS

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Traffic accidents remain a persistent public safety concern with complex underlying causes. This study focuses on the application of statistical modeling techniques – specifically, generalized linear models (GLMs) and generalized additive models (GAMs) – to analyze factors influencing accident frequency. We utilize data from the Fire Rescue Corps of the Czech Republic and the Czech Hydrometeorological Institute, incorporating both qualitative variables (day of the week, month, and region) and quantitative meteorological variables (maximum temperature, wind speed, air pressure, precipitation, snow cover, and sunshine duration). The number of accidents is modeled as a function of these variables. We compare the performance and properties of GLMs and GAMs, and based on the most suitable model, we provide accident frequency predictions under varying conditions.

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UNCERTAINTIES IN DYNAMIC LOGICS

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Uncertainties of various types are inherent in many applied areas. The main types of such uncertainties are ontological, epistemic, and aleatory [1]. Analysis of these types permit to identify such abstract types as partiality and nondeterminism that appear quite naturally in computer science and artificial intelligence. Here we investigate these two uncertainties in program logics, namely, in dynamic logics. Such logics are usually considered as multimodal logics with partial and nondeterministic programs. We add partiality to data and predicates. Such extensions of a logic require additional operations and compositions, in particular, variable unassignment predicate and predicate complement. These extensions complicate logic construction and investigation because nonmonotonicity of functions and compositions appear.

Semantic basis of the considered logics are special many-sorted program algebras. They have four carriers: a class of partial data, a class of partial predicates, a class of nondeterministic ordinary functions, and a class of nondeterministic program functions. Data are treated as partial hierarchical nominative data [2]. Compositions include conventional compositions of imperative programming languages and compositions of the Dijkstra's guarded command language extended with superpositions into predicates and functions.

We study semantic properties of the obtained algebras, define dynamic logics with multimodal compositions based on such algebras, and construct sequent calculi for such logics. Problems of soundness and completeness for the obtained logics are considered, soundness theorems are proved.

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APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATION TO ESTIMATE THE PARAMETERS OF A TWO-REGIME THRESHOLD DISCRETE ORNSTEIN–UHLENBECK PROCESS

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A two-regime threshold Ornstein–Uhlenbeck process defined by a stochastic differential equation is considered:

$$dX_t = \{(\beta_{10} + \beta_{11}X_t)I(X_t \le r_1) + (\beta_{20} + \beta_{21}X_t)I(r_2 > X_t \ge r_1)$$

$$+(\beta_{30} + \beta_{31}X_t)I(X_t \ge r_2)\}dt$$

$$+\{\sigma_1 I(X_t \le r_1) + \sigma_2 I(r_2 > X_t \ge r_1) + \sigma_3 I(X_t > r_2)\}dW_t,$$

W_t is the Wiener process. The model was discretized using the Euler scheme. The process describes state transitions across three ranges, each characterized by its specific set of drift and diffusion parameters $(\beta_{i0}, \beta_{i1}, \sigma_i^2)$. Parameter estimation is performed via a log-likelihood function taking into account regime transition indicators, implemented through an iterative system of equations. Convergence of the method is proved by numerically constructing the Jacobian matrix and computing the spectral radius at each iteration.

The proposed approach is implemented in RStudio and tested on datasets exhibiting various dynamic patterns, including growth, decline, and plateau phases.

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DEMONSTRATING THE POTENTIAL OF FRACTAL DIMENSION IN DEEP LEARNING FOR SKIN CANCER CLASSIFICATION

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Skin cancer is a life-threatening disease affecting millions worldwide. Timely diagnosis is essential for improving patient outcomes, underscoring the need for reliable automated diagnostic tools. This study evaluates the impact of incorporating Fractal Dimension (FD) into a skin lesion classification model using a Vision Transformer (ViT) for feature extraction. Including FD consistently improved key performance metrics. SHAP (SHapley Additive exPlanations) analysis confirmed that FD contributed to the model's decisions, and McNemar's test demonstrated the statistical significance of the improvements. These findings support the potential of Fractal Dimension as a valuable feature for enhancing deep learning-based skin cancer classification.

AUTOMATED SENTIMENT AND PROPAGANDA DETECTION IN NEWS COVERAGE OF THE RUSSIA-UKRAINE WAR: A MACHINE LEARNING APPROACH

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Over 500 multilingual news articles (2023-2024) on the Russia-Ukraine war were analyzed for sentiment and propaganda. Texts were translated, vectorized (TF-IDF, trigrams), and classified using linear SVM.

Sentiment covered five aspects: military, humanitarian, diplomacy, politics, and economy. Annotation combined GPT-based scoring with manual review.

The model achieved 79.7% F1 in propaganda detection and 80%+ accuracy in sentiment classification. Regional media showed divergent framing, notably pro-Ukrainian in the US and critical in Belarus.

The pipeline proves effective for scalable, multilingual analysis of conflict-related media.

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STATISTICAL ANALYSIS OF THE EXPLOITATION HYPOTHESIS OF ALLIANCE MEMBER STATES

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This paper examines the relationship between military expenditures and economic power in selected NATO countries. The aim of this paper is to test the exploitatiton hypothesis using the method of correlation analysis. The analysis considers key decisions made at the 2014 Wales Summit and the 2023 Vilnius Summit, where member states committed to allocating at least 2% of GDP to defense. The data used for testing was obtained from the international databases SIPRI (military spending as a share of GDP) and The World Bank database (GDP). The results of the correlation analysis confirm that there was unequal sharing of the military burden, supporting the hypothesis of exploitation within the Alliance in the analyzed period from 1999 to 2023.

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SOLVING THE STRUCTURALLY FUZZY PROBLEM OF JOBS PRIORITIZATION UNDER LIMITED RESOURCES

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This paper presents an advanced approach to scheduling and prioritizing maintenance and repair tasks under resource constraints and uncertainty, with a focus on structural fuzziness. Traditional job-shop scheduling models in uncertain environments typically rely on expert-based estimates and fuzzy set theory—most often using Triangular Fuzzy Numbers (TFNs) to represent task durations. Some recent studies introduce Type-2 fuzzy numbers to better capture epistemic uncertainty. However, few have explored uncertainty in the structure of the scheduling problem itself.

This research addresses that gap by extending a previously proposed model based on the Fuzzy Flexible Job-Shop Scheduling Problem (FfJSSP) into a Structurally Fuzzy Multi-Objective Job-Shop Scheduling Problem (SF-MOJSSP). In addition to fuzzy task durations and Type-2 fuzzy parameters for one of the objective functions, the model incorporates structural fuzziness—uncertainty in the problem's configuration, such as constraint presence depending on the expert estimation of required capability to execute a particular job. This allows the model to better reflect real-world complexity in maintenance planning, where both data and process logic may be ambiguous.

A structural fuzziness methodology, as introduced in [1], is applied, and a representative scheduling problem is solved to illustrate feasibility and value. The inclusion of structural fuzziness enables a richer and more adaptive decision-support framework for planning under deep uncertainty.

Future research will focus on developing scalable solution techniques, including fuzzy genetic algorithms and swarm intelligence methods, for efficiently handling large-scale, structurally fuzzy scheduling problems. Additional directions include integrating linguistic fuzzy models and real-time decision-support mechanisms for industrial and infrastructure applications.

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COORDINATED AUTONOMOUS NAVIGATION OF AN UNMANNED AERIAL VEHICLES SWARM

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The complex for coordinated autonomous navigation of a unmanned aerial vehicles (UAVs) swarm is based on the integration of two key components: a ground-based center and an onboard AI platform with cognitive services. The ground-based part of the complex serves as a strategic think tank where neural networks used on board drones are trained, tested, and validated. Combat missions are modeled in a simulation environment using a digital twin. The digital twin performs the following functions: modeling tens and hundreds of agents in combat or disaster conditions; training decision-making models for real deployment; validating the effectiveness. The Gazebo, ROS, and PX4 environments are used to simulate and validate coordinated navigation algorithms. Each agent has a virtual twin that goes through a series of missions in a digital environment to test, calibrate, and optimize the algorithms without using a physical drone. The on-board part provides full autonomy for each drone, including the ability to navigate independently, make real-time decisions, avoid obstacles, stabilize flight, and coordinate with other swarm members without the need for constant communication with the ground center.

The system implements mechanisms of swarm self-organization, adaptive task distribution, loss compensation, and adaptation to dynamic environmental changes. For this purpose, hybrid artificial intelligence algorithms are used - from reinforcement learning to stigmergy, SLAM and graph neural networks. In the context of electronic warfare, communication between agents is maintained through secure mesh networks, which allows maintaining synchronized swarm behavior even in the event of partial destruction of the infrastructure. The entire functioning is organized in a decentralized manner, which ensures the survivability and flexibility of the system. Here is an example of a mission implementation.

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IMPLEMENTATION OF DIGITAL TWINS CONCEPT IN SWARM NAVIGATION TASKS FOR DRONES

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The concept of digital twins (DT) is proposed for autonomous swarm navigation tasks of unmanned aerial vehicles controlled by artificial intelligence. It is shown that the effective functioning of a drone's swarm in the absence of stable communication with the ground center is possible thanks to the DT functions distribution of the ground station and AI-agents at the onboard. Mathematical models of the ground DT provide strategic modeling, training, mission simulation and results analysis, while onboard AI-agents focus on local adaptation, diagnostics, environment reconstruction, and cognitive control of drone behavior. The functional division of DT mathematical models into models of swarm, environment, mission, telemetry and AI-agent of drone is proposed.

A good definition of the digital twin notion is given in [1], which distinguishes it from a digital model and a digital shadow by the presence of information flows between the physical system and its digital copy. If there are no automated data flows between the physical system and the digital copy, such a digital object is considered a digital model of the physical system. A digital shadow is understood as a digital object (model) with an automated data flow from the physical system to the digital object. The main task of a digital shadow is to automatically monitor certain changes in the physical system in order to represent its properties. In the case where there is an automated data flow from the physical system to the digital object and vice versa, such a digital object is considered a DT, since changes in the digital object affect the physical system. The interaction examples between a physical system and a digital copy provided above make it possible to distinguish between the categories of digital model, digital shadow, and DT at a qualitative level.

The results were partially supported by the NRFU, Grant no. 2025.06/0022

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ECONOMIC IMPACTS OF MIGRATION IN GEORGIA: A STATISTICAL DECISION THEORY APPROACH TO UNCERTAINTY

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This study applies the theory of statistical decisions and inference to rigorously analyze the economic impacts of migration in Georgia under significant uncertainty. Georgia's migration dynamics are characterized by heterogeneous and volatile flows of emigration and immigration, driven by complex socio-political, demographic, and economic determinants.

To address these challenges, this research constructs a decision-theoretic framework integrating stochastic modeling and statistical inference to quantify uncertainty in migration data, labor market outcomes, and public finance indicators.

Employing Bayesian inference, hypothesis testing, and risk assessment methodologies, the study simulates multiple economic scenarios reflecting variations in migration intensity, workforce participation, and government expenditure.

The approach enables optimization of policy decisions by balancing expected economic benefits against potential risks induced by migration volatility. Empirical results, based on official migration and economic statistics, highlight the efficacy of statistical decision theory in guiding adaptive policy strategies that enhance economic resilience.

This work advances the methodological integration of statistical decision science and migration economics, offering valuable insights for evidence-based policymaking in Georgia's evolving socio-economic environment.

MULTI-AGENT LARGE MODEL FRAMEWORK FOR ADAPTIVE TRANSPORT LOGISTICS OPTIMIZATION

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This paper extends prior research on AI-driven optimization of transport logistics [1] by transitioning from task-specific supervised models to adaptive multi-agent systems powered by large pre-trained models. While previous work demonstrated the effectiveness of XGBoost for route optimization, it requires continuous retraining to adapt to evolving environments. This study explores the integration of intelligent agents built on top of large language models and large multimodal models.

We propose a multi-agent architecture where specialized agents handle demand forecasting, dynamic routing, anomaly detection, and cost–benefit analysis, combining LLM reasoning with domain-specific knowledge via fine-tuned prompts, retrieval-augmented generation, and structured data access. A shared orchestration layer enables collaboration without retraining. In tests using live traffic, historical delivery data, and cost parameters, our prototype outperformed or matched XGBoost-based optimization in route efficiency, while improving adaptability to disruptions.

To assess the effectiveness of the proposed multi-agent architecture, we conducted a comparative evaluation against established optimization methods such as XGBoost-based routing, genetic algorithms for vehicle routing, and reinforcement learning approaches [2]. The results demonstrate that our method achieves comparable or superior routing efficiency while offering greater adaptability to dynamic conditions. These findings align with recent research [3], further supporting the viability of large-model-based multi-agent systems as a next-generation paradigm. The study concludes with a discussion of scalability challenges, and potential for integration with autonomous delivery systems.

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ITERATIVE TWO-SIDED METHOD FOR SOLVING THE CAUCHY PROBLEM

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Considering the Cauchy problem

$$y'(x) = f(x, y(x)), \quad y(a) = y_a \tag{1}$$

on the interval $[a,b]$ we seek the solution in the form of an appropriate functional interval [1] whose bounding functions are piecewise-parabolic. To this end, at each step we reduce the initial problem (1) to a new initial-value problem

$$y'(x) = f(x, y(x)), \quad y(x_i) = y_i, \; i = 0, 1, 2, \cdots, n, \tag{2}$$

where $x_0 = a, x_n = b$, $a < x_1 < x_2 < x_3 < \cdots < x_{n-1} < b$.

It is noted that all values $y(x_i) = y_i$ are the exact values of the wanted solution $y(x)$ of the problem (1) at the points x_i. The arcs of the parabolas of the two-sided approximation thus constructed for the function $y(x)$

$$\overline{p}_{x_i}(x) = 0.5\,\overline{k}\,x^2 + (y'_{x_i} - \overline{k}\,x_i)\,x + 0.5\,\overline{k}\,x_i^2 - (y'_{x_i} - \overline{k}\,x_i)\,x + y_{x_i},$$

$$\underline{p}_{x_i}(x) = 0.5\,\underline{k}\,x^2 + (y'_{x_i} - \underline{k}\,x_i)\,x + 0.5\,\underline{k}\,x_i^2 - (y'_{x_i} - \underline{k}\,x_i)\,x_i + y_{x_i},$$

where $y'_{x_i} = y'(x_i)$, are constructed on the basis of the consistency principle for a functional interval of a function and its derivative [2], ensuring successively on each subinterval $x \in [x_i, x_{i+1}]$ the fulfillment of the double inequality $\underline{k} \le y''(x) \le \overline{k}$, where $\underline{k}, \overline{k}$ are certain constants.

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DIGITAL-TWINS-DRIVEN SEMI-PHYSICAL SIMULATION FOR TESTING OF INDUSTRIAL SOFTWARE

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The industrial software of a smart manufacturing system (SMS) is generally used for industrial control, production, and storing and processing production-related data. To adapt to the urgent needs for digital transformation and upgrading of manufacturing enterprises, the production system urgently needs to be transformed. The traditional workshop production process is not transparent, resulting in production process control being seriously restricted by empirical knowledge. To quickly verify the reliability and adaptability of industrial software in reconfiguring SMS for new or upgraded product orders, a semi-physical simulation method for testing and evaluation of industrial software is proposed based on digital-twins-driven technology. By establishing a semi-physical simulation model of SMS, the reliability and robustness of the software system are verified by running industrial software in various manufacturing scenarios. In recent years, with the development of multidisciplinary modeling and simulation technology, the concept and application of digital twin has become a research hotspot. In this report, the key technologies to carry out semi-physical simulation testing and evaluation of industrial software for SMSs are expounded in detail, including how to synchronize cyber and physical systems, how to conduct semi-physical accelerated simulation testing, and how to identify defects quickly in industrial software used in actual production environments. The robustness of the industrial software for SMS is further verified by conducting fault injection testing, so as to provide implications for fault prognostics or fault-prevention research. Provides a highly simulated and safe operating environment for industrial software with a digital twin system, avoiding possible damage to real production equipment during performance testing and evaluation. The indexes that reflect the performance of industrial software include transaction processing-time, communication interaction time, maximum concurrency. When conducting accelerated tests of industrial software, the reliability assessment verification of industrial software is achieved by accelerating the corresponding speed of digital twin devices and improving the execution time of virtual events.

FUNDAMENTALS OF THE NOOSPHERE THEORY OF INNOVATION

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At the general theoretical level, the expediency of using V. I. Vernadsky's doctrine of the noosphere for the formation of a new paradigm of innovations is substantiated. Innovations are considered not only as a technological or economic factor, but also as a driving force for the transition of the biosphere to the noosphere.

Modern innovation theory focuses primarily on economic aspects and practically ignores the evolutionary, cultural, and spiritual dimensions of this process. Meanwhile, it is innovation that is the main factor in the formation of the noosphere, the sphere of reason in which man becomes a geological and planetary force.

The main problem of man, which arose in the twentieth century and became aggravated at the beginning of the twenty-first century, is to overcome the split between oneself and the outside world by finding a new harmony with nature. At the heart of achieving this new harmony is a different human way of thinking, which will make it possible to change the economic model and, as a result, change the paradigm of interaction with nature to one that is more in line with the ideas of the noosphere.

Such an approach makes it possible to overcome the technocratic limitations of modern theories of innovation and to revise the modern theory of innovation in the context of new spiritual, cultural and ecological principles of harmony of man with the surrounding world. The conceptual foundations of the noospheric rethinking of innovations are formulated: innovative activity is considered in the context of the evolutionary development of the noosphere, which includes a new anthropological model of man, a new quality and an increased role of the collective mind.

The decisive importance of the spiritual and cultural dimension of innovations is emphasized, which opens the way to the integration of spiritual and cultural potential in the process of creating and evaluating innovations. The "introduction" or "introduction" of the noosphere perspective into spirituality will allow for a balancing of tangible and non-material innovations, while promoting ecological, social, and cultural sustainability. This makes it possible to form sustainable, morally motivated and environmentally balanced innovation systems.

BIFURCATIONS OF VORONOI DIAGRAMS

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We consider Voronoi diagrams in the plane defined over a finite set of sites $S=\{s_1,...,s_n\}$, where all sites are distinct. The set of all such diagrams is denoted by $VOR(\mathbb{R}^2,n)$. This set is an open subset of $\mathbb{R}^{2n}$ and inherits the standard Euclidean topology. The *body* of a Voronoi diagram $Vor(S)$, denoted $Vor^b(S)$, is obtained from $Vor(S)$ by removing all unbounded edges.

Two Voronoi diagrams are said to be *isomorphic* if there exists a homeomorphism of the plane that maps one diagram onto the other. A diagram $Vor(S)$ is said to be *structurally stable* if there exists a neighborhood of it in $VOR(\mathbb{R}^2,n)$ consisting entirely of diagrams isomorphic to $Vor(S)$. The set of structurally stable diagrams, denoted by $VOR^0(\mathbb{R}^2,n)$, is open (and has codimension 0) in $VOR(\mathbb{R}^2,n)$.

Diagrams of codimension 1 are structurally unstable and belong to the set:

$$VOR^1(\mathbb{R}^2,n)=VOR(\mathbb{R}^2,n)\setminus VOR^0(\mathbb{R}^2,n).$$

A *typical one-parameter bifurcation* of a diagram $Vor(S)\in VOR^1(\mathbb{R}^2,n)$ is a smooth path in $VOR(\mathbb{R}^2,n)$ that passes through $Vor(S)$ and lies entirely in $VOR(\mathbb{R}^2,n)$ except at that point.

Theorem 1. A diagram $Vor(S)\in VOR^0(\mathbb{R}^2,n)$ if and only if the set S satisfies the following two conditions:

1. The points of S do not lie in a single half-plane whose boundary contains three or more points from S;
2. The points of S do not lie outside a circle whose boundary contains four or more points from S.

Such diagrams have all vertices of degree 3 and do not contain any pair of co-directed unbounded edges. The topological structure of such diagrams is fully determined by their body $Vor^b(S)$.

Theorem 2. A diagram $Vor(S)\in VOR^1(\mathbb{R}^2,n)$ if and only if it is of one of the following two types:

1. The diagram has exactly one vertex of degree 4;
2. The diagram has exactly one pair of co-directed unbounded edges.

In both cases, all other vertices have degree 3, and all other unbounded edges are not co-directed.

If $Vor(S) \in VOR^0(\mathbb{R}^2, 4)$, then $Vor^b(S)$ is either a segment I or a triangle Δ (*I-diagram* or Δ*-diagram*). Diagrams in $VOR^1(\mathbb{R}^2, 4)$ come in two types:

(a) The *X-diagram*, which has a single vertex and four unbounded edges;

(b) The *П-diagram*, which has two vertices and a pair of co-directed unbounded edges (it is isomorphic to the I **-diagram**).

Any bifurcation of an X -diagram results in I **-diagrams**, while a bifurcation of a II **-diagram** can result in either an I **-diagram** or a Δ **-diagram**.

If $Vor(S) \in VOR^0(\mathbb{R}^2, 5)$, then the body $Vor^b(S)$ has one of the following types:

(a) A quadrilateral;

(b) A quadrilateral and a triangle sharing a common edge:

c) Two segments with a common endpoint;

(d) A triangle and a segment sharing a common vertex.

Diagrams in $VOR^1(\mathbb{R}^2, 5)$ fall into the following types:

(a) Diagrams isomorphic to those in $VOR^0(\mathbb{R}^2, 5)$, but with 4 or 5 unbounded edges, two of which are co-directed;

(b) Diagrams for which $Vor^b(S)$ is a segment, with one endpoint of degree 3 and the other of degree 4;

(c) Diagrams for which $Vor^b(S)$ is a triangle with one vertex of degree 4.

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ODOO ERP INTEGRATION WITH GOOGLE WORKSPACE SERVICES

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The use of ERP systems is a modern condition for the successful functioning of business processes in an enterprise. Popular ERP systems include ERP from Odoo, which provides business process management (sales, purchasing, logistics, human resources management, accounting, etc.). Google Chat is widely used as a corporate messenger. This raises the question of integrating the ERP system with the messenger, since employees are often forced to switch between systems for task management, communication, etc. Time costs and duplication of information create the need for a combination of tools, which will also contribute to increasing the integration of the business environment and automation of business processes.

This study proposes building integration modules for Odoo ERP with Google Chat and Google Task services to improve internal communication and project management: a basic functional module for working with APIs and event processing, two application modules for integration with the module of projects and internal discussions in Odoo.

The architecture of the integration modules is divided into two parts: the part that concerns the built-in entities of the Odoo system; and the part that concerns the technical side of the integration. The first part contains models that are interconnected by standard Odoo relationships. The second part contains service models that provide data exchange, connection status control, and access rights management. Integration modules perform the following actions: create a new Google Chat space from the Odoo interface; connect existing chats to relevant projects or channels; perform two-way synchronization of messages, attachments, reactions and users; automatically create and update personal Google Tasks when Odoo tasks change; support all types of chats in Google Chat; provide access to the functionality to users from corporate Google Workspace and external participants with personal accounts.

Implementing an approach based on the use of Google Workspace Events and Google Cloud Pub/Sub allowed us to achieve full interactivity and minimize event losses in the implementation of the integration.

As a result of the work, a reliable and functional solution for use in a business environment was proposed.

ON APPROACHES TO CONTROL IN RETRIAL QUEUEING SYSTEMS

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In this paper, we consider a two-dimensional continuous-time Markov process $Q(t)=(Q_0(t),Q_1(t))$, $t \geq 0$ in the phase space $S=\{0,1,...,c\}\times Z_+$, where $Q_0(t)$ denotes the number of busy servers at time t, $Q_1(t)$ denotes the number of repeated-attempt sources (**customers in orbit**) at time t. The process describes the service dynamics of a multi-channel queueing system with a single retrial attempt to initiate service, where the rate of input flow depends on the number of repeated-attempt sources. This system is characterised by the following parameters: λ_j is the input flow rate, μ is the service rate and ν is the rate of repeated attempt.

For this system, we consider the situation when the input flow rate is controlled according to threshold and hysteresis strategies. In the threshold strategy, if at some moment of time the total number of customers in the system's orbit does not exceed the value H, the system operates in the first mode with the input flow rate h_1. If the number of customers exceeds H, the system switches to a second mode with an input flow rate h_2. Alternatively, a hysteresis strategy is built using two thresholds H_1 and H_2, $H_1 \leq H_2$. If the number of customers in the orbit does not exceed H_1, the system operates in the first mode with input flow rate h_1. If it exceeds H_2, the system operates in the second mode with the input flow rate h_2. When the number of customers is between H_1 and H_2, the system remains in the mode it occupied in the previous moment.

According to these strategies, multi-criteria optimization problems for the effective operation of the system are formulated and solved, with the quality functionals expressed in terms of the system's stationary probabilities [1].

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SUPPRESING DISTURBANCES IN THE COGNITIVE MAP IMPULSE PROCESS OF DEMOGRAPHIC SECURITY OF UKRAINE USING LINEAR MATRIX INEQUALITIES

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In this report a cognitive map (CM) of demographic security of Ukraine is developed, based on which the dynamic model of impulse processes of this CM in the form of a system of difference equations (Roberts' equations) is introduced. Concepts (nodes) of the CM are: state support of families with children, average salary of a worker, export volume, import volume, consumer price index, population in Ukraine, real GDP of Ukraine, birth rate, inflation rate, migration out of Ukraine, unemployment rate, death rate, military events. The first four of them can be varied by decision-maker (actions of Ukrainian authorities) and thus can be considered as controls. The last five nodes can be considered as disturbances which negatively affect the demographic security (catastrophic depopulation). Here we set a goal to ensure stability and quick stabilization of the system on some appropriate levels of the CM nodes under the effect of these disturbances. To achieve this we apply the methodology developed in [1] which is based on applying the idea of invariant ellipsoid minimization to the CM impulse process (after decomposition into stabilizable subsystem and subsystem of disturbances) using linear matrix inequalities. Under the suggested control actions, the simulation demonstrates that we are able to stabilize the process of depopulation. Main control action the authorities should apply are: export increase and import decrease, average salary increase and support of the families with children.

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AUGMENTED REALITY IN EDUCATION: TRANSFORMING TRADITIONAL LEARNING ENVIRONMENTS

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This paper explores how Augmented Reality (AR) technology is transforming traditional education by making learning more interactive and engaging. AR allows digital information to be overlaid onto the real world, creating new opportunities for students to understand complex topics through hands-on experiences. AR technology helps students visualize difficult concepts by bringing abstract ideas to life. For example, students can examine 3D models of the human heart in biology class, walk through ancient Rome in history lessons, observe and talk about different plants, animals in Science or Language classes. This approach makes learning more interesting and helps students remember information better because they can see and interact with what they are studying. Research shows that students using AR technology demonstrate higher levels of engagement and motivation compared to traditional teaching methods. AR accommodates different learning styles by providing visual, auditory, and hands-on experiences that appeal to various types of learners. Teachers benefit from AR as well, gaining access to new tools for presenting lessons and assessing student progress. The technology provides immediate feedback and allows educators to track student understanding in real-time, enabling more personalized instruction.

However, implementing AR in schools faces several challenges. Schools need adequate technology infrastructure, sufficient funding for equipment, and trained teachers who understand how to use AR effectively. Technical issues and the learning curve for new technology can also create barriers to successful implementation. Despite these challenges, AR shows great promise for transforming education. The research suggests that successful AR integration requires careful planning, teacher training, and institutional support. The study concludes that AR technology has the potential to make education more engaging, interactive, and effective for students of all ages. While challenges exist, the benefits of AR in education make it a valuable tool for modern learning environments.

METHODS OF USING DIGITAL TWINS WITH THE INTERNET OF THINGS IN HIL-MODELLING

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Modern technological trends make the synergy of digital twins, the Internet of Things (IoT) and HIL modeling (Hardware-in-the-Loop) not just promising, but necessary for the development of both the industry. This approach allows you to create virtual-physical testing circuits, where real equipment interacts with a dynamic digital model updated in real time due to IoT sensors. A digital twin is a “live” digital copy of an object or system that receives data from connected IoT devices. In production, these are vibration, temperature, load sensors, etc. Thanks to the IoT, the data flow in the digital twin is continuous, which makes it possible to model and optimize in real operating conditions. HIL modeling complements this bundle, allowing you to test control systems on real equipment interacting with a virtual environment. For example, an automobile company can test an engine's electronic control unit by connecting it to a digital twin of the engine. IoT sensors provide real-time feedback, recording deviations, breakdowns or loads.

Use methods include equipment condition monitoring and predictive maintenance (predicting failures before they occur), optimization of production lines, and resource management in production. In addition, a safe environment is created for training and testing scenarios that would be too risky in reality.

The value of this technology is in the transition from a reactive to a proactive approach. The organization does not wait for equipment failure, but models the system’s behavior in advance. This reduces costs, reduces downtime, increases safety and provides flexibility in decision-making.

Thus, the combination of digital twins, IoT and HIL modeling forms the foundation for a new industrial, where virtual simulation becomes an integral part of real processes. This not only accelerates innovation, but also takes the management of complex systems to a whole new level. Using digital twins in conjunction with IoT in HIL simulation is the key to safe, accurate and cost-effective real-time testing of complex systems.

STATISTICAL EVALUATION OF ADOLESCENTS' PHYSICAL FITNESS IN THE CONTEXT OF ADMISSION TO A MILITARY UNIVERSITY

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The University of Defence, as the only military university in the Czech Republic, admits several hundred new students each year into full-time military study programs. These are typically young individuals either immediately after completing secondary education or within a few years of graduation an age at which the human body is generally at its peak physical performance.

This group is highly diverse, including both active athletes and individuals who do not engage in regular physical activity. It also encompasses a wide range of career preferences from applicants interested in serving in special forces to those aiming for office-based roles or specialist positions in communication and information technologies. Given the demanding nature of military service, physical readiness is one of the key requirements for students. The University of Defence therefore includes its own entrance examinations, which also assess physical fitness. The aim of this article is to map the current level of physical fitness among applicants in relation to the established criteria for each discipline and, based on the analysis, to propose recommendations for adjusting the admission process as well as for systematically improving the physical preparedness of students at the University of Defence.

As part of the admission process, data were collected in 2022, 2023, and 2024 from the following disciplines: the 12-minute run, sit-ups, and among selected applicants pull-ups. These data were subsequently analyzed and statistically evaluated. The results may serve as a basis for revising physical fitness standards and for modifying the entrance examination itself to better reflect the physical capabilities of today's younger generation and align with current recruitment needs of the Armed Forces of the Czech Republic.

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TRAFFIC EQUILIBRIUM MODELING WITH SELF-ADAPTIVE ALGORITHMS FOR VARIATIONAL INEQUALITIES

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The problem of traffic equilibrium modeling on the road network [1] is considered. Congestions within road networks continue to be relevant, and in situations where there is no systematic approach to its solution, losses due to the inefficiency of the infrastructure can be very large.

Three modified modern algorithms for solving variational inequalities applied to the user-optimal traffic equilibrium search problem. For that, mathematical model of traffic equilibrium corresponding to the first principle of Wardrop is provided, with formulation in form of variational inequality [1]. The algorithms are adaptive versions of extragradient algorithm [2], extrapolation from the past [3], and Malitsky-Tam algorithm [4]. We do not need to know Lipschitz constant of operator to select the step size. All three algorithms are extended with step size

modification rule which allows both step increasing and decreasing. Numerical experiments are conducted with the toy problem and with the well-known Sioux Falls transportation network.

The talk is based on the paper [5].

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A NEW RESULT ON THE DESCENT PROPERTY OF THE EXTRAGRADIENT ALGORITHM

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Variational inequalities provide a convenient general framework for formulating relevant problems in optimal control, operations research, and machine learning. Certain non-smooth convex optimization problems can be efficiently solved by reformulating them as saddle-point problems and applying algorithms for solving variational inequalities [1, 2].

The simplest method for solving variational inequalities is an analogue of the gradient descent method, which, in the case of saddle-point problems, is known as the gradient descent-ascent method (Arrow-Hurwicz method). However, for general Lipschitz continuous monotone operators, this scheme may fail to converge.

The most well-known modification of this scheme for variational inequalities is the extragradient method. Its iteration requires two evaluations of the operator and two metric projections onto the feasible set.

A new result on the descent property of the extragradient algorithm is proved, and for unconstrained problems, an estimate of the algorithm speed in terms of natural residuals is obtained under weaker conditions than known ones.

Finaly, we verify our theoretical results through numerical experiments.

The talk is based on the paper [3].

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THE ALGORITHM FOR SOLVING A MULTICRITERION OPTIMIZATION PROBLEM ON COMBINATORIAL CONFIGURATIONS OF POLYPERMUTATIONS

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A mathematical model of a multi-criteria problem $Z(F,X)$ on combinatorial configurations of polypermutations $P_{nk}(A)$ is proposed [1,2] $Z(F,X):\min\{F(x)\mid x\in X\}$ as the minimization of a vector criterion $F(x)=(f_1(x), f_2(x),..., f_\ell(x))$, on the set X with R^n, $X\neq\varnothing$, where each point $a\in P_{nk}(A)$ corresponds to a point $x\in X$: $X=\operatorname{vert}\Pi_{nk}(A)$, $\Pi_{nk}(A)=\operatorname{conv}P_{nk}(A)=\Pi(A)$, here is $\Pi(A)$ a polyhedron describing the convex hull of the combinatorial configuration of polypermutations.

The features of the problem are analyzed taking into account the properties of combinatorial configurations of their polyhedra and structural graphs. A new algorithm for the horizontal method of solving such problems for finding a set of Pareto-optimal solutions is developed, which is based on adaptive combinatorial search algorithms and heuristic methods. The operation of the proposed algorithm is illustrated by solving a test problem. Based on the conducted numerical experiments, the effectiveness of the algorithm on Euclidean configurations of polypermutations and permutations with repetitions is substantiated, taking into account modern approaches in combinatorial and vector optimization [1].

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MODELING DECISION-MAKING IN MILITARY AND CIVIL SPHERES USING ARTIFICIAL INTELLIGENCE TOOLS

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Contemporary challenges in both international and domestic security demand new approaches to understanding and modeling decision-making processes. Artificial intelligence (AI) offers valuable tools for forecasting scenarios in military and civilian domains, analyzing risks, and improving management efficiency. However, the growing use of AI also raises ethical and humanitarian concerns, particularly regarding transparency, accountability, and security. This study explores the potential benefits and risks of AI applications in military and civil contexts and suggests ways to ensure their responsible and ethically grounded deployment.

In a world marked by increasing global instability and uncertainty, intelligent technologies are playing an ever more central role in decision-making. AI algorithms can process vast amounts of data, identify hidden patterns, and anticipate the behavior of political, military, and social

systems, opening up new possibilities for both military and civilian management.

Military Dimension. In the military sphere, AI supports risk assessment, conflict scenario modeling, and evaluation of the strategic consequences of managerial decisions. It is particularly useful for simulating potential crises, predicting state responses to sanctions, and modeling cyber threats. At the same time, the development of autonomous weapons systems carries the risk of sparking a new arms race, posing additional challenges to global and regional security.

Civil Dimension. In civilian applications, AI is increasingly used to model socio-economic trends, forecast election outcomes, manage crises, and optimize public administration. A key challenge in this domain is the spread of deepfakes and manipulative technologies that can undermine public trust in democratic institutions. At the same time, AI can support diplomacy and peacekeeping, where accurate predictive models facilitate more effective dialogue between parties.

Humanitarian and Social Context. The integration of AI into decision-making processes raises important questions about ethics, transparency, and accountability. Establishing international standards for the responsible use of intelligent technologies, which balance innovation with humanitarian security, is critical. Equally important is enhancing public media literacy to reduce the impact of manipulative practices.

Conclusions. AI is a powerful tool for modeling and forecasting in both military and civilian spheres. Its effective use can enhance the quality of decision-making, but this requires robust ethical and legal frameworks. Future development in this area should prioritize the integration of technological innovation with the protection of humanitarian and social values.

MODIFIED DIFFERENTIAL EVOLUTION ALGORITHM FOR DYNAMIC SPATIALLY DISTRIBUTED MODELS WITH PARAMETERS

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The desire to avoid local optima and the accumulation of error in solving spatially distributed dynamic problems is one of the reasons for turning to differential evolution algorithms. Differential evolution

methods are a powerful set of global search techniques in multidimensional spaces, which makes them attractive for developing new efficient approaches for numerical consideration of system models represented by partial differential equations, as well as for studying the influence of parameters on the behavior of the system or even their determination.

The purpose of this study is to create a software system based on the constructed modified differential evolution algorithm for numerical modeling of dynamic natural processes described by partial differential equations. The modified differential evolution algorithm uses the foundations of the DE/rand/1/bin algorithm in the spatial component and the finite-difference scheme in time. As an example of a process for the study, the process of the spread of infectious diseases, described by a compartmental model of the SIRS type, was chosen. The selected model has a variable parameter that takes into account seasonal fluctuations in the rate of infection transmission ($\beta(t)$). In the model, for each point of a given space $x\epsilon\Omega, t\epsilon[0,T]$, the fractions of those susceptible to the disease $S(x,t)$, infected $I(x,t)$ and those who recovered, $R(x,t)$ sum to 1, and the model itself can be written as:

$$\frac{\partial S}{\partial t} = \mu(1-S) - \beta(t)SI + \delta R + \sum_i D_S \frac{\partial^2 S}{\partial x_i^2},\ x\epsilon\Omega, \Omega \subseteq \mathrm{R}^2, t\epsilon[0,T],$$
$$\frac{\partial I}{\partial t} = \beta(t)SI + (\mu+\gamma)I + \sum_i D_I \frac{\partial^2 I}{\partial x_i^2},$$
$$\frac{\partial R}{\partial t} = \gamma I - (\delta+\mu)R + \sum_i D_R \frac{\partial^2 R}{\partial x_i^2},$$

where μ, γ, δ – birth rates of individuals S, recovery of individuals I, loss of immunity by individuals R; D_S, D_I, D_R – diffusion rates, as well as initial and boundary conditions in the presence of flow through the boundary $\partial\Omega$.

The implementation of the constructed algorithm allowed us to search for the values of the model parameters; to investigate the short-term and long-term dynamics of the process under various changes in the parameters.

CONVERGENCE ANALYSIS OF THE TWO-STEP METHOD FOR SOLVING NONLINEAR EQUATIONS

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Let us consider a nonlinear equation [2]

$$H(x) = 0, \tag{1}$$

where H is a nonlinear operator, defined on a convex domain D of the Banach space X with values in the Banach space Y. This operator can be represented as the sum of two operators, i.e. $H = F + G$, and it is known that F is exactly Fréchet-differentiable operator and G is a continuous operator.

To find an approximate solution of the equation of this type, we use a differential-difference method that combines the derivative of the differentiable part with the divided difference of the continuous part of the operator. This approach is based on the two-step Newton method and a two-parametric secant-type method [1], and is formulated as follows:

$$\begin{aligned} x_{n+1} &= x_n - A_n^{-1} H(x_n), \\ y_{n+1} &= x_{n+1} - A_n^{-1} H(x_{n+1}), \quad n \geq 0, \end{aligned} \tag{2}$$

where $x_0, y_0 \in D$, $A_n = F'\left(\frac{x_n + y_n}{2}\right) + G(u_n; v_n)$, $u_n = x_n + a_n(y_n - x_n)$, $v_n = x_n + b_n(y_n - x_n)$, a_n and b_n are different real numbers. The particular case with $a_n = 0$ and $b_n = 1$ was previously studied in [2].

We provide a convergence analysis of method (2) under certain conditions on the derivatives and divided differences. We establish the radius of the convergence ball and obtain error estimates. Numerical experiments confirm the efficiency and practical applicability of the method (2), and also confirm the theoretical results.

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STABILITY AND CONVERGENCE OF DYNAMIC PROCESSES IN NEURODYNAMICS PROBLEMS

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Introduction. The field of knowledge in which neural networks are considered as nonlinear dynamical systems and the main focus is on the problems of stability and convergence of processes is called neurodynamics. Classical stability theory understands stability as the stability of a separately chosen trajectory (usually an equilibrium state). Therefore, models are considered that are described by dynamic systems (differential or difference) that have one equilibrium state. If we consider systems with several equilibrium states, then there is the concept of "coarse systems". Unfortunately, if for nonlinear systems on the plane there are necessary and sufficient conditions of "coarseness", then for systems of higher dimensions they are absent. Systems of equations that describe dynamic processes of neurodynamics have "very high dimensionality". Therefore, the direct use of traditional methods of studying stability here encounters certain difficulties. First of all, classical research methods encounter fundamental difficulties of high dimensionality. And in systems that describe dynamic processes of neurodynamics, the concept of "systems with weak nonlinearity" is often used, that is, systems with a separate linear part and nonlinearity contained in some rather small sector. Obviously, with a small value of "weak nonlinearity", the stability of the equilibrium state of a nonlinear system will be global and will follow from the asymptotic stability of the linear approximation system. Hence, in particular, convergence of learning processes will take place.

In the proposed report, conditions for stability and convergence of solutions of systems of differential equations describing neurodynamic processes are obtained. The apparatus of the second Lyapunov method is used in the study. The Hopfield network consists of a set of neurons that form a system with a set of inverse connections. The number of inverse connections is equal to the number of neurons. The output of each neuron is closed through a unit delay element to all other neurons of the network. And the neuron of this network does not have inverse connections with itself. In this case, the dynamics equation can be rewritten as

$$C_i \frac{dv_i(t)}{dt} = -\frac{v_i(t)}{R_i} + \sum_{j=1}^{n} \omega_{ij} \phi_j \left(v_j(t) \right) + I_j.$$

Let the Hopfield network be given. It will have dynamic equilibrium if its input is an image that is reflected back into itself. From the point of view of physics, the total energy of the system at this point will be minimal. The energy function (essentially the Lyapunov function) can be given as the sum of the kinetic and potential energies

$$V(y) = \frac{1}{2} \sum_{i,j=1}^{n} w_{ij} y_i y_j + \sum_{k=1}^{n} \delta_k y_k.$$

And, as follows from the theorems of the second Lyapunov method, the steady state of the network will be asymptotically stable if the Lyapunov function is positive definite and its total derivative along the solutions of the system is negative definite.

Main result. We consider a dynamic model of a neural network in space R^n. It is a system of nonlinear ordinary differential equations of the form

$$\dot{y}_1 = -a_{11} y_1 + b_{11} f_{11}(y_1) + b_{12} f_{12}(y_2) + ... + b_{1n} f_{1n}(y_n) + c_1,$$
$$\dot{y}_2 = -a_{22} y_2 + b_{21} f_{21}(y_1) + b_{22} f_{22}(y_2) + ... + b_{2n} f_{2n}(y_n) + c_2,$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\dot{y}_n = -a_{nn} y_n + b_{n1} f_{n1}(y_1) + b_{n2} f_{n2}(y_2) + ... + b_{nn} f_{nn}(y_n) + c_n.$$

We believe, that $a_{ii} > 0$, functions $f_{ij}(y_j)$, $i, j = n$ is continuously differentiable, and the system of differential equations has a single point of rest $M_0(y_1^0, y_2^0, ..., y_n)$, which is the solution to the system of equations

$$-a_{11} y_1 + b_{11} f_{11}(y_1) + b_{12} f_{12}(y_2) + ... + b_{1n} f_{1n}(y_n) + c_1 = 0,$$
$$-a_{22} y_2 + b_{21} f_{21}(y_1) + b_{22} f_{22}(y_2) + ... + b_{2n} f_{2n}(y_n) + c_2 = 0,$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$-a_{nn} y_n + b_{n1} f_{n1}(y_1) + b_{n2} f_{n2}(y_2) + ... + b_{nn} f_{nn}(y_n) + c_n = 0.$$

Let's replace the "parallel transfer" of the rest point $M_0(y_1^0, y_2^0, ..., y_n)$ at the origin

$$y_i = x_i + y_i^0, i = 1, 2, ..., n.$$

Then the system will reduce to the form of a “system of perturbation equations”

$$\dot{x} = -a_{11} x_1 + b_{11} F_{11}(x_1) + b_{12} F_{12}(x_2) + ... + b_{1n} F_{1n}(x_n),$$
$$\dot{x}_2 = -a_{22} x_2 + b_{21} F_{21}(x_1) + b_{22} F_{22}(x_2) + ... + b_{2n} F_{2n}(x_n),$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dot{x}_n = -a_{nn}x_n + b_{n1}F_{n1}(x_1) + b_{n2}F_{n2}(x_2) + ... + b_{nn}F_{nn}(x_n),$$

where $F_{ij}(x_j) = f_{ij}(x_j + y_j^0) - f_{ij}(y_j^0)$.

And, because

$$F_{ij}(0) = 0,\ i, j = 1, n,$$

then the study of the stability of the equilibrium state $M_0(y_1^0, y_2^0, ..., y_n^0)$ and the convergence of solutions to this state is reduced to the study of the stability of the zero equilibrium state $O(0, 0, ..., 0)$ systems of perturbation equations.

Two cases of nonlinearities are considered. In the first case, the function $F_{ij}(x_j), i, j = 1, n$ satisfy the so-called "linear constraint conditions"

$$\left|F_{ij}\left(x_j\right)\right| \le K_{ij}\left|x_j\right|, i, j = 1, n.$$

And the asymptotic stability of the zero equilibrium state of system can be investigated using “classical” quadratic Lyapunov functions of the form

$$V\left(x_1, x_2, ..., x_n\right) = \sum_{i=1}^{n} h_i x_i^2,\ \ h_i > 0, i, j = 1, n.$$

In the second, more “rigorous” case, the functions $F_{ij}(x_j)$ satisfy the so-called "sector conditions"

$$0 < F_{ij}(x_j) < L_{ij}x_j \ \text{ at } \ x_j > 0\,,$$

$$L_{ij}x_j < F_{ij}\left(x_j\right) < 0 \ \text{ at } \ x_j < 0\,,$$

$$F_{ij}\left(0\right) = 0, i, j = 1, n.$$

These conditions can be written in a more compact form

$$\left[L_{ij}x_j - F_{ij}(x_j)\right]F_{ij}(x_j) > 0,\ \ i, j = 1, n.$$

When fulfilling the "sector conditions", obtaining the conditions of asymptotic stability and convergence estimates can be carried out using the Lurie-Postnikov type Lyapunov function. In some cases, it can give more accurate results. Since the linear part has a diagonal form, the Lyapunov function is constructed as the sum of the squares of the phase variables and integral terms with constant coefficients

$$V\left(x_1, x_2, ..., x_n\right) = \sum_{i=1}^{n} h_i x_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{x_j} \gamma_{ij}F_{ij}\left(s\right)ds,\ \ h_i > 0,\ \ i = 1, n\,.$$

AUTHENTICATION INTERFACE CONCEALMENT FOR WEBSITES: CONCEPT AND APPLICATIONS

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This paper introduces Authentication Interface Concealment (AIC), a concept for hiding website authentication mechanisms. Unlike "Security through Obscurity" [1] or "Steganoauthentication" [2,3], AIC offers a structured, multi-layered approach. Current methods for concealing admin access, such as dedicated login pages or unlinked "orphan pages," are flawed as they often rely on unreliable obscurity [1].

The proposed AIC concept operates on two levels. Level 1 conceals the login interface, revealing it only after specific user actions are detected by obfuscated JavaScript [4]. Level 2 conceals the authentication algorithm by analyzing keystroke dynamics (a user's unique typing rhythm), offering resilience against threats like keyloggers. This data can be verified on the server side using techniques like neural networks [5]. For enhanced protection, the system can be augmented with a High-Interaction Honeypot (HIHs).

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CLIENT-SERVER TOOLKIT FOR COMPARTMENTAL MODELING AND OPTIMAL CONTROL

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We present *CompLab* a client–server toolkit that automates the workflow for compartmental models, including interactive model graph construction, automatic generation of the ODE system, numerical integration and/or optimal control. The goal is to lower the technical barrier for researchers who need rapid prototyping of non-standard compartmental structures and interventions.

A user-defined model graph is translated on the fly into a first-order ODE system. Modeling is performed with SciPy's implementation of LSODA (adaptive stiff/non-stiff ODE solver). The package implements Pontryagin's Maximum Principle via a forward-backward sweep; bang-bang or relaxed control updates are chosen adaptively; only user-inputted integral objective is used; admissible control trajectories are further restricted through user-defined upper and lower bounds. Auxiliary features include, but not limited to interactive graph editing with auto-layout, workspaces, import/export, reuse of results from previous runs, and rich information visualization.

The architecture is intentionally simple: heavy computation remains server-side (Python + Flask), while all interaction stays in the client (Angular), facilitating future extensions and development. *CompLab* accepts arbitrary closed models with constant parameters, thereby giving users a high degree of flexibility. Applicability of the proposed software was demonstrated using examples of epidemic diseases.

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FEATURES OF STRATEGIC DECISION-MAKING IN BUSINESS USING THE BOSTON MATRIX AND DATA MINING

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In today's economy, strategic decisions – investment, development, risk minimization – require tools that combine accuracy and predictive value. One such tool is the Boston Matrix (BCG), which allows you to analyze product positions by market share and growth rate. The analysis allows you to choose areas of development, identify priority areas of investment, find profitable niches, and assess the profitability of specific categories of goods and market areas. Therefore, the strategic decisions obtained with its help fundamentally affect the entire business. Hence such attention to the reliability of the information used in it.

However, BCG only records the current state of the business. To take into account the future, it is proposed to supplement it with Data Mining methods that use behavioral, financial and marketing characteristics: frequency of purchases, seasonality, average check, loyalty, etc.

Data mining turns data into empirical regularities (EMS), which replace guesses, assumptions and can improve a person's awareness of something, based on data. EMS can be considered empirical knowledge, which allows you to determine the method of action, create methods, instructions, rules, give superficial assessments, in some cases predict, that is, obtain meaningful information necessary for decision-making. At the same time, only the initial level of understanding (cognition) is provided.

This approach allows us to replace assumptions with a reasoned opinion based on data. In fact, we get a new type of information that is used in economics and business – a well-grounded opinion and, obviously, making strategic decisions taking it into account improves their forecasting characteristics. This allows for improved management decisions, but remains limited in long-term strategies.

To increase reliability, it is necessary to move from inductive (through Data Mining) to deductive analysis, including the formulation and testing of hypotheses, the establishment of cause-and-effect relationships. In this case, it becomes possible to establish and substantiate cause-and-effect relationships related to the questions: "why?", "how does it happen?", "why does it happen this way?" This is the only way to achieve a level of knowledge that can provide strategic advantages.

EVALUATION OF THE INFORMATIVENESS OF DISCRETE FEATURES IN CLASSIFICATION PROBLEMS

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When analyzing data, detecting patterns in them and extracting them, one has to face the problem of dimensionality. The dimensionality of the stored data, determined by the number of various features describing the state of the objects under consideration, is very large and sometimes reaches several tens and hundreds of indicators. Therefore, the problem of reducing the dimensionality of the feature space and highlighting the most informative features is very relevant for various classification systems. In classification tasks, it is important to understand which features are most significant for building a model. Uninformative or redundant features can reduce the quality of classification by introducing noise into the model, increase computational costs, complicate the training process, and cause overfitting if the model remembers noise rather than patterns.

In this case, the assessment of the informative value of features is a key step in building machine learning models. The problem is especially relevant for multidimensional data, for example, in bioinformatics, text processing, computer vision. In such cases, it is important to select only key features, which makes the task of assessing their informative value critical.

Statistical methods for assessing the informativeness of discrete features used in classification problems are considered and studied in detail: the Shannon method, the Ginney method, and the chi-square. A nonparametric approach based on relationship estimates is proposed, taking into account the interaction of features and ensuring interpretability.

The application of these methods is demonstrated by a number of practical examples, including medical data, with the construction of a correlation matrix and interpretation of the results. A developed web service that implements the Ginny method and the nonparametric method is described. The web service provides the user with a convenient interface for assessing the informative value of features describing the state of the object, including arbitrary groups of features. This complex can be built into specialized information systems for diagnosing various processes.

MODELING AND ANALYSIS OF SEISMICALLY ACTIVE AREAS USING CELLULAR AUTOMATA FRAMEWORKS

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Seismic data plays a crucial role in both civil life and scientific research. Consequently, numerous approaches have been developed to analyze seismic activity (for example [1]–[2]).

Cellular automata offer an effective framework for modeling the spatiotemporal dynamics of seismic data. This approach provides valuable tools for data visualization, particularly in extrapolation and interpolation tasks.

As a result we propose the python software tool for visualize and perform analysis of dynamic seismic activity data.

The application enables the spatiotemporal analysis of seismic events by integrating geographic and statistical information.

The following technologies were used: pandas, numpy, matplotlib.pyplot and FuncAnimation.

In future plan we want to work with mathematics models for Seismic hazard analysis, for example, algorithm of optimal estimation ([3]-[4]).

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MULTIPLE-EXPERT MODAL LOGICS OF PARTIAL PREDICATES WITH SEPARATE DOMINANCE OF EXPERTS

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The apparatus of modal logics is successfully used in information and software systems, artificial intelligence systems. In this paper, we propose new class of modal logics of epistemic type – multiple-expert modal logics of partial quasiary predicates. These logics are based on the concept of multiple-expert modal system (MEMS). A characteristic feature of MEMS is the separate dominance relation on the set of intelligent agents, or intelligent experts. Such a relation is necessary for the coordination of experts' decisions. In the known [1] multiple-expert systems, the dominance typically requires the preservation of *true*: if a dominant expert considers a statement Q to be true, the subdominant expert must also consider Q to be true. However, in the proposed MEMS, both *true* and *false* must be preserved under dominance. In this case, an incorrect situation may arise: expert a is directly dominated by experts b and c, such that b considers a statement Q to be true, while c considers Q to be false. To avoid such issues, we introduce a separate dominance relation on a set of experts: one expert cannot be directly dominated by ≥ 2 experts.

We define the language of pure first-order MEMS, and describe the interpretation mapping for formulas on states of the world and experts. The attention is focused on the features of the relation of separate dominance. We formulate conditions for truth values preservation for atomic formulas under dominance and prove that these conditions can be extended to all formulas. On the set of formulas specified by the names of experts and states, a number of relations of logical consequence are introduced.

In the future, it is planned to continue the study of logical consequence relations in MEMS and to construct sequent calculi for MEMS.

The proposed multiple-expert modal logics can be used in information and software systems that require accounting for partiality and incompleteness of information, in artificial intelligence systems and expert systems.

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COINCIDENCES OF A SEMI-MARKOV STOCHASTIC PROCESS WITH A MARKOV ONE

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Let us find the conditions under which the semi-Markov random process coincides with the Markov process defined by the system of differential equations

$$\frac{dP(t)}{dt} = AP(t), \quad A = \left\| a_{ks} \right\|_{k,s=1}^{n}. \tag{1}$$

For a Markov process, jump transitions from a state θ_k to the same state θ_k are not considered. Therefore, we assume

$$q_{kk}(t) \equiv 0 \quad (k = 1,...n).$$

Let us pass from the Markov process (1) to the Markov chain

$$P(t_k + h) = P(t_k) + hAP(t_k), \quad t_k = kh \quad (k = 1,...n)$$

On the main diagonal of the matrix $E + hA$ are the elements $1 + ha_{kk}$ $(a_{kk} < 0)$. Therefore, the probability of remaining in a state θ_k over time $t = nh$ is defined by the expression

$$P\left\{\zeta(t_j) = \theta_k\right\} = (1 + ha_{kk})^{th^{-1}} \xrightarrow[h\to 0]{} e^{a_{kk}t}.$$

Therefore, for the Markov process we have the equality

$$\psi_k(t) = e^{ta_{kk}} \quad (k = 1,...,n;\ a_{kk} < 0).$$

From the system of equations (1) it follows, that from state θ_k to another state θ_s $(s \neq k)$ Markov chain goes over time $\Delta t = h$ with probability ha_{sk}. Based on this, we assume

$$q_{sk}(t) = a_{sk}\psi_k(t) = a_{sk}e^{ta_{kk}} \quad (s \neq k;\ k, s = 1,...,n).$$

Final intensity $q_{sk}(t)$ for the Markov random process $a_{kk} < 0$ $(k = 1,...n)$ we get in the form

$$q_{kk}(t) \equiv 0,$$
$$q_k(t) = -a_{kk} e^{ta_{kk}},$$
$$\psi_k(t) = e^{ta_{kk}} \quad (k = 1,...n)$$
$$q_{sk}(t) = a_{sk} e^{ta_{kk}} \quad (s \neq k; k, s = 1,...,n). \qquad (2)$$

Formulas (2) determine the intensities for the Markov process (1), defined as a semi-Markov random process.

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OPERATOR EQUATIONS FOR PARTIAL DENSITIES

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We consider a system of linear difference equations with semi-Markov coefficients.

$$X_{k+1} = A(k, \zeta_k) X_k, \quad (k = 0,1,2,...), \qquad (1)$$

where $\zeta(t)$ is a semi-Markov chain that takes a finite number of values $\theta_1,...,\theta_n$ with intensities $q_{jS}(k)$ $(j, S = 1,...,n)$ of transition from state θ_S to state θ_j at the moment k. The intensities satisfy the conditions:

$$q_{jS}(k) \geq 0, \quad \sum_{j=1}^{n} q_{jS}(k) = q_S(k), \quad (S = 1,...,n),$$
$$\sum_{k=1}^{\infty} q_S(k) = 1, \quad \sum_{k=1}^{\infty} q_{jS}(k) = \pi_{jS}, \quad (j, S = 1,...,n) \qquad (2)$$

$\Pi = \left\| \pi_{jS} \right\|_1^n$ – stochastic matrix, $\sum_{j=1}^{n} \pi_{jS} = 1$.

Functions are also introduced $\psi_S(k)$

$$\psi_S(k) = \sum_{j=k+1}^{\infty} q_S(j); \quad \psi_S(0) = 1, \quad (S = 1,...,n; k = 0,1,2,...)$$

$$\lim_{k\to\infty} \psi_S(k) = 0, \quad (S = 1,...,n).$$

We assume that the system of difference equations of the system of linear differential equations

$$\frac{dX(t)}{dt} A(t, \zeta(t)) X(t),$$

has jumps of solutions that occur simultaneously with jumps of the random chain ζ_k.

Consider a continuous-discrete random process (X_k, ζ_k) and write the distribution density in the form

$$f(k, X, \zeta) = \sum_{k=1}^{n} f_k(k, X) \delta(\zeta - \theta_S).$$

We introduce the vector of partial distribution densities

$$F(k, X) = \begin{pmatrix} f_1(k, X) \\ ... \\ f_n(k, X) \end{pmatrix} \quad (k = 0,1,2,...)$$

and find a stochastic operator $L(s)$ $(s = 0,1,2,...)$ such that

$$F(s + k_j, X) = L(s) F(k_j, X) \quad (j, s = 0,1,2,...)$$

In this case, we assume that the operator $L(s)$ has the form of a matrix with operator elements

$$L(s) = \begin{pmatrix} L_{11}(s) & L_{12}(s) & ... & L_{1n}(s) \\ L_{21}(s) & L_{22}(s) & ... & L_{2n}(s) \\ ... & ... & ... & ... \\ L_{n1}(s) & L_{n2}(s) & ... & L_{nn}(s) \end{pmatrix}.$$

we introduce stochastic operators $R_S(k)$, S_{Sk} defined by the formulas

$$R_S f(X) \equiv f(N_S^{-1}(k)X) \cdot \left|\det N_S^{-1}(k)\right| \quad (S = 1,...,n)$$

$$S_{Sk} f(X) \equiv f(C_{Sk}^{-1}X) \cdot \left|\det C_{Sk}^{-1}\right| \quad (S = 1,...,n)$$

In this case, we obtain a system of operator equations

$$L(k)F(0,X) = \psi(k)R(k)F(0,X) + \sum_{j=1}^{k} L(k-j)S(j)R(j)F(0,X),$$

where the designations are used

$$\psi(k) \equiv \left\|\delta_{rl}\psi_l(k)\right\|_1^n,$$

$$R(k) \equiv \left\|\delta_{rl}R_l(k)\right\|_1^n,$$

$$S(k) \equiv \left\|q_{rl}S_{rl}\right\|_1^n, \quad (k = 0,1,2,...)$$

The operator equation for the stochastic operator $L(k)$ takes the form

$$L(k) = \psi(k)R(k) + \sum_{j=1}^{k} L(k-j)S(j)R(j), \qquad (3)$$

The solution of equation (3) can be found by the method of successive approximations.

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EXPLICIT AND RECURRENT FORMS OF REPRESENTATION FOR PARAMETER ESTIMATE BY THE METHOD OF LEAST SQUARES OF RELATIVE ERRORS UNDER NON-CLASSICAL ASSUMPTIONS FOR SYSTEMS UNDER UNCERTAINTIES

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The estimation problem of unknown parameters for mathematical models of systems is considered, in which the main attention is concentrated not on the absolute values of model errors, but on their relative ones. This need often arises in areas such as finance, economics, chemistry, physics, environmental and Earth sciences, etc.

To solve this problem, it is proposed to use the method of least squares of relative errors. The estimate of the vector of unknown parameters of the regression model by this method in the classical case, when it is unique, has already been obtained earlier.

In this work, the explicit expression of this estimate of the regression model parameter vector is spread to the case when the classical assumption that guarantees the uniqueness of this parameter estimate may be violated. In obtaining this estimate, it became necessary to use the Moore-Penrose matrix pseudo-inversion operator.

In order to use this parameter, estimate in a non-classical situation in real time mode, there is a need to construct an appropriate recurrent form of representation for it. For this purpose, the estimate with the least Euclidean norm was chosen as the unique estimate on the set of all these estimates in this case for further applying.

The recurrent form of representation is obtained for this unique estimate of the method of least squares of relative errors. The constructed recurrent algorithm no longer requires the use of either the Moore-Penrose matrix pseudo-inversion operator or even the usual matrix inversion operation.

For the current quality control of the obtained mathematical model, it is useful to be able to calculate the value of the corresponding weighted sum of squares of the residuals. A recurrent procedure is developed to recalculate the values of the latter. It also does not require the use of pseudo-inversion or ordinary inversion to the matrices.

The proposed recurrent forms of representation for the parameter estimate by the method of least squares of relative errors and the corresponding weighted sum of squares of residuals allow significantly

speed up the processing of incoming information during the experiment and reduce the load on the used computing devices.

Applications of obtained results are discussed too.

SUBDIFFUSIVE OPTION PRICING VIA FRACTAL DUPIRE EQUATION

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The classical diffusion models for continuous time like Black-Scholes-Merton are incapable of adequately modelling illiquidity for real-life asset dynamic and evaluate derivatives.

The paper focuses on subdiffusive models with local volatility that extend classical approaches to capture illiquidity effects and trapping events through stochastic time changes.

The main option pricing tool in this paper is a fractional extension of the Dupire equation.

We apply the equation for subdiffusive in Brownian setting in the case of the inverted tempered stable subordinator and demonstrate numerical results for real financial data.

The study finds that subdiffusive models are more accurate and sensitive, especially in capturing market behaviors like periods of price motionless for illiquid markets.

The results of comparing the studied model with the classical one show that the classical B-S model demonstrates better results in the short-term period, while the sub-diffusive model is more effective in the long-term perspective. Moreover, the accuracy of sub-diffusive model with local volatility prevails over sub-diffusive model with constant volatility

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SOFTWARE AND HARDWARE FOR REAL-TIME MODELING AND CONFIGURING OF DIGITAL TWINS

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Digital twins are an important tool in modern technologies, allowing creating accurate models of physical objects for their monitoring, analysis and optimization in real time. Software and hardware tools for modeling digital twins for industrial applications have been investigated, suitable for different types of applications, such as real-time modeling applications, analytical applications and applications with artificial intelligence that use and fill the content of the digital twin.

The main software tools include platforms for creating digital twins: 3D modeling software (Autodesk, SolidWorks); simulation platforms (MATLAB, Simulink, Ansys); integrated platforms for IoT (PTC ThingWorx, Siemens MindSphere); data collection and processing tools: software modules for big data (Hadoop, Apache Spark); tools for real-time data analysis (StreamSets, Kafka). Artificial intelligence and machine learning: algorithms for forecasting and automating processes, tools for training models (TensorFlow, PyTorch, Scikit-learn). The main hardware includes sensors and transducers: sensors for temperature, pressure, humidity and other physical parameters; cameras for visual monitoring; computing devices: servers for real-time data processing, graphics processing units (GPUs) for processing complex models; data transfer devices: network devices for fast data transfer (5G, LoRaWAN); cloud services for data storage and synchronization (AWS, Azure).

Digital twins are created according to the types of real-world analogues, its contain data about their physical copy. Instances will be based on templates of their types in accordance with the environment configuration. To display, understand and predict the operating states and behavior of twins, the digital copy includes computational or analytical models, as well as models used to assign actions based on the business logic and goals of the corresponding physical world object. To access its data and exploit its capabilities, a digital twin contains a set of service interfaces for industrial applications or other twins. Software and hardware for digital twins provide significant potential for optimizing processes and increasing efficiency in many industries. Further development of the technology is essential to ensure greater accuracy and integration.

FUZZY IMPLICATIONS IN MODELING INTELLIGENT SYSTEMS

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Approximate reasoning techniques are a fundamental human ability and skill that allows us to cope with solving real-world problems under conditions of uncertainty. They help us better and faster understand physical phenomena, various social behaviors, or complex technological processes. Uncertain knowledge representation has attracted the attention of artificial intelligence researchers for many years, who try to make knowledge obtained directly from the real world suitable for automatic processing by computing machines.

There are many formalisms that enable effective and efficient reasoning under uncertainty automatically, without human intervention. These also include approaches based on fuzzy Petri nets. In fuzzy Petri nets modeling the inference process, each transition of the net represents one production rule of the IF.. THEN type describing an elementary step of such a process. The role of a single rule in such a process is to transform imprecise information from the premise of the rule into its conclusion, knowing in advance the degree of truth of the rule. Each production rule can be interpreted in a theoretical sense as a function of two variables mapping the unit square to the unit segment and satisfying additional conditions in the logical sense, where the first argument represents the degree of truth of the premise, the second – the degree of truth of the conclusion, and the value of such a function represents the degree of truth of the rule. To correctly describe the process of transforming information from the input of a transition to its output, the function representing the production rule associated with the transition should be inverted with respect to the second argument when modeling the forward reasoning process and with respect to the first argument when modeling the backward reasoning process.

Mathematicians have proven that there are an infinite number of functions describing production rules, i.e. satisfying the definitional requirements of the so-called fuzzy implication. For many years, they have been providing examples of such functions and information about their properties, which have not only cognitive but also practical significance. Engineers select from theoretical proposals those functions that may be useful to them in practice. Despite the existence of many

functions that satisfy the definition of fuzzy implications, some of them are: (1) irreversible with respect to the required argument; (2) even if they are, their inverse functions are not always defined on the entire unit square; (3) both functions before and after inversion are represented by rather complex mathematical formulas, which makes the given functions less useful in practice. Even considering the noticeable progress in this direction, it should be noted that there is still an urgent need for further searches for appropriate functions that will make fuzzy Petri net models of complex fuzzy inference systems, in particular control systems, expert systems or decision support systems operating under uncertainty, simpler, more efficient and more effective.

In this lecture we will address the problem of inverting fuzzy implications with respect to the second argument, show examples of functions before and after inversion along with their properties, and illustrate their application in the framework of automatic modeling of intelligent systems operating under uncertainty.

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ON THE ALGEBRAIC CHARACTERIZATION OF WEAKLY LOCALLY COMPACT GROUPS

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A. Hulanicki [1] provided an algebraic characterization of compact abelian groups. Specifically, he identified the algebraic commutative groups on which a compact topology can be defined.

In this paper, we establish the existence of infinite, torsion-free, connected commutative groups that admit a weakly locally compact topology [2].

Definition. A commutative topological group G is called weakly locally compact if the zero component of G group - G_0 is locally

compact, and the factor group G / G_0 is locally weakly linearly compact.

Just as in the class of locally compact abelian groups,where the classes of compact and discrete groups are distinguished, so in the class of weakly locally compact groups, the classes of quasi-compact and quasi-discrete groups are distinguished. These are related to each other from the viewpoint of the theory of characteristics.

There is concept of local direct product of topological groups (direct product with the mentioned subgroups).

Let $(G_i)_{i\in J}$ be the set of weakly locally compact groups for which every $i \in J$, H_i is a subgroup of $G_i, i \in J$.

If $(X_i)_{i\in J}$ is the set of characteristic groups of given groups $(G_i)_{i\in J}$, then it is proved that groups G and X are characteristic groups of each other.

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CONSTRUCTING A MINIMUM AREA POLYGON USING CONVEX HULL LAYERS

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The paper proposes a method for constructing a simple polygon of minimal area over a set of points in the plane using a layered convex hull approach.

The problem of generating a minimum-area polygon over a set of *n* points is relevant to applied optimization tasks in systems analysis, particularly where spatial resource constraints are critical. Its solutions support coverage minimization, object clustering, boundary condition formulation in mathematical models, and reduction of geometric approximation complexity. This enhances decision-making efficiency in distributed and resource-constrained systems.

It is known that the problem of minimum area polygonization (MAP) belongs to the set of NP-Hard problems. Therefore, various

approximation algorithms were developed. This paper proposes an approach based on the layered construction of convex hulls, followed by optimization of contour selection to form a simple polygon of minimal area. To connect the hulls, a visibility graph is employed to detect and eliminate self-intersections. This yields an approximate algorithm with a computational complexity of $O(n^6 \log^2 n)$. We also developed a software implementation of the proposed method in Python.

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COMPUTATIONAL METHODS FOR DECISION MAKING IN UNCERTAIN DYNAMICAL SYSTEMS USING RETRIEVAL-AUGMENTED GENERATION AND CHUNK REARRANGEMENT

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In this work, we investigate advanced computational methodologies for decision making in uncertain dynamical systems across various applied domains. The proposed approach integrates large-scale computation, statistical modeling, and AI-driven reasoning to address challenges arising from incomplete or noisy information. Special attention is given to Retrieval-Augmented Generation (RAG) techniques, which combine vector-based information retrieval with neural sequence generation, enabling the formulation of informed and robust decisions grounded in domain-specific knowledge.

Let $D = \{d_1, ..., d_n\}$ be a document collection, and let q be a query representing the decision context. The retrieval stage computes relevance scores $s_i = \varphi(q_i, d_i), i = 1, ..., N$, where φ is a similarity function (e.g., cosine similarity in an embedding space). The top-k documents $C_k = \{d_{i1}, ..., d_{ik}\}$, with $s_{i1} \geq s_{i2} \geq ... \geq s_{ik}$ are selected as context chunks. The generation stage uses a parametric model G_θ to produce a decision

recommendation:

$$y^* = \arg\max_y P_\theta(y \mid q, C_k). \quad (1)$$

To improve decision accuracy, we propose a chunk re-arrangement optimization before feeding context to the generator.

Let π be a permutation of $\{1,...,k\}$ We define the decision utility function:

$$U(\pi) = E[R(G_\theta(q, \{d_{i\pi(1)}, ..., d_{i\pi(k)}\}))], \quad (2)$$

where $R(\cdot)$ is an evaluation metric (e.g., accuracy, F-score, or domain-specific cost function). The optimal chunk order is:

$$\pi^* = \arg\max_{\pi \in S_k} U(\pi), \quad (3)$$

which can be approximated via heuristic search, simulated annealing, or reinforcement learning. This reordering ensures that the most contextually critical information appears earlier in the model's context window, improving reasoning under limited token budgets.

The proposed framework is tested on datasets from financial forecasting, and legal document analysis, showing measurable improvements in decision quality compared to baseline RAG without chunk optimization. Computational experiments demonstrate that the added rearrangement step increases decision accuracy by up to 8-12% while keeping inference latency acceptable for real-time applications.

The integration of retrieval-augmented generation with mathematically grounded chunk rearrangement optimization provides a flexible, domain-independent methodology for decision making in diverse fields. This approach benefits from both statistical retrieval theory and sequence generation modeling, while the chunk ordering optimization contributes to better use of limited context size in large language models.

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MOBILE MEDICAL AND REHABILITATION COMPLEXES

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Solution Architecture: A compact wearable device based on Xilinx programmable logic has been developed that captures several vital signs (heart rate, ECG, blood oxygen level, temperature, activity, etc.) in real time and immediately processes these signals using built-in edge AI. The hardware FPGA architecture with parallel processing and a neural network accelerator allows for on-site detection of dangerous conditions (e.g., arrhythmias, hypoxia) instantly, without the need to transmit raw data to external servers. The built-in dual-core ARM processor controls the device, provides data collection from sensors, energy-efficient battery operation, and wireless transmission of results.

Functionality and capabilities: The device functions as a personal "medical assistant". It continuously monitors the user's health, records key parameters, and can issue an alarm when abnormalities are detected (via sound, light indicator, or phone notification). The received data is displayed in a convenient form on the mobile application: the user sees current indicators and receives advice (for example, sit down to rest, drink water, consult a doctor). The application also automatically sends data to the cloud processing center, where it is stored and analyzed. Doctors have access to this data via a secure web portal and can monitor the dynamics of the patient's condition remotely. The system supports modern medical standards, so the data can be integrated directly into the electronic patient record, which simplifies the use of information in clinical practice.

Usage scenarios: For home patients, it is an opportunity to safely undergo rehabilitation or control of chronic diseases outside the hospital walls: the device transmits indicators to the doctor, minimizing the number of visits, and at the same time providing the patient with peace of mind that his condition is under control. In clinics, the complex can be used for wireless monitoring of the intensive care unit or during diagnostic procedures, quickly notifying staff about changes in condition. In the field (ambulance, combat, remote areas) it is a portable tool that provides medical workers with on-site diagnostics, with the ability to consult with specialists via telemetry. In all scenarios, the system reduces delays in obtaining medical information and contributes to a faster response to potentially dangerous events.

TOKEN-LORA FUSION: EFFICIENT PERSONALIZATION OF TEXT-TO-IMAGE DIFFUSION MODELS

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Personalizing text-to-image models to generate novel images of a specific subject from only a few reference images remains a fundamental challenge in generative modeling. Prior works such as Textual Inversion and DreamBooth offer two distinct approaches: the former learns a pseudo-token in the embedding space of a frozen model, while the latter fine-tunes the entire model to tightly bind a subject's identity to a unique textual identifier. While effective, these methods trade off between sample efficiency, training cost, and generalization.

In this work, we propose a hybrid personalization framework that bridges these two paradigms. Our method begins by learning a pseudo-token to encode a novel concept (person, object, or style) from a few images for half the training. We then fine-tune the base diffusion model with LORA for the remaining half without updating the learned token. This progressive adaptation strategy leverages the semantic grounding of token-based methods and the expressive power of model-level fine-tuning, while maintaining parameter efficiency and fast convergence.

We evaluate our method across standard benchmarks for subject-driven generation and compare it with recent baselines in fidelity, diversity, and editability. Quantitative and qualitative results show that our approach preserves subject identity more robustly than Textual Inversion, while being more efficient and composable than DreamBooth. Our method enables creative recontextualization of personalized subjects across diverse scenes and prompts, with minimal computational overhead.

This work contributes a practical and flexible solution for efficient personalization of diffusion models, opening new directions for subject-driven generation in resource-constrained settings.

GENERATING AND APPLYING RANDOM MAZES

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Generation of random mazes is a current research area in the fields of search algorithms, computer graphics, and procedural data generation. Mazes used to optimize navigation strategies, simulate environments in computer games, and test artificial intelligence.

Maze generation algorithms are based on various approaches, among which Prim's algorithm, the recursive space division algorithm, and the depth-first search (DFS) algorithm attract particular attention.

All three algorithms implemented in Python using standard libraries. The generation results were output in a text format, allowing for a visual assessment of the quality of the generated mazes.

A comparative analysis of the implementations conducted based on execution time, structure complexity, and the number of dead ends. All algorithms run on mazes of size 21×21, enabling a comparison of their efficiency with the same data set. Each algorithm executed 10 times to average the execution time and assess the stability of the results.

Examples of implementation results can be seen in Fig. 1-3.

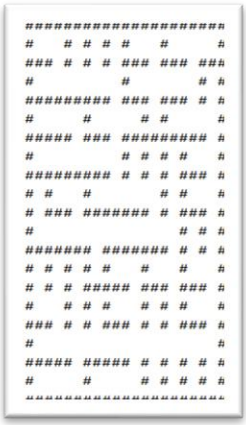

Fig. 1. Maze, generated by the Prim algorithm

Fig. 2. Maze, generated by the Recursive Division

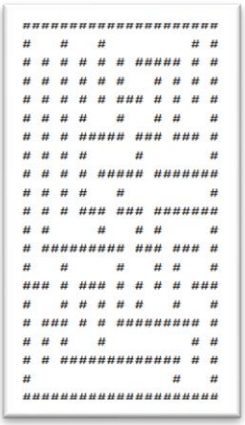

Fig. 3. Maze, generated Depth-First Search algorithm

It has been established that each of the considered algorithms has its optimal scenarios for application. The Depth-First Search algorithm is the most effective for generating complex, convoluted mazes. It is suitable for gaming environments where a high number of dead ends and winding paths increases the difficulty of navigation. DFS is also applied in training artificial intelligence agents to operate in environments with a high branching factor, improving their pathfinding and decision-making capabilities. In addition, DFS-generated mazes are used in cognitive psychology experiments to study human and animal spatial navigation under challenging conditions.

Prim's algorithm produces balanced and logically structured mazes, making it suitable for testing search and navigation algorithms in robots and autonomous systems. It is often used in warehouse for autonomous robotics, where robots must navigate between shelves in a structured but non-linear layout to optimize path planning. In disaster response simulations, such mazes can model partially obstructed building interiors, helping to evaluate search-and-rescue algorithms.

The Recursive Division algorithm is best for rapid generation of simple, symmetrical mazes. It is frequently applied in basic navigation strategy testing for autonomous robots or drones, where the emphasis is on verifying movement algorithms rather than solving complex puzzles. Beyond robotics, this algorithm is also valuable in architectural layout generation and urban planning simulations, where fast prototyping of partitioned spaces is essential.

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METHODS OF USING DIGITAL TWINS TOGETHER WITH THE INTERNET OF THINGS IN HIL -MODELING

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Digital twins (DT) have evolved from static models into dynamic, data-driven replicas of real-world objects. Their metrics come from the Internet of Things (IoT): distributed sensors that transmit temperature, vibration, pressure, or other signals to the cloud in real time. When integrated into a hardware-in-the-loop (HIL) test rig, they provide the ability to validate designs, predict failures, and optimize operations. In a classic HIL rig the controller under test interacts with an emulated plant in real time. Replacing that plant model with a DT fed by live IoT data adds fidelity the moment a physical parameter drifts. The model remains phase-aligned with its real counterpart because the same sensor data that informs the controller also refresh the twin . Consequently, algorithms can be stressed under authentic, time-varying boundary conditions without endangering the actual machine.

DT require a constant stream of data to effectively mirror their physical counterparts. And IoT devices embedded with sensors can help with this. These sensors collect data in real-time from the environment, user interactions, or the object's performance. As IoT devices transmit data, Digital Twins update in real-time. This ensures that the virtual model is always in sync with the physical entity.

A data-rich DT does more than mirror; it forecasts. Trend deviations detected in the twin (e.g., rising bearing temperature) trigger maintenance events inside the HIL loop before the fault propagates to the hardware. Such predictive maintenance schemes have already cut turbine and motor downtime in manufacturing by double-digit percentages. Because IoT streams capture usage diversity, the DT can replay thousands of “what-if” scenarios inside the HIL. Controller code is compiled once, flashed to the real ECU, and iteratively tuned against the evolving twin until KPIs. This shortens physical prototyping cycles and slashes material waste.

Coupling Digital Twins with IoT streams places a real-time, self-updating model at the heart of HIL-modelling. The resulting closed loop lets designers observe, predict, and act on cyber-physical systems long before faults escape to the field

SELF-ADJUSTING ALGORITHMS THAT CHANGE THE STRUCTURE OF THE PROBLEM

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Self-adjusting systems are divided into two large groups:

• self-adjusting, in which only the values of the problem parameters change;

• self-organized, in which the structure of the problem itself changes.

To solve the situation of uncertainty of various types in combinatorial optimization, self-adjusting algorithms are developed [1]. They are used to solve the situation of uncertainty, which is associated with incomplete input and current information, as well as with the ambiguity that arises due to the special structure of combinatorial sets (the argument of the objective function). In the process of their work, they automatically change certain parameters or generate additional current information in order to find the optimal solution to the problem.

There are problems in which in the process of their solution it is necessary to change their structure. For example, in some problems from scheduling theory, the optimal distribution of problems in a swarm of unmanned aerial vehicles, the problem of reducing (or increasing) machines arises in the process of their solution. The properties of combinatorics are used to automatically change the structure of such problems. Applied problems in which the argument of the objective function is a combination without repetition or with repetitions to fulfill the given constraints by selecting elements from a given base set, the number of objects changes, that is, the structure of the problem is changed (the knapsack problem, Johnson's problem in scheduling theory, etc.). In the process of solving the problem by changing the structure of the base set, the number of their elements automatically changes. That is, a self-adjusting process of changing the structure of such problems is organized.

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A CONJUGATE GRADIENT ALGORITHM FOR AN OPTIMAL INITIAL CONTROL PROBLEM OF A TIME-FRACTIONAL WAVE EQUATION

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Let $\Omega \subset R^n$ be a bounded domain with a smooth boundary $\partial\Omega$.

Let us consider the following optimal control problem

$$J(y,u) = \frac{1}{2}\left\|y(x,T) - y_d^0\right\|^2_{L^2(\Omega)} + \frac{1}{2}\left\|\frac{\partial y}{\partial t}(x,T) - y_d^1\right\|^2_{L^2(\Omega)} + \\ + \frac{\lambda}{2}\|u\|^2_{H_0^1(\Omega)} \to \inf, \quad (1)$$

$$u \in U_\partial, \quad (2)$$

where $\lambda > 0$, $U_\partial \subset H_0^1(\Omega)$ is a nonempty closed convex set of admissible controls, and $y = y(x,t,u)$ is a solution of the system

$$ {}_0^C D_t^\alpha y(x,t) = Ay(x,t),\ x \in \Omega,\ t \in (0,T), \quad (3)$$

$$y(x,t) = 0,\ x \in \partial\Omega,\ t \in (0,T), \quad (4)$$

$$y(x,0) = 0,\ x \in \Omega, \quad (5)$$

$$\frac{\partial y}{\partial t}(x,0) = u(x),\ x \in \Omega. \quad (6)$$

Here A is a symmetric uniformly elliptic operator

$$Ay(x) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial y}{\partial x_j}(x)\right) + a_0(x)y(x),$$

where $a_{ij} = a_{ji} \in C^1(\overline{\Omega})$, $a_0 \in C(\overline{\Omega})$, $a_0 \le 0$, and there exists such a constant $\mu > 0$ that $\sum_{i,j=1}^n a_{ij}(x)\xi_i\xi_j \ge \mu\sum_{i=1}^n \xi_i^2$ for each $x \in \overline{\Omega}$ and $\xi_1, \dots, \xi_n \in R$.

${}_0^C D_t^\alpha f(t)$ is a left-sided Caputo fractional derivative of order $\alpha > 0$:

$${}_0^C D_t^\alpha f(x,t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n f}{\partial \tau^n}(x,\tau)\,\mathrm{d}\tau,$$

where $n = [\alpha] + 1$, and $\Gamma(\cdot)$ is the Euler gamma function.

Based on the existence and uniqueness results from [1], we apply the conjugate gradient algorithm to the optimal control problem (1)-(6).

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METHODS OF ESTIMATION UNDER UNCERTAINTIES

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The following problem is considered: we have a vector of observations $\overline{\xi} = (\xi(0), \xi(1), \ldots, \xi(n))$ of the stochastic process $\xi(t)$, which has the following structure $\xi(t) = \xi_{x(t)}^{(t)}$. $x(t)$ denotes a homogeneous Markov chain with a discrete time $t = 0,1,\ldots$, finite set of states $E = \{1,2,\ldots,M\}$ and known transition probabilities $P = [p_{ij}]$, $i, j \in E$. The random variables $\xi_i^{(k)} \Leftrightarrow N(m_i, \sigma^2)$, $k = 0,1,\ldots$, $i \in E$, have a normal distribution. The following assumptions regarding for the set of random variables are made: $\xi_i^{(k)}$ are independent of each other for different $k = 0,1,\ldots$, are independent of the chain $x(t)$, the variance σ^2 of the distribution is known, and the mathematical expectations m_i, $i \in E$, are unknown. We also assume that we are dealing with a hidden Markov model of observations, and the values $(x(0),\ldots,x(n))$ of the chain are unknown. It is necessary to construct estimates $m*_i^{(n)} \approx m_i$, $i \in E$, of mathematical expectations. The paper shows that the consistent estimate $\overline{m}*^{(n)} = (m*_1^{(n)}, \ldots, m*_M^{(n)})$ of the vector $\overline{m} = (m_1, \ldots, m_M)$ is among the solutions of the system:

$$\sum_{s=1}^{M} \sum_{l=1}^{M} \pi_s p_{sl}^{(m)} x_s x_l = H_m^{(n)} \; ; \; H_m^{(n)} = 1/n \sum_{t=1}^{n} \xi(t)\xi(t+m) \; ; \; m = \overline{1,M} \; .$$

Where $p_{sl}^{(m)} = P\{x(t+m) = l / x(t) = s\}$ denotes the transition probability of a stochastic process $x(t)$ over m steps. In the case, where a random process $x(t)$ represents a mathematical model of a stochastic hierarchical structure [1], an explicit form of the solutions of this system has been found. In the work, a simulation model of a stochastic hierarchical structure $x(t)$ was built for $M = 3$. Using model data $\overline{\xi}$, the analysis and empirical verification of the obtained solutions were carried out.

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ON THE APPLICATION OF COMPUTERS TO NATURAL SCIENCE

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In the middle of the twentieth century, electronic computers began to be used to speed up the solution of problems with a large volume of calculations. With the development of information technology, it became possible to simulate such a human property as his intelligence. This task is complex and resource-intensive, requiring a whole range of characteristics from computing facilities. The main ones are: a large amount of RAM, since models simulating intelligence can include millions or billions of parameters; high-performance computing, since it is necessary to simulate the processes of training, recognition, planning, prediction; access to cloud infrastructure, since these resources provide the ability to train models on clusters and fast access to data from different sources; large data warehouses (Data Lakes, knowledge bases), since modeling requires knowledge, which is huge amounts of information; high I/O and network bandwidth for fast data exchange between modules in the case of simulating memory, perception, decision-making; support for multi-module systems for simulating attention, learning, emotions, consciousness.

Turning to the history of the emergence of computers, it should be noted that it was scientific research that was the main source of their development. Requirements were put forward for the calculation results: they had to be very accurate (with a large number of digits) and considered in a wide range. However, studies have shown that many concepts and models of natural processes at the present stage of science do not always reflect the full scope of natural phenomena and need to be refined and supplemented. Thus, mathematical models built on the basis of Newton's law are only an approximate "description" of natural phenomena; studies of such concepts as energy, force, body mass, distance in space are ongoing with the aim of further clarifying them. This indicates that the abstract mathematical basis should be revised, taking as a basis one that would allow obtaining more reliable information about nature. After all, the results of calculations obtained by applying inaccurate formulas and unreliable models with the help of modern information technologies will lead to erroneous actions, distorted conclusions and the departure of future research into a dead end.

RANDOM EVOLUTION ON THE REGENERATIVE PROCESS IN NONLINEAR APROXIMATION

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Let $x_1(t), x_2(t), ..., x_d(t)$ – independent regenerating processes specified in phase space (E, B). Then every of processes $x_i(t)$ moments of regeneration will be marked $\tau_i^{(k)}$.

Let's consider these processes on a time scale $\frac{t}{g(\varepsilon)}$ with normalizing factor $g(\varepsilon) \to 0$ when $\varepsilon \to 0$. Then

$$x_i^{(k)}\left(\frac{t}{g(\varepsilon)}\right) = x_i\left(\tau_i^{(k)} + \frac{t}{g(\varepsilon)}\right)$$

when

$$0 \le \frac{t}{g(\varepsilon)} < \tau_i^{(k+1)} - \tau_i^{(k)},$$

independent copies of the process $x_i\left(\frac{t}{g(\varepsilon)}\right)$ on the interval $[0; \tau_i^{(k+1)} - \tau_i^{(k)})$.

Let $\left(v_n\right)_{n=0}^{\infty}$ – homogeneous Markov chain with a finite set of states $\{1, 2, ..., d\}$. In addition, regenerating processes and the Markov chain are independent.

The purpose of the work is the asymptotic analysis of this process in nonlinear approximation

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SECURE SYSTEM FOR DATA STORAGE AND ANALYSIS OF BIOMEDICAL SIGNALS FROM HOLTER MONITORS AND ECG DEVICES

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Electrocardiography (ECG) is an important method for diagnosing cardiovascular diseases, allowing the detection of arrhythmias and other heart diseseas pathologies. ECG is widely used both in hospitals and in outpatient practice. However, traditional methods of reading and analyzing ECGs require significant involvement of medical personnel, which limits their effectiveness in large-scale or when patients are been treatened remotly via telemedicine solutions. In this regard, the automation of ECG signal acquisition, processing, and interpretation is becoming increasingly important, especially in the context of the vast development of telemedicine and digital medical services. A study was conducted [1], and a method for detecting R-peaks in ECG signals was developed and tested on recordings obtained from portable Holter devices taken from the MIT-BIH database [2]. An architecture was proposed for a software solution that allows integrating this method into the telemedicine workflow, both in real-time mode and in standard applications. To ensure compatibility with existing healthcare information systems and standardize data exchange, it is advisable to use the HL7 FHIR (Fast Healthcare Interoperability Resources) protocol. In addition to technical compatibility, it is important to comply with national legislation on personal data protection. To ensure the confidentiality and integrity of sensitive medical data, a set of security mechanisms has been implemented. By using pseudonymization, it is possible to avoid transmitting personal data together with biosignals, instead using unique patient identifiers. Data transmission is carried out using secure protocols (TLS 1.3), and critical records in the database can be encrypted at the field level. Special attention is given to integration with external analytical modules, such as for other medical data processing algorithms. For this purpose, a secure REST API with authentication and controlled access is used. The data structure is based on HL7 FHIR principles, ensuring compatibility with hospital IT systems. The proposed system meets the key requirements of national and international legislation — including GDPR or HIPAA [3].

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CLUSTERING IN GBDT IN THE CONTEXT OF BUILDING-TECHNICAL EXPERTISE PROCESS

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As a result of the russian invasion of Ukraine thousands of buildings have suffered partial or complete destruction. Given limited human resources and the urgent need of rapid and reliable buildings technical state assessment, there is a growing demand for automated systems to support the building-technical expertise process. One of the core tasks of such systems is to classify buildings by damage level based on a set of their input features.

In this context, gradient boosted decision trees (GBDT) is an effective classification method. They combine high accuracy, relatively low training data samples size, and interpretability [1], a crucial advantage when supporting expert assessments. However, conventional implementations rely on weak learners with strictly binary splits. This leads to deeper trees and reduces transparency in decision-making logic. This fact significantly limits the usability of the system from the expert point of view. This work proposes replacing binary splits with multi-way splits derived from clustering algorithms [2,3]. It was demonstrated how input features can be grouped into semantically meaningful clusters during weak tree construction. This allows us to get shallower decision trees, better handling of complex feature dependencies and more interpretable decision paths. However, picking the optimal algorithm is a highly domain-dependent task and thus it remains a subject for future

research.

The primary goal of this study is to demonstrate the value of clustering-based multi-way splits in weak learners within gradient boosted decision trees. Unlike traditional binary splits, the proposed approach potentially enhances both performance and interpretability in a critical real-world application – supporting the building-technical expertise process.

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MIXTURE OF ERGODIC MARKOV CHAINS

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Markov chains is a powerful tool for data analysis when it comes to sequence data modelling. With it help we can perform classification and prediction tasks.

This paper addresses how to represent an ergodic Markov chain as a linear combination of arbitrary ergodic Markov chains. Main assumption is the ergodicity of Markov chains of interest. Transition matrix of the represented Markov chain can be estimated using observed data and transition matrices of the Markov chains which are elements of the linear combination are given.

The methodology of finding the coefficient of the linear combinationusing properties of Ergodic theorem is presented in the paper.

INTEGRATION OF IMMERSIVE INTERFACES AND DIGITAL TWINS INTO SEMI-NATURAL MODELING SYSTEMS

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The introduction of immersive interfaces, digital twins, and the Internet of Things into semi-natural (hardware-in-the-loop) modeling systems opens up new opportunities for improving efficiency and safety across various industries. A digital twin of a complex product is a cyber-physical system in which a mathematical model receives physical data from a real object and influences it through feedback signals. The development of a modern digital twin typically unfolds in three sequential stages: Model-in-the-Loop (MiL), which involves the creation and testing of control algorithms and simulation models; Software-in-the-Loop (SiL), where the developed software interacts with the simulated environment; and Hardware-in-the-Loop (HiL), where the software is executed on the actual hardware platform and connected to the simulation model via telemetry. At each of these stages, the use of immersive technologies transforms the model into a visual, interactive environment where a human can intuitively perceive and manipulate system behavior. In the MiL stage, virtual reality (VR) helps eliminate design blind spots and allows multidisciplinary teams to collaboratively assess the product at full scale. At the SiL stage, VR becomes an interactive testbed for debugging and verifying software logic. At the HiL stage, a combination of VR and augmented reality (AR) turns real-time operational data into spatial guidance and training tools, enhancing both safety and predictability during the product's deployment and use. Technically, this integration is implemented through a three-layer architecture: a field data layer, a simulation and analytics layer, and a visual-immersive front-end. The integration of immersive interfaces with digital twins significantly reduces development time and accelerates every phase of the product lifecycle. It cuts both capital and operational costs by reducing reliance on physical prototypes and minimizing downtime during configuration. Product quality improves due to early detection of design flaws in the virtual environment and real-time integration of sensor data into the operator's interface. Personnel productivity increases during the operation phase, while extended reality enables safe, repeatable training scenarios without the risks associated with real equipment. Such integration of technologies reduces uncertainty in solving problems of high technical complexity.

INVESTIGATION OF THE STABILITY OF REACTION-DIFFUSION DYNAMIC SYSTEMS

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The investigation of the stability of dynamic systems is an extremely important task. The theory of input-to-state stability was developed in the works of Sontag [1] and was subsequently adapted to study the robust stability of systems [2], [3], [4]. Further research focused on extending this theory to the case of infinite-dimensional systems [5] and separately for systems known as reaction-diffusion systems [6].

In bounded domain $\Omega \subset \mathbb{R}^n$ we consider the next system of reaction-diffusion

$$\begin{cases} y_t = a\Delta y - f(y) + h(x) + d,\ x \in \Omega,\ t > 0, \\ y\big|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

where $y = y(t,x) = (y^1(t,x),...,y^N(t,x))$ is unknown vector function, $f = (f^1,...,f^N), h = (h^1,...,h^N)$ are given vector functions, a is real matrix of dimension $N \times N$ such that $\frac{1}{2}(a + a^*) \geq \mu I, \quad \mu > 0,$ $d = (d^1,...,d^N)$ is input disturbance.

It is assumed that the function $h \in (L^2(\Omega))^N$, $g(y) = (g^{ij})_{i,j=1}^N$ is a given matrix-valued function, and there exist constants $C_1, C_2, C_3, C_4 > 0,$ $\gamma_i > 0,\ p_i \geq 2,$ where $i = \overline{1,N},$ such that for any vectors $v \in \mathbb{R}^N$, the following properties hold:

$$\sum_{i=1}^N \left| f^i(v) \right|^{\frac{p_i}{p_i - 1}} \leq C_1 \left(1 + \sum_{i=1}^N \left| v^i \right|^{p_i} \right), \ \sum_{i=1}^N f^i(v) v^i \geq \sum_{i=1}^N \gamma_i \left| v^i \right|^{p_i} - C_2, \tag{2}$$

$$\left\| g(v) \right\|^2 := \sum_{i,j=1}^N \left| g^{ij}(v) \right|^2 \leq C_3, \quad Df(v) \geq C_4. \tag{3}$$

Let $H = (L^2(\Omega))^N$, $\left\| d \right\|_\infty = ess\sup_{t \geq 0} \left| d(t) \right|_H$, Θ be a global attractor generated by the semigroup $S : \mathbb{R}_+ \times H \mapsto H,$ where $S(t, y_0) = \{ y(t) \mid y(\cdot)$ *is a global weak solution of* (1) *with* $d \equiv 0, y(0) = y_0 \}$, $\left\| y \right\|_\Theta = \inf_{\theta \in \Theta} \left\| y - \theta \right\|_H$, and the following well-known comparison function classes [7]: $\mathcal{K} = \{ \gamma \in \mathbb{C}(\mathbb{R}_1) \mid \gamma(0) = 0, \gamma$ *is strictly increasing* $\}$, $\mathcal{KL} = \{ \beta \in \mathbb{C}(\mathbb{R}_+ \times \mathbb{R}_+) \mid \beta(\cdot, t) \in \mathcal{K}, \beta(s,\cdot) \searrow 0 \}$.

In the case when $d \equiv 0,$ problem (1) is autonomous, and thus, in the

phase space of the system, there exists a stable equilibrium position – a global attractor Θ, which is a compact, invariant, uniformly attracting set of states. However, in the presence of a non-zero perturbation d, problem (1) is no longer autonomous, and to estimate the deviation of its trajectories from the equilibrium position, two methods are applied: 1) for local estimation in the sense of ISS (input-to-state stability) – the theory of multivalued semiflows and Lyapunov-type functions; 2) for global estimation in the sense of AG (attracting globally) – the theory of multivalued semiprocesses, the theory of uniform attractors, and the result on the upper semicontinuous dependence on the parameter.

Theorem. Problem (1)

(i) [9] with $f \in C(\mathbb{R}^N;\mathbb{R}^N)$, $d = d(t,x) \in L^\infty(\mathbb{R}_+, H)$, and under the fulfillment of conditions (2) possesses the asymptotic gain (AG) property with respect to the attractor Θ, i.e., there exist $R_0 > 0, \gamma \in \mathcal{K}$ such that for all $y_0 \in H, \forall \|d\|_\infty \le R_0$

$$\overline{\lim}_{t\to\infty} \|y(t, y_0, d)\|_\Theta \le \gamma(\|d\|_\infty);$$

(ii) [10] with $f \in C^1(\mathbb{R}^N;\mathbb{R}^N)$, $d = g(y)d(t), d(t) \in L^\infty(\mathbb{R}_+, H)$, and under the fulfillment of conditions (2), (3), possesses the local input-to-state stability (local ISS) property with respect to the attractor Θ, i.e., there exist $r > 0, \beta \in \mathcal{KL}$, and $\gamma \in \mathcal{K}$ such that for all $\|y_0\|_H \le r$, $\forall \|d\|_\infty \le r$

$$\|y(t, y_0, d)\|_\Theta \le \beta(\|y_0\|_\Theta, t) + \gamma(\|d\|_\infty), \quad t \ge 0.$$

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AN ITERATIVE TWO-SIDED METHOD FOR SOLVING INTEGRAL EQUATIONS

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Based on the results on functional interval mathematics [1], an iterative method is developed for computing two-sided approximations to the solution of the integral equation

$$y(x) = f(x) + \int_a^x k(x, s, y(s))\, ds \qquad (1)$$

where $f(x)$ is a known function, $y(x)$ is a wanted function, and $x \in [a, b]$ denotes the given interval, which is guaranteed to contain the solution of the equation (1).The method implements the ideas set out in [2]. For this purpose, the equation (1) is preliminarily interpreted as a general initial value problem.

Let the function $y(x)$ be twice continuously differentiable on the interval $[a, b]$. Then the initial two-sided approximation to the solution of the equation (1) is constructed in the form of a linear functional interval $Y([a,b])$. The point $\tilde{b}$ is defined taking into account that the double inequality $\underline{k}' \le y'(x) \le \bar{k}'$ shall be satisfied on the interval $[a, \tilde{b}]$, where $\underline{k}', \bar{k}'$ are constants. After that, at any point $b \in [a, \tilde{b}]$, the previously constructed piecewise-parabolic functional intervals are successively contracted to the point representing the value of the solution $y(x)$ at this point.

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EFFICIENT DATA STORAGE FORMATS FOR DATASET DISTILLATION

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Dataset Distillation aims to find a synthetic dataset $S = \{X_s, Y_s\}$ based on the original dataset $D = \{X, Y\}$ so that $|S| \ll |D|$ and model trained on the S gives the same performance as if it were trained on D. Among the existing methods of Dataset Distillation, those that generate a dataset based on the original one, are the most promising.

The standard format for deep learning model weights is **float32**, while original images are most often stored as **uint8**. Therefore, when an original image is fed into the model, it contains four times less information than it potentially could. Moreover, to maintain consistency, distilled data also undergoes casting from float32 to uint8, thereby losing part of the information it originally contained. We attempted to evaluate this loss.

For evaluation, we selected two dataset distillation approaches: *Trajectory Matching*-based **DATM [1]** and *Stable Diffusion*-based **CaO2 [2]**. As a test dataset, we chose **ImageNetWoof**, while **ResNet18** was selected as the model. The evaluation was carried out on the test split of the original dataset.

	DATM [1]	CaO2 [2]	Original dataset
uint8	66.5%	**76.20%**	92.16%
float32	**69.39%**	69.92%	-

The results indicate the promise of preserving images in the **float32** format for approaches such as *Trajectory Matching*. However, for *Stable Diffusion*–based approaches, we observe the opposite outcome.

The reason for this difference may lie in the underlying principle of dataset distillation employed by each method. In the case of **DATM**, the target model directly participates in the data distillation process, which allows the resulting dataset to adapt to working with **float32**. In contrast, **CaO2** does not interact with the model directly and does not generate redundant information that could only be preserved in the **float32** format.

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USING A GUARANTEED APPROACH TO FIND RMS ESTIMATES OF LINEAR FUNCTIONALS FROM SPECTRAL FUNCTIONS

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Let at points $t_1 < t_2 < ... < t_N$ and on the interval $[s_1, s_2]$ the realization of a scalar random process

$$y(t) = SpC^T(t)S(t) + \eta(t), \qquad (1)$$

is observed, where the matrix functions $C(t), S(t)$ are continuous on $[s_1, s_2]$ and take on values in the space $H_{m,n}$ of real matrices of size $m \times n$ with the scalar product $\langle A_1, A_2 \rangle = Sp\left(A_1 A_2^T\right)$, $A_j \in H_{m,n}$, $j \in \overline{1,2}$; $Sp(\cdot)$ is a trace of a square matrix; $\eta(t)$ are implementation of a random process with zero mean continuous in the mean square on the interval $[s_1, s_2]$; $\eta(t_k)$, $k \in \overline{1,N}$ are random variables with zero mean. Suppose that $S(t)$ has the form $S(t) = \int_{R^1} \exp(i\omega t)\tilde{S}(\omega)d\omega$, where the matrix complex-valued function $\tilde{S}(\omega)$ of $m \times n$ size is continuous on every finite interval and integrated with the square in R^1. Let the matrix $\tilde{S}(\omega)$ be unknown and belong to a certain set G, and the unknown correlation function $R(t,s)$ belong to the set V,

$$V \triangleq \left\{ R(t,s) : \int_{s_1}^{s_2} R(t,t)dt < \infty; E\eta^2(t_k) < \infty, k \in \overline{1,N} \right\}.$$

Let $l_j\left(\tilde{S}\right), j = \overline{1,r},$ be linear functionals of the form $l_j\left(\tilde{S}\right) \triangleq \mathrm{Re}\int_{R^1} \left\langle L_j(\omega), \tilde{S}(\omega) \right\rangle d\omega, j \in \overline{1,r}$ (here $L_j(\omega), j \in \overline{1,r}$ are known matrix complex-valued functions), which are integrated with the square

in R^1, and f is the vector $f \triangleq \left(l_1\left(\tilde{S}\right),...,l_r\left(\tilde{S}\right)\right)^T$.

Definition 1. We call the linear estimate of the vector f

$$\hat{f} = \sum_{k=1}^{N} z^k y(t_k) + \int_{s_1}^{s_2} z(t) y(t) dt + b, \qquad (2)$$

where b and $z^k, k \in \overline{1,N}$ are vectors from the space R^r; $z(t)$ is a vector function that takes on value in the space R^r and for which the condition $\int_{s_1}^{s_2} \left|z(t)\right|^2 dt < \infty$; $y(t_k), k \in \overline{1,N}$ are satisfied and $y(t), t \in [s_1, s_2]$ are given by formula (1).

Let (τ_1, τ_2) be the smallest interval that contains the points $t_1,...,t_N$ and the interval $[s_1, s_2]$. Next, we will write the linear estimate (2) in the form $\hat{f} = \int_{\tau_1}^{\tau_2} u(t) y(t) \mu(dt) + b$, where $u(t)$ is a measurable vector function on (s_1, s_2) for which the inequality $\int_{\tau_1}^{\tau_2} \left|u(t)\right|^2 dt < \infty$ hold; $\mu(\cdot) = \mu_1(\cdot) + \mu_2(\cdot)$, $\mu_1(\cdot)$ the Lebesgue measure on (τ_1, τ_2), $\mu_2(\cdot) = \sum_{k=1}^{N} \mu_2^{(k)}(\cdot)$, $\mu_2^{(k)}(\cdot)$ is concentrated at the points $t_k, k \in \overline{1,N}$, and such that the equalities $\int_{\tau_1}^{\tau_2} \psi(t) \mu_2^{(k)}(dt) = \psi(t_k)$, $k \in \overline{1,N}$ hold for any continuous function $\psi(\cdot)$ on the interval (τ_1, τ_2).

Definition 2. The guaranteed mean square estimate (GMSE) of the vector f is the vector $\hat{\hat{f}}$ of the form: $\hat{\hat{f}} = \int_{\tau_1}^{\tau_2} \hat{u}(t) y(t) \mu(dt) + \hat{b}$, where $\left(\hat{u}, \hat{b}\right) \in A \arg\min_{u,b} \sigma_g(u,b)$, $\sigma_g(u,b) \triangleq \left\{\sup_{G,V} E\left|f - \hat{f}\right|^2\right\}^{1/2}$, and the value $\sigma_g\left(\hat{u}, \hat{b}\right)$ is the guaranteed mean square error of the estimate $\hat{\hat{f}}$.

The task is to find a guaranteed root-mean-square estimate of linear functionals from the spectral functions of signals and its error. We assume that the following inclusions are fulfilled: $G_1 \subseteq G \subseteq G_2, V_1 \subseteq V \subseteq V_2$, where $G_j = \left\{\tilde{S}(\omega) : \int_{R^1} q_j^2(\omega) \left\|\tilde{S}(\omega)\right\|^2 d\omega \le 1\right\}$, $j \in \overline{1,2}$; $V_j = \left\{R(t,s) : \int_{\tau_1}^{\tau_2} \delta_j^2(t) R(t,t) \mu(dt) \le 1\right\}$, $j \in \overline{1,2}$; $q_j^2(\omega), q_j^{-2}(\omega), j \in \overline{1,2}$ are known continuous positive functions on an arbitrary bounded interval;

$\delta_j^2(t), \delta_j^{-2}(t), j \in \overline{1,2}$ are known continuous positive functions on the interval $[\tau_1, \tau_2]$.

Theorem 1. The guaranteed estimate of the functional $l_1(\tilde{S}) \triangleq \mathrm{Re}\int_{R^1} \langle L_1(\omega), \tilde{S}(\omega) \rangle d\omega$ can be written in the form $\hat{\hat{l}}_1(\tilde{S}) = \mathrm{Re}\int_{R^1} \langle L_1(\omega), \hat{S}(\omega) \rangle d\omega$, where the matrix function $\hat{S}(\omega)$ has the form $\hat{S}(\omega) = \int_{\tau_1}^{\tau_2} V(\omega,\tau)\theta(\tau)\mu(d\tau)$, the matrix function $V(\omega,\tau)$ is given by the formula $V(\omega,\tau) \triangleq q^{-2}(\omega)\int_{\tau_1}^{\tau_2} C(\tau)\exp(-i\omega t)\mu(dt)$, and the scalar function $\theta(\tau), \tau \in [\tau_1, \tau_2]$, is found as a solution to the integral equation

$$\delta_1^{-2}\theta(\tau) + \int_{\tau_1}^{\tau_2} \theta(t)\langle C(t), C(\tau)\rangle R_1(t-\tau)\mu(dt) = y(\tau),$$

$$R_1(\tau) \triangleq \int_{R^1} q_1^{-2}(\omega)\cos(\tau\omega)d\omega.$$

Let us denote by W a linear operator acting in the Hilbert space $L_2(s_1, s_2)$ of functions integrated with a square on the interval (s_1, s_2) according to the rule $(Wv)(t) = \delta_1^{-2}(t) + \int_{s_1}^{s_2} \langle C(t), C(\tau)\rangle R_1(t-\tau)v(\tau)d\tau$ and $\hat{u}_0(t), t \in (s_1, s_2)$ is a solution of the operator equation $(W\hat{u}_0)(t) = \mathrm{Re}\langle L(t), C(t)\rangle$, and by $g_j(t), j \in \overline{1,N}$ we denote the solutions of the system of equations $(Wg_j)(t) = \langle C(t_j), C(t)\rangle R_1(t_j - t), j \in \overline{1,N}$. Let the quantities $\overline{u}_k, k \in \overline{1,N}$ be solutions of the system of linear algebraic equations $\delta_{1k}^{-2}\overline{u}_k + \sum_{j=1}^{N} a_{kj}\overline{u}_j = \overline{f}_k, k \in \overline{1,N}$, where $\delta_{1k}^{-2} = \delta_1^{-2}(t_k)$;

$$\overline{f}_k \triangleq \mathrm{Re}\langle L(t_k), C(t_k)\rangle - \int_{s_1}^{s_2} \hat{u}_0(\tau)\langle C(t_k), C(\tau)\rangle R_1(t_k - \tau)d\tau;$$

$$a_{kj} = \langle C(t_k), C(t_j)\rangle R_1(t_k - t_j) - \int_{s_1}^{s_2} g_j(\tau)\langle C(t_k), C(\tau)\rangle R_1(t_k - \tau)d\tau.$$

Theorem 2. The GSME of the functional $l_1(\tilde{S})$ has the form

$$\hat{\hat{l}}_1(\tilde{S}) = \sum_{k=1}^{N} \overline{u}_k y_k + \int_{s_1}^{s_2} \hat{u}_0(t)y(t)dt - \sum_{j=1}^{N} \overline{u}_j \int_{s_1}^{s_2} g_j(t)y(t)dt.$$

We conducted computer experiments to calculate estimates of linear functionals for a test example: scalar values $y_k = S_1(t_k) + \chi t_k S_2(t_k) + \eta_k$, $k \in \overline{1,N}$ are observed, where $t_k = 1 + \delta \cdot k, k \in \overline{1,N}$; signals $S_p(t), p \in \overline{1,2}$

are given by formulas

$$S_1(t)=\begin{cases} t-1,\ t\in[1;2], \\ -t+3,\ t\in[2;3], \\ 0,\ t\in(-\infty;1)\cup(3;\infty), \end{cases} \qquad S_2(t)=\begin{cases} 2t-6,\ t\in[3;4], \\ -2t+10,\ t\in[4;5], \\ 0,\ t\in(-\infty;3)\cup(5;\infty), \end{cases}$$

random variables $\eta_k, k\in\overline{1,N}$ satisfy the conditions $E\eta_k=0, k\in\overline{1,N}$; $E\eta_k\eta_j=\delta_{kj}$, $k\in\overline{1,N}$, $j\in\overline{1,N}$, δ_{kj} is the Kronecker symbol; $E\eta_k^2=\sigma_k^2, k\in\overline{1,N}$; parameters $\delta>0$ and $\chi>0$ are small numbers.

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