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OLEKSANDR NAKONECHNYI'S 80th ANNIVERSARY!

The renowned scientist, an honored member of the education community, Laureate of the Ukrainian State Prize in Science and Technology, Distinguished Professor of Kyiv University of Taras Shevchenko, Doctor of Science, Professor Oleksandr G. Nakonechnyi is celebrating his 80th anniversary in January 9, 2026.

All his life has been dedicated to Taras Shevchenko National University of Kyiv. Oleksandr has been working at the Faculty of Cybernetics since 1969; here from a software engineer he reached the status of a Professor, the head of the department.

Both teaching and doing scientific research, he successfully defended his PhD degree in physics and mathematics in 1973; the topic of the dissertation was "Moment functions of senior orders of random processes" and his scientific adviser was the Academician of the Ukrainian Academy of Sciences Anatoliy Skorokhod. Oleksandr Nakonechnyi defended his doctoral degree with "Minimax estimation of functionals of equation solutions with partial derivatives" in 1981.

Professor Nakonechnyi dedicates much efforts and energy to education, scientific and methodical activities. He was the head of the department of system analysis and decision-making theory since 1996 to 2025, which is the base of systems analysis. As many as seven Doctoral and twenty PhD degrees were defended under the scientific supervision of Professor Nakonechnyi. He is the author of more than 420 scientific works; among them are ten monographs and eleven training manuals. Professor Nakonechnyi is the President of the Ukrainian Academy of Sciences of Higher School of Ukraine.

Professor Nakonechnyi is in the prime of his life when it comes to scientific research. His scientific and educational activities inspire colleagues and students, ensuring respect of both Ukrainian and foreign scientists. The PDMU Organizing Committee congratulates the Program Committee Chairman Professor Nakonechnyi and warmly wishes him good health, happiness and more scientific achievements for the benefit of Ukraine.

GURAM CHACHANIDZE
80th ANNIVERSARY

Guram Chachanidze is a Doctor of Sciences, Professor; Director of the European Education Center; UNESCO Expert in the field of language development in cyberspace; President of the Global Union of Scientists for Peace (GUSP) for Georgia; President of the International Academy of Sciences of Education Informatization; Academician of the International Informatization Academy of the UN; Academician of the International Academy of Science and Innovation Technologies; He is Founder and President of the Georgian Sciences and Society Development Foundation "Intellect"; Founder, Editor in Chief and Publisher of the International Scientific Journal "Intellect".

Mr. Guram holds numerous awards and certificates: He was awarded the State Award of Georgia - the Order of Honor; Honorary Doctor of Zaman University of Cambodia for the great scientific researches in the field of Informatics; the first degree Diploma of "SofTEC 99"; Medal of Honor since 1998; Ilia Chavchavadze Award and Diploma in the field of Interdisciplinary Methodology by the Georgian-European Institute of Paris; he is awarded the Medal by Ukrainian of Yaroslav the Wise and the jubilee medal of the Academy of Sciences of the Higher School of Ukraine.

For many years, Mr. Guram was the Head of Dissertation Council at I. Gogebashvili Telavi State University; Supervisor of the Doctoral and Master's Program of "Computer" and "Education Sciences"; Head of the Department of Informatics, Computational Methods and Method of Teaching Mathematics at Kutaisi State University; Head of the Department of Information Technology, Analysis and Forecasting at the Kutaisi City Hall. Mr. Guram has organized numerous of international scientific symposiums with the participation of European and worldwide scientists in the field of computer technologies and education sciences; he was supervising about 70 post-graduate and doctoral dissertation. He, together with Professor Oleksandr Nakonechnyi, is a member of the International Program Committee of the Scientific Conference on Problems of Decision Making under Uncertainty (PDMU). He edited about 200 scientific works, manuals, guides, monography in Georgia and internationally.

The Organizing Committee of the PDMU Conference sincerely congratulates Mr. G. Chachanidze on his anniversary!

IBRAIM DIDMANIDZE
70th ANNIVERSARY

Ibraim Didmanidze, Doctor of Sciences, Professor at Batumi Shota Rustaveli State University. Head of the Information Technologies division at the Center for Languages and Information Technologies, Faculty of Exact Sciences and Education, Batumi Shota Rustaveli State University; Academician of the Georgian Engineering Academy; Academician of the International Academy of Sciences of information of Education; Academician of the International Academy of Science and Innovation Technologies; Expert for the QS World University Rankings; Chair of the Adjara Republican Organization of the Georgian Pedagogical Union named after Iakob Gogebashvili.

Mr. Ibraim Didmanidze has received numerous awards and certificates. Among them, he is a laureate of the Iakob Gogebashvili Gold Medal Prize and has been awarded the Yaroslav the Wise Medal of Ukraine and the Jubilee Medal of the Academy of Sciences of Higher Education of Ukraine.

For many years, Mr. Ibraim Didmanidze held a number of senior academic and administrative positions, including: Head of the Computing Center, Dean, and Rector of the Batumi Polytechnic Institute of the Georgian Technical University; Head of the Department of Computer Sciences at Batumi Shota Rustaveli State University; Professor at Iakob Gogebashvili Telavi State University; Professor at St. Tbel Abuseridze Teaching University of the Georgian Patriarchate.

Mr. Ibraim Didmanidze has participated in numerous international scientific symposia and conferences. He has served as Chair or Member of the International Program Committees, International Organizing Committees, and Local Organizing Committees of numerous international scientific and practical conferences. He has also supervised more than 50 master's theses and doctoral dissertations.

He has published more than 180 scientific works, including textbooks and monographs. He is a member of the editorial boards of numerous international scientific journals.

The Organizing Committee of the PDMU Conference sincerely congratulates Mr. Ibraim Didmanidze on his anniversary!

JAROSLAV MICHÁLEK
80th ANNIVERSARY

On July 8, 2026 Jaroslav Michálek, long term employee and Associate Professor of the University of Defence, Brno celebrated his 80th birthday anniversary.

In 1968 he graduated from the Faculty of Mathematics and Physics, Charles University in Prague, and later on, he received the degrees RNDr and PhD there. After the graduation, he joined the Faculty of Science, Masaryk University in Brno where he as well worked as the Vice-Dean for Science and Research in 1990-91. Then based on his initiative the Department of Applied Mathematics was established there and he was the Head of the Department for 5 years. That time, he participated in preparation of the inter-faculty Master´s degree program in Mathematics and Economics, and together with colleagues from the Faculty of Mechanical Engineering, Brno University of Technology, they prepared the interdisciplinary Master´s degree program in Mathematical Engineering.

In 1994 he took the opportunity to work abroad (Zimbabwe, Botswana, Malta) and shortly after his return from abroad he joined Faculty of Mechanical Engineering, where he served as the head of the Department of Statistics and Optimization. Simultaneously, since 1990 he has worked at the University of Defence in Brno.

J. Michálek supervised 10 PHD students, four of them have already been habilitated. He has published more than 120 scientific papers and he also worked as an external examiner at foreign universities.

J. Michálek worked for a long time for the PDMU´s Programme Committee and he collaborated with prof. O. G. Nakonechnyi whom he met during his first internship in Brno in 1976.

In 2026, he terminated his permanent employment and is now actively engaged only in occasional work at the University of Defense and in supervising doctoral students.

The Organizing Committee of the PDMU Conference sincerely congratulates Mr. Jaroslav Michálek on his anniversary!

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IMPLEMENTING FORENSIC LINGUISTICS AS AN INTERDISCIPLINARY UNIVERSITY COURSE

Neli Akhvlediani, Ibraim Didmanidze, Ia Turmanidze
Batumi Shota Rustaveli State University, Georgia

The expansion of digital communication, the proliferation of anonymous and threatening texts, and the growing complexity of interrogation, confession, and online discourse have revealed the limitations of purely legal or intuitive approaches to textual interpretation.

The proposed framework is based on systematic cooperation between linguists and legal scholars, where disciplinary roles remain functionally distinct: legal professionals formulate juridical questions, while linguists analyze the same materials as linguistic evidence through empirically controlled methods.

In contemporary security environments, forensic linguistic analysis contributes to intelligence practices, authorship attribution, and the early detection of linguistically encoded threats. Within military education, forensic linguistics extends beyond post-factum evidential analysis and functions as a preventive analytical tool capable of identifying linguistic indicators of potential risk in digital communication.

Furthermore, forensic methodological approaches support the analysis of cyber discourse and the decoding of adversarial propaganda narratives, demonstrating the broader relevance of linguistic expertise in security-related contexts. The proposed curriculum is based on the analysis of authentic and pedagogically adapted case materials, including threatening and anonymous communication, disputed authorship texts, interrogation discourse, digital interaction, and propagandistic language. A central pedagogical objective is to transform linguistic theory into transferable analytical competencies, enabling students to apply linguistic expertise across legal, institutional, and security-oriented domains.

The paper concludes that the introduction of forensic linguistics as an interdisciplinary university course represents a timely and sustainable response to contemporary communicative and security challenges.

THE PEDAGOGICAL TRANSFORMATION OF ARTIFICIAL INTELLIGENCE IN THE FOREIGN LANGUAGE TEACHING PROCESS

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Traditionally, foreign language teaching has been based on the transmission of knowledge and the provision of feedback derived from the information acquired by students. However, the rapid development of artificial intelligence (AI) has introduced new challenges to the educational process. Increasingly, students complete assignments without engaging in their own cognitive effort, instead copying AI-generated responses, a practice that contradicts the principles of academic integrity.

The aim of this study is to demonstrate that artificial intelligence can be employed not only as a tool for obtaining ready-made answers but also as an instrument for fostering analysis, critical thinking, and language development.

The study is based on a mixed-methods methodology and constitutes a small-scale experimental case study integrating both quantitative and qualitative analyses. Data were collected from students studying the Turkish language at Batumi Shota Rustaveli State University in accordance with the principles of action research. The intervention involved the critical editing of AI-generated texts, communicative interaction with AI, and the application of the **flipped learning** approach, in which students themselves evaluated and explained the responses generated by AI.

The findings demonstrate that the thoughtful and purposeful integration of AI into the teaching and learning process is more effective than prohibiting its use. Such an approach promotes the development of students' critical and metacognitive thinking, strengthens their linguistic autonomy, and enhances their sense of academic responsibility. Accordingly, this paper proposes a practical pedagogical model that presents artificial intelligence not as a source of academic dishonesty but as an effective tool for linguistic and cognitive development.

RELIABLE FINITE ELEMENT IMPLEMENTATIONS WITH DEEP LEARNING

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Finite element and boundary element methods and their combinations are the basis of the proposed adaptive algorithms for solving problems of the axisymmetric theory of elasticity. The construction and implementation of adaptive approaches is one of the ways to achieve this goal. We have proposed an h-adaptive approach for solving problems of the theory of elasticity, which is based on different orders of accuracy obtained using FEM and indirect boundary element method (IBEM) schemes based on mortar approximations for constructing nonconforming discretizations. In [1], training data for a neural network are generated by uniform domain refinement. In our case we use local adaptive mesh refinement to generate training solutions for a predictor that maps data from a coarse mesh to stresses from a locally refined solution, reducing dataset generation cost.

At Gauss points, FEM stresses are compared with stresses obtained by an IBEM. Refinement is driven by the elementwise relative indicator:

$$\eta_e = \frac{\frac{1}{\sqrt{|\Omega_e|}}\sqrt{\int_{\Omega_e}\sum_{i=r,\phi,z}(\sigma_{ii}^F-\sigma_{ii}^B)^2+2(\sigma_{rz}^F-\sigma_{rz}^B)^2 d\Omega}}{\frac{1}{\sqrt{|\Omega|}}\sqrt{\int_{\Omega}\sum_{i=r,\phi,z}(\sigma_{ii}^B)^2+2(\sigma_{rz}^B)^2 d\Omega}}.$$

Elements above a prescribed threshold are subdivided, with mortar constraints enforcing weak continuity of displacement across nonconforming interfaces. Repeating the adaptive cycle yields a nonuniform mesh hierarchy with refinement concentrated where the discrepancy is high.

Each sample pairs coarse-mesh information around a target point with the corresponding stress from a locally refined FEM solution. A feed-forward neural network learns the correction. Training data are generated for different test cases, with refinement concentrated near loaded segments, reentrant corners, and high stress gradients. The goal is a surrogate model that reproduces stresses from locally refined FEM

solutions using coarse-mesh data, without performing every adaptive cycle.

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CONVERGENCE OF BRANCHING STOCHASTIC PROCESSES WITH MANY TYPES OF PARTICLES AND EMIGRATION

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Branching processes with a single particle type constitute a special case of Markov processes with a countable state space. Branching processes with a finite number of particle types are a particular case of Markov processes with a countable set of states. However, whereas in the single-type case the states are naturally indexed by the non-negative integers representing the total number of particles, in the more general multi-type case the state of the process is specified by the vector $\alpha = (\alpha_1, \ldots, \alpha_n)$, whose coordinates are non-negative integers.

In this work, we study the convergence of a sequence of multi-type branching processes with emigration.

Let $\xi_n(\varepsilon) = \left(\xi_n^1(\varepsilon), \ldots, \xi_n^d(\varepsilon)\right)$, $n = 0,1,2,\ldots$ be a sequence of discrete time branching processes with d-types of particle and generating function $F_\varepsilon(S) = \left(F_\varepsilon^1(s), ..., F_\varepsilon^d(s)\right)$, where ε - ceries parameter, $s = (s_1, \ldots, s_j)$, $F_\varepsilon^j(s) = M_j s_1^{\xi_1^1(\varepsilon)} \ldots s_d^{\xi_1^d(\varepsilon)}$. Here ξ_n^j is interpreted as a number of particles j-type in n-th generation and M_j is conditional

expectation on condition that at the beginning there was only one particle of j-th type.

Let $Q(s)$ be generating function of emigrating particles.

The average number of j-type desendants from one particle of i-type denote by $a_{ij}(\varepsilon)$, i.e.

$$a_{ij}(\varepsilon) = M\left[\xi_1^j(\varepsilon)\middle|\xi_0(\varepsilon) = e_i\right], \quad e_i = \left(\underbrace{0,...,0}_{i-1},1,\underbrace{0,...,0}_{d-i}\right)$$

Denote $A(\varepsilon) = \left\|a_{ij}\right\|_{i,j=1}^{d}$ and suppose that the following expansion holds:

$$A(\varepsilon) = E + \varepsilon C + o(\varepsilon),$$

where $E = \left\|\delta_{ij}\right\|_{i,j=1}^{d}$, is the identity matrix, and $C = \left\|c_{ij}\right\|_{i,j=1}^{d}$ is a matrix of constants c_{ij}.

Under this expansion and certain conditions imposed on the generating functions $F_\varepsilon(s)$ and $Q(s)$ limit theorems on the convergence of branching stochastic processes with many types of particles and emigration have been obtained.

ON THE PROBLEM OF İDENTİFYİNG A LİNE SOURCE FOR A THİRD-ORDER HYPERBOLİC EQUATİON WITH NONLOCAL TIME-INTEGRAL CONDITIONS OF THE SECOND KIND

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Let $D_T = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$. Next, let $\varphi_0(x)$, $\varphi_1(x)$, $\varphi_2(x)$, $\omega(x)$, $h_1(t), h_2(t)$, $f(x,t)$, $g(x,t)$, $p_0(t)$, $p_1(t), p_2(t)$ are given functions.

Consider the following inverse boundary value problem: It is

required to find the triple $\{u(x,t),a(t),b(t)\}$ of the functions $u(x,t),a(t),b(t)$ related by the equation [1]:

$$u_{ttt}(x,t)-u_{txx}(x,t)+u_{tt}(x,t)-\alpha u_{xx}(x,t)= \\ =a(t)u(x,t)+b(t)g(x,t)+f(x,t) \tag{1}$$

under the following conditions:

$$u(x,0)=\int_0^T p_0(t)u(x,t)dt+\varphi_0(x),\quad u_t(x,0)=\int_0^T p_1(t)u(x,t)dt+\varphi_1(x), \\ u_{tt}(x,0)=\int_0^T p_2(t)u(x,t)dt+\varphi_2(x)\ \ (0\le x\le 1) \tag{2}$$

$$u_x(0,t)=u(1,t)=0\ \ (0\le t\le T), \tag{3}$$

$$\int_0^1 \omega(x)u(x,t)dx=h_1(t)\ \ (0\le t\le T) \tag{4}$$

$$u(0,t)=h_2(t)\quad (0\le t\le T), \tag{5}$$

where $0<\alpha<1$ are given number.

Definition 1. The quadruple $\{u(x,t),a(t),b(t)\}$ is said to be a classical solution of the inverse boundary value problem (1)-(5), if the following conditions are satisfied: the function $u(x,t,)\in \tilde{C}^{2,3}(D_T)$ $a(t)\in C[0,T]$, $b(t)\in C[0,T]$, satisfying equation (1) in D_T , condition (2) in $[0,1]$, continuous (3) - (5) in $[0,T]$.
where

$$\tilde{C}^{2,3}(D_T)=\{u(x,t):u(x,t),u_{xx}(x,t),u_{txx}(x,t),u_{ttt}(x,t)\in C(D_T)\}.$$

In this work, we finally prove the unique existence of a classical solution of the problem under consideration.,

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EXTENDING THE APPLICABILITY OF A PARAMETRIC SIXTH-ORDER CONVERGENT METHOD FOR SOLVING NONLINEAR EQUATIONS

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Let X and Y be Banach spaces, and let Ω be a convex and nonempty subset of X. We consider a nonlinear equation $F(x)=0$, where $F:\Omega\subset X\to Y$ is a Fréchet-differentiable operator. For solving such equations and systems of equations, a highly efficient parametric family of three-step methods with sixth order of convergence was introduced in [1]. For an initial approximation $x_0\in\Omega$ and each $n=0,1,2,\ldots,$ it is defined by

$$y_n = x_n - \frac{2}{3}F'(x_n)^{-1}F(x_n),\ z_n = x_n - A_n^{-1}B_nF'(x_n)^{-1}F(x_n),$$

$$x_{n+1} = z_n - \Big[(a+\tfrac{5}{2})I + F'(x_n)^{-1}F'(y_n)\big((-\tfrac{3}{2}-2a)I + aF'(x_n)^{-1}F'(y_n)\big)\Big]F'(x_n)^{-1}F(z_n),$$

where $H_n = F'(x_n)^{-1}F'(y_n)$,

$$A_n = 2\left(q_5I - 3q_6H_n + 9q_7H_n^2 - 27q_8H_n^3\right),$$

$$B_n = q_1I + 3q_2H_n + 9q_3H_n^2 - 27q_4H_n^3,$$

and $q_1,\ldots,q_8$ are real polynomials in a parameter $p\in\mathbb{R}$. The analysis in [1] was carried out in $\mathbb{R}^m$ using Taylor series expansions and was limited by its reliance on derivatives up to order six, although only the first derivative appears in the method. We extend the applicability of the method to the Banach space setting by proving convergence under weaker assumptions, using only generalized continuity conditions on the first Fréchet derivative. Our work provides a complete local and semi-local analysis: computable radii of the convergence ball and error bounds are derived, and the semi-local theory, based on majorizing scalar sequences, guarantees convergence without prior knowledge of a solution; the domain of uniqueness is also found. Numerical

experiments validate our theoretical findings and show that the method converges even when the standard differentiability conditions are violated.

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PRIORITISING MULTI-STEP ATTACK PATHS IN CONTAINERIZED CLUSTERS

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Containerized platforms support many public and business-critical services, but their security cannot be assessed as a simple list of isolated vulnerabilities. A real attack may pass through workloads, service accounts, orchestration APIs, cloud identities, worker nodes, and the control plane. The value of each step depends on the privileges already obtained, trust relations, the current cluster configuration, and the information accumulated by the attacker. A previously reached position may therefore become useful again after new credentials or knowledge have been acquired [1].

To represent this behaviour, an attack is described as a multi-domain finite-state process with memory. The cluster state includes nodes, APIs, subjects, resources, authorization relations, network reachability, trust links, and the deployment mode. The attacker's memory stores obtained credentials, discovered environment data, and evidence of control. Transitions denote offensive actions or observations and are enabled by explicit predicates reflecting vulnerabilities, authorization conditions, and required knowledge.

The main decision problem is how to rank the transitions that are currently available. The proposed priority assessment separates feasibility from usefulness. An inadmissible transition receives zero priority, while an admissible action is evaluated by its novelty, exploitability, immediate effect on confidentiality, integrity and availability, and progress toward a valuable terminal state.

Two formal properties make the model suitable for automated search. First, dominated operational configurations may be safely pruned. If two configurations have the same abstract automaton state, while one contains at least as much attacker memory and does not restrict any transition available from the other, the poorer configuration cannot produce a unique attack continuation. Second, under complete transition information and unit transition cost, a progress-dominant rule reaches a terminal compromise state in the minimum possible number of steps.

The model was used to create a machine-readable knowledge base of Kubernetes attacks and configuration weaknesses [2]. In five Kubernetes security assessments, it supported the discovery of four previously unmodelled critical chains leading to full cluster compromise and eight high-impact weaknesses. Structured descriptions of attack steps, impact, recommendations, and mappings to MITRE ATT&CK [3] also reduced report preparation time by approximately 23-25%.

Conclusion The approach provides a formal basis for prioritizing and automating attack-path analysis while retaining information often lost in static graphs. Further work may represent incomplete configuration data by fuzzy priority estimates and combine the automaton with an adapted A* search procedure. This would make it possible to find high-priority paths efficiently without discarding attacker memory or changes in cluster state.

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NUMERICAL SOLUTION OF THE OPTIMAL CONTROL PROBLEM FOR THE FINANCIAL FLOW

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This paper considers the following optimal control problem for the financial flow of minimizing the functional:

$$J_\alpha(u)=\sum_{l=1}^{m}\left\|x^l-y^l\right\|^2_{L_2^{(n)}(0,T)}+\alpha\left\|u-\omega\right\|^2_{L_2^{(m)}(0,T)} \qquad (1)$$

on the set:

$$U\equiv\left\{u=u(t):u\in L_2^{(m)}(0,T),\ \left\|u\right\|_{L_2^{(m)}(0,T)}\le b_0\right\}$$

under conditions:

$$\dot{x}^l(t)=A^l(t)x^l(t)+B^l(t)u(t)+f^l(t),\quad l=\overline{1,m},\ 0\le t\le T,$$
$$x^l(0)=x_0^l,\ l=\overline{1,m}, \qquad (2)$$

where $T>0$, $\alpha\ge 0$, $b_0>0$ are given numbers and $\omega=\omega(t)=\left(\omega_1(t),\omega_2(t),...,\omega_m(t)\right)$ – a pre-planned financial flow vector at the time t; $y^l=y^l(t)=\left(y_1^l(t),y_2^l(t),...,y_n^l(t)\right)$, $y_j^l(t)$ is pre-planned financial flow received by the j-client from the l-financial flow of the bank at time t; $x_0^l=\left(x_{01}^l,x_{02}^l,...,x_{0n}^l\right)$, x_{0j}^l, $j=\overline{1,n}$ are specified numbers describing the initial state of the financial flow in the enterprise; $u=u(t)=\left(u_1(t),u_2(t),...,u_m(t)\right)$, $u_l(t)$, $l=\overline{1,m}$ is financial flow of the l-source of finance at the time point t; $x^l=x^l(t)=\left(x_1^l(t),x_2^l(t),...,x_n^l(t)\right)$, $y_j^l(t)$, $j=\overline{1,n}$ is the financial flow received by the j-client from the bank from the l-financial flow at the time point t; $A^l(t)=\left\{a_{ij}^l(t)\right\}$, $i,j=\overline{1,n}$, and $B^l(t)=\left\{b_{jk}^l(t)\right\}$, $j=\overline{1,n}$, $k=\overline{1,m}$, $l=\overline{1,m}$ are given matrices; $f^l=f^l(t)=\left(f_1^l(t),f_2^l(t),...,f_n^l(t)\right)$, $l=\overline{1,m}$ are given functions. Matrices of $A^l(t)$, $B^l(t)$ and vector-functions of $f^l(t)$, $l=\overline{1,m}$ are determined by the internal laws of financial process

in the enterprise. The elements of matrices of $A^l(t)$ and $B^l(t)$, $l=\overline{1,m}$ are measurable limited functions and the elements of vector-functions $f^l(t)$, $l=\overline{1,m}$ are the quadratic integrable functions.

A difference method is used to numerically solve problem (1), (2). To this end, a difference approximation of problem (1), (2) is first constructed and estimates for the solution of the corresponding difference scheme and its error are established. Next, the convergence of the difference approximations with respect to the functional is proven. Finally, an algorithm for numerically solving the optimal control problem for the financial flow is constructed, as in [1].

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WORLDVIEW ALGORITHMS OF DECISION-MAKING AND PHILOSOPHICAL MATHEMATICS: PERSPECTIVES OF INTERDISCIPLINARY EDUCATION

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Aim and Objectives of the Report: The primary aim of this report is to investigate the theoretical and practical foundations through which philosophical mathematics and worldview algorithms transform modern decision-making models within the context of interdisciplinary education. To achieve this aim, the paper sets forth several key objectives: first, to define the phenomenon of the "worldview algorithm" as a synthesis of logical-mathematical structures and philosophical ethics; second, to analyze how the philosophy of mathematics fosters the development of critical thinking and complex choice-making skills among students; and third, to establish a conceptual framework that demonstrates the efficacy of interdisciplinary educational models in shaping strategic decisions within crisis-ridden and multifaceted environments.

Significance and Theoretical Framework: The significance of this research stems from the necessity of making multilayered decisions in an era of global **digital** transformation and systemic crises, where technocratic or purely technical approaches often prove insufficient to address ethical and existential challenges, as the “Union of Existence and Essence” [2]. The theoretical framework of the paper is grounded in the historical and contemporary paradigms of the philosophy of mathematics - ranging from the logicism of Gottlob Frege and Bertrand Russell to Leibniz’s concepts of a universal symbolic language and calculus, which can be viewed as precursors to worldview algorithms. This theoretical foundation is integrated with modern theories of interdisciplinary education and Epistemology, allowing us to conceptualize mathematical formalism not as an abstract isolation, but as a structural and philosophical anchor for the decision-making process.

Methodology and Scientific Novelty: The methodological foundation of this paper consists of complex theoretical analysis, along with methods of synthesis and comparative research, which allow us to interlink the formal apparatus of mathematical logic with philosophical hermeneutics. Given the specific nature of the research topic, we utilize interdisciplinary and systems approaches to examine the processes of structuring thought within the **educational** environment. The scientific novelty of the work lies in the original conceptualization of the term "worldview algorithm," which has not previously been analyzed as a decision-making tool within the framework of interdisciplinary education. This research introduces, for the first time, a model where the philosophy of mathematics is applied not merely as a theoretical discipline, but as a practical guide for shaping strategic and ethical choices at the crossroads of the humanities and exact sciences.

Introduction: In the modern era, the decision-making process has transcended the boundaries of narrow rationalism. The primary challenge of **interdisciplinary** education is to demonstrate that mathematics is not merely a science of numbers, but a tool for structuring thought. The concept of a "worldview algorithm" implies a synthesis of philosophical and mathematical logic that aids individuals in making sound choices within crisis-ridden and multifaceted situations, because "the algorithm exists not just in code" [4]. Philosophical mathematics thus becomes the bridge that unites ethical values with precise analytical models. To illustrate the process of

reflection and decision-making, we utilize a modern interpretation of Leibnizian logical calculus *(Calculus Ratiocinator)* and probabilistic weights, where choice is not merely numerical, but fundamentally saturated with worldview values. Accordingly, “they do not give a qualitative change in the situation in education” [3]

On a practical level, the worldview algorithm of decision-making can be expressed through a functional model $D = f(A, V)$, where D represents the final optimal decision, A is the set of purely mathematical, rational algorithms, and V is the vector of the subject's philosophical and ethical values. For instance, when a student or researcher assesses risks during a complex crisis, formal probability theory provides a **distribution** of probabilities *(P)*, yet the weight of the final choice is determined by an ethical coefficient *(a)*. By expressing this relationship as $W = a \cdot P$, we observe that the mathematical probability *(P)* loses its significance or shifts fundamentally if the philosophical value coefficient *(a)* is zero or negative [5]. The objective of interdisciplinary education is to train students to manage both components of this formula simultaneously.

Epistemological Foundations of Philosophical Mathematics: The first chapter is dedicated to analyzing the historical intersection of mathematical formalism and philosophical thought, wherein mathematics is conceptualized not as an isolated tool, but as a universal language of **cognition**. This section of the paper focuses on the interrelation between ancient Pythagoreanism, Leibniz's rationalist project, and twentieth-century logicism, all of which laid the groundwork for structuring thought through calculus. Within this chapter, it is argued that philosophical mathematics serves as an epistemological framework that enables individuals to transform abstract logical rules into tangible, worldview categories [6]. This approach establishes the necessary foundation for understanding how a matrix structured by pure reason becomes the very basis for decision-making.

Worldview Algorithms in the Decision-Making Structure: The second chapter **shifts** the focus to the theoretical deconstruction of the concept of the "Worldview Algorithm," defining it as a multidimensional strategy for making complex choices, however, “considering it that had an impact on conception” [1] This section of the paper argues that classical algorithms, which focus solely on linear logic and optimization, are insufficient for resolving the existential and social dilemmas facing modern individuals. The concluding section of the

chapter demonstrates that a philosophically grounded algorithm transforms decision-making from a mechanical, automated act into a deeply reflexive and conscious worldview choice.

Findings and Conclusion: The core finding is that the efficacy of the decision-making process in the modern era directly depends on the synthesis of mathematical formalism and philosophical values. The paper demonstrates that "worldview algorithms" **serve** not merely as a theoretical abstraction, but as a practical educational methodology providing students and researchers with a tool to navigate multilayered choices during crisis situations.

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EVOLUTION OF ORCHESTRATED MULTI-AGENT SYSTEMS

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Orchestrated multi-agent systems (OBAS) based on large language models represent an evolutionary extension of classical multi-agent systems. While preserving the fundamental principles of autonomy, distributed problem solving, and agent interaction, OBAS introduce a qualitatively different level of system organization through orchestration, adaptive reasoning, and functional specialization.

From a systems perspective, modern OBAS should not be viewed simply as sets of autonomous intelligent agents. Instead, they are organized computational systems whose capabilities emerge from the coordinated interaction of architectural components responsible for orchestration, functional specialization, workflow organization, memory and knowledge management, and interaction with external computational resources. Consequently, system performance depends not only on the capabilities of individual agents but also on the architecture that control their interaction.

Last researches show that the principal innovation of OBAS is not the forming the set of specialized agents, which has long been a characteristic feature of classical multi-agent systems, but the integration of role specialization with LLM-based reasoning, adaptive coordination, and dynamic task allocation. This combination enables greater flexibility in solving complex, evolving and uncertainty problems.

Another important conclusion is that orchestration should be understood as a comprehensive architectural principle rather than merely a workflow management technique. It determines how coordination responsibilities are distributed among architectural components and therefore directly influences scalability, robustness, adaptability, and operational efficiency.

Finally, contemporary OBAS increasingly rely on hybrid orchestration strategies that combine different principles according to task requirements and operational conditions. This trend reflects a broader transition from static coordination structures toward adaptive organizational architectures capable of supporting heterogeneous agents, external computational resources, and continuously evolving collaborative processes.

COMBINATORIAL OPTIMIZATION PROBLEMS OF CRITICAL INFRASTRUCTURE PROTECTION

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General approaches to the formulation and solving the combinatorial optimization problems of critical infrastructure (CI) facilities protection in frontline cities are considered, taking into account the evolution of both attack and defense tools. It is shown that combinatorial optimization naturally corresponds to the discrete nature of many real problems and relevant decisions (selection of types and quantities of defensive tools, their placement, and the allocation of discrete resources among defense echelons, etc.). At the same time, classical formulations of problems have significant limitations: insufficient consideration of partial damage, recovery costs and time, cascading effects, attacks on the protection systems themselves, continuous components, and the high uncertainty aspects of modern hybrid warfare.

The evolution of threats (combined mass attacks by ballistic and cruise missiles, low-cost drones, including those with AI coordination elements and fiber-optic guidance, KABs, etc.) and new defensive tools (laser and microwave systems, low-cost interceptors, multi-layer systems) requires an update of conceptual models. A framework is proposed that covers static, dynamic, multi-layer, and hybrid formulations with multi-criteria objectives (survival probability, expected losses, CVaR, resilience, and cost).

Among the promising approaches there are hybrid methods that combine a combinatorial core with fortification–interdiction games, distributionally robust optimization (DRO) machine learning, and network models. Special attention is given to the protection of distributed and moving CI objects.

It is concluded that further research should focus on hybrid models for new types of threats, decision-dependent uncertainty, systemic analysis of interdependent networks, real-time online optimization, and empirical validation based on the experience of Ukraine and the Middle East.

ON UNIFORM CONVERGENCE OF REARRANGED FOURIER SERIES

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In 1905 H. Lebesgue has shown that the Fourier series of a continuous periodic function f may not converge uniformly even it converges point-wise to f. In 1964 B.S. Stechkin has found a continuous periodic function f, whose Fourier series diverges at some points, but the rearranged Fourier series converges uniformly to f . Later D.V. Pecherskij [1988] and Sz.Gy. Révész [1994] independently have found an additional condition on the Fourier series of a continuous periodic function f, which guarantees that some rearrangement of it converges uniformly to f.

We plan to discuss the question whether Pecherskij-Révész's condition is always satisfied for a differentiable function.

IOT-BASED INFORMATION SYSTEM FOR MONITORING FRESH FRUIT QUALITY AND THE SUPPLY CHAIN

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Fresh fruit is highly perishable, with quality and shelf life strongly influenced by environmental and handling conditions throughout the supply chain. Continuous monitoring is therefore a key technological challenge. This paper proposes a conceptual IoT-based information system for fresh fruit quality and supply-chain monitoring, integrating multi-sensor measurements, Edge Computing, wireless communication, cloud-based data processing, and a Decision Support System (DSS).

The five-layer architecture integrates sensor acquisition, Edge/ESP32 processing, communication, cloud analytics, and application/DSS functions. It monitors key quality and environmental parameters, including temperature, humidity, CO_2, ethylene-related indicators, color,

mass, volatile organic compounds (VOC), and mechanical impacts. Edge processing enables filtering, timestamping, and anomaly detection before data transmission to an IoT platform for analysis, visualization, and quality assessment.

A conceptual Quality Index (QI) is proposed for multi-parameter quality assessment, combining normalized indicators through predefined weights for seven parameters: temperature, humidity, color, ethylene-related indicator, mass, CO_2, and VOC. Using illustrative data, the QI equals 0.878 (≈0.88), demonstrating the model's operation rather than an experimental result.

The monitoring mechanism combines QI assessment with parameter-specific thresholds to evaluate overall product condition and detect critical changes. Based on QI values and threshold violations, the DSS can classify product condition and generate alerts, recommendations, and operational decisions.

The proposed architecture is conceptual and requires experimental validation. Future work will include sensor calibration, real-time data collection under varied storage and transportation conditions, laboratory quality assessment, optimization of QI weights, and model validation using MAE, RMSE, R^2, accuracy, and related performance metrics.

Keywords: Internet of Things (IoT), fresh fruit, quality monitoring, supply chain, multi-sensor system, Edge Computing, Quality Index (QI), Decision Support System (DSS), real-time monitoring.

SOFT SKILLS OF FUTURE SPECIALISTS IN THE CONDITIONS OF INDUSTRY 5.0

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The modern world is becoming increasingly technological. Machine learning, Big Data, artificial intelligence and automation of routine processes and operations are increasingly used by organizations. Industry 5.0 changes the old technological paradigm - Industry 4.0. This change symbolizes the transition from a human-centric model to a model focused on coordination between people and machines, sustainability and environmental friendliness [1, 2].

The use of advanced digital technologies allows organizations to analyze large amounts of information for decision-making and forecasting, optimize delivery routes and time, personalize customer

relationships. The implementation of the Industry 5.0 model allows for the use of environmentally friendly practices in production, increases social impact, and promotes process resilience to failures [2].

However, despite digitalization and technology, the most important element of Industry 5.0 is human skills, in particular soft skills.

In 2022-2023, a joint study was conducted by Ukraine and Poland on the necessary skills and readiness to work among young graduates (1155 respondents, 45% and 55% in Ukraine and Poland, respectively) under Industry 4.0 conditions [3]. It showed that the most demanded skills were: cognitive (adaptability, ability to learn), interpersonal (communication skills, cooperation), self-leadership (self-improvement, result orientation). In 2026, a study of the skills of young specialists who will work in the conditions of Industry 5.0 began. And although the study only started in 2026 (so far 100 respondents), the following trend can be identified. Among the soft skills that Ukrainian respondents possess sufficiently to work in the conditions of Industry 5.0 are: ecological thinking, flexibility, openness to change, critical thinking. Research by Polish colleagues [2] shows that with a high degree of mastery are the following skills: critical and abstract thinking, risk management, cognitive skills.

Thus, for Ukrainian specialists, communication skills, flexibility and adaptability are more inherent; for Polish ones – critical thinking, management skills.

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ON SOME STEEFENSEN METHOD MODIFICATIONS

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Let's consider the problem

$$P(x)=0, \quad \text{where} \quad P: R^n \to R^n . \qquad (1)$$

There are several algorithms for solving problem (1). We will consider a particular new algorithm, namely:

$$u_k = \Phi(x_k); \;\; x_{k+1} = x_k - P(x_k, u_k)^{-1} P(x_k) . \; k = 0,1,... \qquad (2)$$

Theorem. Let's follow the conditions that are satisfied:

1. $\Phi(x)$ operator that satisfied the condition $\|\Phi(x) - x_*\| \le C \|x_k - x_*\|^{\tau}$, where $C > 0$, $1 \le \tau \le 2$, x_* - solution of the problem (1).

2. For $x, y \in X$, $X = \left\{ x \in R^n : \|P(x)\| \le \|P(x_0)\| \right\}$: $\left\| P(x,y)^{-1} \right\| \le B$, where $B > 0$.

3. For $x, y, z \in X$: $\|P(x,y) - P(x,z)\| \le M \|y - z\|$, where $M > 0$.

4. The initial approximation x_0 is chosen to satisfy the condition $q^{\tau} = BMC \|x_0 - x_*\|^{\tau} < 1$.

Then the sequence x_k is defined correctly by (2) and the next evaluation of convergence can take place $\|x_k - x_*\| \le q^{(1+\tau)^k - 1} \|x_0 - x_*\|$.

Propose considering some cases of choosing formulas for $\Phi(x)$ such as

$$u_k = x_k - P(x_{k-1}, u_{k-1})^{-1} P(x_k), \; x_{k+1} = x_k - P(x_k, u_k)^{-1} P(x_k),$$
$$u_0 = x_0 - \alpha P(x_0), \; k = 0,1,...$$

The rate of convergence for this case is $1 + \sqrt{2}$.

NORMALLY SOLVABLE BOUNDARY VALUE PROBLEMS WITH IMPULSE ACTION

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Solvability conditions are obtained for a normally solvable boundary value problems for a system of ordinary differential equations with a degenerate impulse action at fixed moments in time, under the assumption of an infinite number of impulse perturbations [1]. Differences are established between the structure of solutions of a system of ordinary differential equations with a nondegenerate [2] impulse action at fixed moments (with an infinite number of impulse perturbations), and the solutions of a system with a finite number of degenerate impulse perturbations [3].

The conditions for the boundedness of solutions to systems of ordinary differential equations with degenerate impulse action at fixed moments of time, obtained authors, generalize the results of [1,2] to the case of a finite number of degenerate impulse perturbations [3].

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A GENERALIZATION OF THE NEWTON-KANTOROVICH THEOREM FOR NONLINEAR OPERATOR EQUATIONS

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We constructed a modification of the classical Newton-Kantorovich [1] method for nonlinear operator equations, the linearization of which leads to normally solvable equations [2]. We established an iterative scheme with quadratic convergence for finding the solution to a nonlinear operator equation [3]. In particular, we obtained the solvability conditions and constructed an iterative scheme for finding the solution to a generalized Riccati equation [4].

The relevance of studying the nonlinear operator equations in infinite-dimensional Banach spaces is linked to the numerous applications of such equations in solving infinite systems of differential equations [5,6].

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ON THE CONSTRUCTION OF LOCALLY OPTIMAL STRATEGIES IN GAME PROCESSES

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Locally optimal controls are constructed for game systems

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t), p_i, S_i, t),\ x_i(\bar{t}) \in X_i^0 \subset R^{m_{i0}^x},\ i \in I^x,$$

$$\dot{x}_{ij}(t) = f_{ij}(x_{ij}(t), u_{ij}(t), p_{ij}, t),\ x_{ij}(\bar{t}) \in X_{ij}^0 \subset R^{m_{i0}^x},\ j \in J_i^x,$$

$$\dot{x}_{ij}(t) = f_{ij}(x_{ij}(t), u_{ij}(t), p_{ij}, t),\ t \geq t_{is},$$

$$x_{ij}(t_{is}) = r_{ij}(S_{i-1}, S_i, x_i(t_{is})) \in X_{ij}^0 \subset R^{m_{is}^x},\ j \in S_i \setminus S_{i-1},$$

$$\dot{y}_i(t) = g_i(y_i(t), v_i(t), q_i, t),\quad y_i(t_0) \in Y_i^0 \subset R^{m_{i0}^y},\ i \in I^y,$$

$$w_{ij} = (p_{ij},\ \{u_{ij}(t)\}) \in W_{ij},\quad w_i = (p_i,\ \{u_i(t)\},\ \{(t_{is}, S_{is})\}) \in W_i,$$

$$z_k = (q_k,\ \{v_k(t)\}) \in Z_k.$$

Based on the initial partition (selected after the starting moment $t = \bar{t}$) of the set I^x into subsets I_k^1 ($\bigcup_{k \in I^y} I_k^1 = I^x$, $k \neq l \to I_k^x \cap I_l^x = \varnothing$) and of the set $\bigcup_{i \in I^x} (i, J_i^x)$ into subsets I_k^2 ($\bigcup_{k \in I^y} I_k^2 = \bigcup_{i \in I^x} (i, J_i^x)$, $k \neq l \to I_k^2 \cap I_l^2 = \varnothing$), the minimizers $(i^1(k),\ w_{i^1}^1(k))$ and $(i^2(k) j^2(k),\ w_{i^2 j^2}^2(k))$ of the functionals

$$T_1(k) = \min_{i \in I_k^x} \max_{z_k \in Z_k} \min_{w_i \in W_i} \min_{F_i^1(x_i(t), y_k(t), c_{ik}) \leq 0} t,$$

$$T_2(k) = \min_{ij \in J_{ik}^x} \max_{z_k \in Z_k} \min_{w_{ij} \in W_{ij}} \min_{F_i^2(x_{ij}(t), y_k(t), c_{ijk}) \leq 0} t$$

are calculated. By time $\bar{t} + h$, values of k are removed from the set I^y if $T_1(k) < \bar{t} + h$ (with the value $i^1(k)$ being added to the set I_k^x associated with the largest value of $T_1(k)$) or if $T_2(k) < \bar{t} + h$ (with the value $i^2(k) j^2(k)$ being added to the set J_{ik}^x associated with the largest value of $T_2(k)$). We set $\bar{t} = \bar{t} + h$, calculate the updated optimal values of $x_i(\bar{t})$ and $x_{ij}(\bar{t})$, and if $I^Y \neq \varnothing$, the process continues. The required minimizers are calculated based on the necessary and sufficient conditions we have established for their local optimality on the sets W_{ij},

W_i, and Z_k, which are defined by the constraints $p_{ij} \in P_{ij}$, $u_{ij}(t) \in U_{ij}(t)$, $p_i \in P_i$, $u_i(t) \in U_i(t)$, $t_{i0} \geq t_0$, $t_{i(s+1)} > t_{is}$, $S_{i(s+1)} \subset S_{is}$, $q_k \in Q_k$, $v_k(t) \in V_k(t)$, $c_{ijk} \in C_{ijk}$.

Taking the human factor into account, we develop "three-point" strategies incorporating "situationally optimized" random variables to assess key indicators of (tennis) player proficiency and optimize the training process, thereby maximizing the probability of future victories.

A COMPARTMENTAL MODEL OF TUBERCULOSIS TRANSMISSION DYNAMICS WITH INCOMPLETE TREATMENT AND REINFECTION

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Tuberculosis remains one of the leading infectious diseases worldwide, and mathematical models provide an important tool for understanding its transmission dynamics and assessing the impact of control strategies. This work studies a deterministic Susceptible–Latent–Infectious–Treated–Recovered ($SLITR$) compartmental model of tuberculosis transmission. The total population $N(t)$ is divided into five epidemiological compartments: susceptible individuals (S), latently infected individuals (L), infectious individuals (I), individuals undergoing treatment (T), and recovered individuals (R). The model incorporates latent infection, treatment, successful treatment completion or treatment interruption followed by a return to the infectious compartment, as well as reinfection of recovered individuals.

The model is given by

$$
\begin{aligned}
dS/dt &= \Lambda - \beta SI - \mu S,\\
dL/dt &= \beta SI + \omega\beta RI - (\mu + \kappa)L,\\
dI/dt &= \kappa L + \eta(1-p)T - (\alpha + \mu + \delta_1)I,\\
dT/dt &= \alpha I - (\eta + \mu + \delta_2)T,\\
dR/dt &= p\eta T - \omega\beta RI - \mu R.
\end{aligned}
$$

The initial conditions are $S(0) = S_0 > 0$, $L(0) = L_0 \geq 0$, $I(0) = I_0 \geq 0$, $T(0) = T_0 \geq 0$, $R(0) = R_0 \geq 0$. The positivity and boundedness of solutions are established, and a positively invariant region is identified. The basic reproduction number $\Re_0$ is derived using the next-generation matrix method. The local and global stability of the disease-free equilibrium are investigated. The analysis demonstrates that incomplete treatment increases the infectious burden, whereas successful treatment contributes to reducing tuberculosis transmission. Numerical simulations illustrate the temporal evolution of the epidemiological compartments and support the analytical results.

OPTIMAL TWO-THRESHOLD STRATEGY FOR A FINITE RETRIAL QUEUE

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We are dealing with a retrial queue of $M/M/1/\infty$ type (see. [1]) with intensity λ of the input Poisson flow of the primary calls, and ν is a parameter of the repeated calls. It is supposed the number of places on the orbit to be equal to $m < \infty$. Let $\xi(t)$ stand for the number of the customers on the orbit at the instant t, and $1 \leq a < b \leq m - 1$ be two fixed numbers (thresholds). Let at the moment τ a customer (primary or from the orbit) hit the server. If $\xi(\tau) \leq a$, then that customer will be served with μ_1, if $a + 1 \leq \xi(\tau) \leq b$, then it will be served with rate μ_2 and if $b + 1 \leq \xi(\tau) \leq m$ it will be served with rate μ_3.

We put $\eta(t) = i$, $i = 1,2,3$ if the service rate at the time t is μ_1, and $\eta(t) = 0$ if at the time t the server is free. The process $(\eta(t), \xi(t))$ is a homogeneous Markov process.

In the report the formulae for steady-state distribution of the system were obtained. Using that formulae we considered some optimization problems for such a system.

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FOR DEMOGRAPHIC PROCESS MANAGEMENT

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Thin the spectrum of national migration challenges, the organized management of the multi-stage repatriation process for migrated individuals remains a critical issue. Resolving this complex problem is virtually impossible without systematic analysis and rigorous mathematical modelling.

The model developed in this study optimizes the demand-and-supply dynamics between emigrants and the state, constructing an efficient, automated reintegration mechanism.

To ensure objective evaluation, and optimize the distribution of limited state resources based on individual priority coefficients, the system executes **Chachanidze's Inference Rules:**

- **Chachanidze's Rule 1** (Full Intersection with Absence of Property - $\neg F_{(x)}$): $\exists_{(i,j,f)} \in M_{i\,j\,f}\ [\neg F_{(x)} \wedge (S_s \cap D_{i\,j\,f\,(g)} = D_{i\,j\,f(g))}] \rightarrow D_{i\,j\,f(g))} \wedge$ Add To Return List (Mijf).
- **Chachanidze's Rule 2** (Critical Need for Housing - $\neg F1$): $\forall_{(i,j,f)} \in M_{i\,j\,f}\ [\neg F1_{(i,\,j,\,f)} \wedge (S_s \cap D_{i\,j\,f(g)} = D_{i\,j\,f(g))}\ ] \rightarrow$ Add To Return List ($M_{i\,j\,f}$).
- **Chachanidze's Rule 3** (Need for Commercial Space / Business Support - $\neg F2$)**:** $\forall_{(i,j,f)} \in M_{i\,j\,f}\ [\neg F2_{(i,\,j,\,f)} \wedge (S_s \cap D_{i\,j\,f(g)} = D_{i\,j\,f(g))}] \rightarrow$ Add To Return List (Mijf).
- **Chachanidze's Rule 4** (Need for Industrial Facility - $\neg F3$): $\forall(i,j,f)$ [Mijf $\wedge$ $\neg F3$ $\wedge$ (Ss $\cap$ Dijf(g) = Dijf(g))] $\rightarrow$ Dijf(g) $\wedge$ Add To Return List (Mijf).
- **Chachanidze's Rule 5** (Need for Land Plot - $\neg F4$): $\forall(i,j,f)$ [Mijf $\wedge$ $\neg F4$ $\wedge$ (Ss $\cap$ Dijf(g) = Dijf(g))] $\rightarrow$ Dijf(g) $\wedge$ Add To Return List (Mijf).
- **Chachanidze's Rule 6** (Discrepancy and Interactive Dialogue Regime): $\forall(i,j,f)$ [Mijf $\wedge$ (Ss $\cap$ Dijf(g) $\neq$ Dijf(g))] $\rightarrow$ Dijf(g) $\wedge$ Go To Dialogue (Mijf).

The logical-mathematical model formulated via **Chachanidze's Inference Rules** minimizes subjective and bureaucratic vulnerabilities within the emigrant reintegration process.

THE MATHEMATICAL AND SYSTEMIC ARCHITECTURE OF THE NATION'S ENERGY AND DEVELOPMENT POTENTIAL INDEX (NEDPI)

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The Nation's Energy and Development Potential Index (NEDPI), or the "National Regenerative Energy," serves as a socio-biological and economic model. To transform NEDPI into a functional scientific instrument for decision-making under uncertainty, this study develops dynamic linkages and practical computational algorithms between key indicators.The basic structural equation of NEDPI that we developed has the following form:

$$NEDPI = \left(\sum_{i=1}^{n} \omega_i . I_i \right) . \prod_{k=1}^{m} C_k \ ,$$

where I_i is the normalized score of each category (scaled 0–100), ω_i – represents the relative weight of the category $\sum \omega_i$ =1.00 and C_k denotes Chachanidze-Mamaladze's critical multiplier indicators $\left(0 \le C_k \le 1\right)$. For high mathematical precision, bi-directional Min-Max normalization is applied.

1. Positive (Constructive) Indicators: $I_i^+ = \dfrac{X_i - X_{\min}}{X_{\max} - X_{\min}} \cdot 100$.

2. Negative (Destructive) Indicators: $I_i^- = \dfrac{X_{\max} - X_i}{X_{\max} - X_{\min}} \cdot 100$.

Synthesizing catastrophe theory and systemic dynamics, the core equation governing national vitality is established through Chachanidze-Mamaladz's Vitality Index:

$$E_R(t) = \left[E_C \cdot e^{\lambda \cdot t} + \psi_A \right] \cdot \frac{1}{1 + \Phi_P} \ ,$$

where E_R is the Nation's Ethno-Regenerative Energy Index, E_C represents the core energy of cultural codes, λ is the speed of psycho-social regeneration following a catastrophe (war, invasion, or crisis), ψ_A denotes the potential energy of strategic and geopolitical adaptation, and Φ_P measures the force of external environmental

pressure. The model eliminates subjective biases in evaluating state sustainability, offering a robust tool for demographic security planning.

PROBLEMS OF APPROXIMATION OF MANY VARIABLES FUNCTIONS FROM OBSERVATIONS AT RANDOM POINTS

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Let $\varphi(x)$ be a continuous bounded function in a region D of n-dimensional space. Suppose that at random points $x_k \in D, k=\overline{1,N}$ are given the values $y_k = \varphi(x_k) + \eta_k, k=\overline{1,N},$ where η_k are random errors of the data, independent of $x_k \in D, k=\overline{1,N}$. In 1964, the Georgian mathematician E.A. Nadaraya proposed a nonparametric estimate of the function $\varphi(x)$ for $n=1$ with observations $(y_k, x_k), k=\overline{1,N},$ that has found wide application.

In our work, we assume that x_k are unknown, and their distribution is given.

The unknown function $\varphi(x)$ belongs to some set of the Sobolev space. The estimate of the function $\varphi(x)$ is in the class of estimates of the form

$$\hat{\varphi}(x) = \sum_{k=1}^{N} y_k u_k(x) + u_0(x),$$

where functions $u_k(x), k=\overline{0,N}$ are chosen from the condition of the minimum of the guaranteed mean square error.

In special cases, there is an explicit form of the guaranteed estimates.

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OPTIMAL QUALITY MANAGEMENT DECISION-MAKING IN CAREER DEVELOPMENT AND LEADERSHIP SKILLS FORMATION

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Engineering high-level institutional quality loops to accelerate leadership skills formation requires stochastic frameworks capable of mapping multi-stage competence maturation profiles. This paper develops an adaptive stochastic decision-making model rooted in discrete-time Markov chains, ensuring conceptual variation and complete scientific independence among institutional tracks.

Let us consider an institutional education quality optimization and competency engineering problem under fluctuating job-market conditions. We model the professional progression of students using a finite, discrete-time Markov decision process (MDP) defined by the tuple $\langle S, A, P, R \rangle$.

Let S denote the finite set of student leadership and competency states, and A be the set of quality management administrative decisions. The transition probability matrix P governing career development tracking is formalized as:

$$P_{ij}(a) = P(X_{i+1} = j \mid X_i = i,\ A_i = 0),$$

where $a \in A$ is the specific quality control policy.

To maximize institutional excellence and career readiness, the decision-making loop determines an optimal policy π^* maximizing the expected cumulative reward function R. The local and semi-local convergence properties of the policy iteration algorithm are verified under **Lipschitz** conditions applied to the **Bellman** transition operator. The geometric domain of uniqueness for the optimal educational configuration is established. Numerical experiments confirm the model's systemic stability.

The computational convergence performance perfectly mirrors theoretical assumptions, proving this Markovian design to be an efficient model for automated educational quality management and automated leadership career decision-making.

A DIGITAL ASSET MODELLING APPROACH FOR OPTIMAL INVESTMENT DECISION-MAKING IN GLOBAL MARKETS

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The scale of the international development of the securities market has put on the agenda the inevitability of regulating this process in order to ensure the smooth, safe and efficient operation of the market. Market regulation implies the existence of certain rules, prohibitions and control mechanisms and principles.

Let us consider a modern digital asset portfolio optimization problem under global market uncertainty. Let $X_t \in R^n$ represent the state vector of a digital security price dynamic at a discrete time step t. We model the decision-making process using a controlled discrete-time linear system with stochastic perturbations:

$$X_{t+1} = A_t X_t + B_t U_t + \epsilon_t, \quad t = 0,1,...,N-1,$$

where A_t is the structural market transition matrix, B_t is the investment control multiplier, U_t represents the vector of investment decisions, and ϵ_t captures bounded market noise and volatility.

We study the local convergence and computational stability of the decision-making sequence under classical Lipschitz conditions applied to the cost-benefit operator. To optimize the digital security valuation model, we utilize a restricted multi-attribute target region approach. The semi-local convergence of the algorithm is established using majorizing scalar sequences. Furthermore, the exact domain of uniqueness for the optimal portfolio configuration is determined. Numerical experiments based on actual global financial trends validate the efficiency of this simplified model.

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MATHEMATICAL MODELING AND DECISION-MAKING OPTIMIZATION UNDER BANK CUSTOMER CHURN RISK

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We consider the binary classification problem of commercial bank customer churn under economic uncertainty. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of customers, where each $x_i \in \mathbb{R}^d$ is a feature vector encompassing demographic and engineered financial metrics. To estimate the conditional probability of churn $\hat{p}_i = P(y_i = 1 | x_i)$, we apply ensemble machine learning methods (XGBoost, Random Forest), optimized via 5-fold RandomizedSearchCV. The primary predictive model, XGBoost, minimizes the objective function at step t as follows:

$$Obj^{(t)} = \sum_{i=1}^{n} l\left(y_i, \hat{y}_i^{(t-1)} + f_t(x_i)\right) + \Omega(f_t),$$

where l is a differentiable loss function and Ω penalizes model complexity to prevent overfitting.

Standard classifiers apply a heuristic decision threshold of $t = 0.5$. However, due to asymmetric financial risks, this yields sub-optimal business outcomes. We propose a dynamic Cost-Sensitive optimization model to calculate the Total Expected Cost of a retention strategy:

$$Cost(t) = (FN_t \times CLV) + (FP_t \times C_{int})$$

where FN_t and FP_t are False Negatives and False Positives at threshold $t \in (0,1)$, CLV is the Customer Lifetime Value (penalty for a Type II error), and C_{int} is the marketing intervention cost (penalty for a Type I error). The globally optimal threshold is defined as $t_{opt} = \arg\min_{t \in (0,1)} Cost(t)$.

To evaluate economic viability, a prescriptive Return on Investment (ROI) simulator is formulated. The expected Net Profit is calculated as:

$$NetProfit = (N_{target} \times S_{rate} \times CLV) - (N_{target} \times C_{int})$$

where N_{target} is the subset of customers satisfying $\hat{p}_i \geq t_{opt}$ and S_{rate} is the stochastic probability of campaign conversion. This prescriptive

framework enables risk managers to dynamically construct a cost minimization curve by varying conversion rates and intervention constraints. Consequently, static algorithmic outputs are successfully transformed into actionable, ROI-driven retention strategies.

The results are presented in a fig. 1:

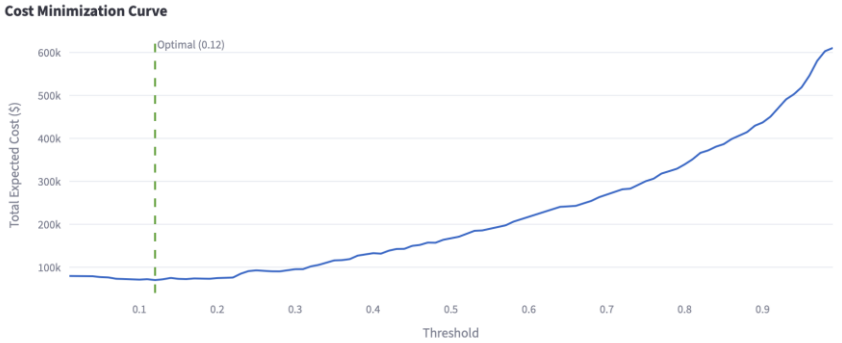

Fig. 1 – Cost minimization curve for retention strategy optimization

Conclusion Experimental results on a dataset of 10,000 records demonstrate that XGBoost yields superior discriminative capacity (ROC-AUC = 0.867), while Random Forest ensures optimal probability calibration (Brier Score = 0.104). Substituting the standard $t = 0.50$ threshold with the mathematically derived $t_{opt} \approx 0.12$ drastically minimizes aggregate financial losses. Integrating corporate economic parameters directly into the machine learning inference pipeline allows financial institutions to prioritize retention targets strictly based on ROI maximization.

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LEXICAL FEATURES OF THE KLARJIAN DIALECT

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The Klarjian dialect is one of the most important varieties of the Southwestern Georgian dialects spoken in the historical region of Klarjeti. Due to its geographical location and historical development, the dialect has preserved numerous archaic lexical elements while simultaneously reflecting the effects of prolonged language contact, particularly with Turkish. Consequently, the lexical corpus of the Klarjian dialect constitutes a valuable source for investigating the historical development of the Georgian language and the evolution of its dialects.

The vocabulary of the Klarjian dialect is characterized by the preservation of archaic lexical items that have either disappeared from Standard Georgian or undergone considerable semantic change. In addition, the dialect contains a rich layer of indigenous vocabulary associated with traditional rural life, agriculture, animal husbandry, craftsmanship, and the natural environment.

As a result of centuries of linguistic and cultural interaction, many lexical borrowings have become fully integrated into the dialect. These loanwords have undergone phonological and morphological adaptation in accordance with the structural principles of Georgian and frequently coexist with native Georgian equivalents.

The analysis of the lexical features of the Klarjian dialect reveals the interaction between inherited Georgian vocabulary and later contact-induced innovations. The investigation of these lexical layers contributes to a deeper understanding of the historical development of Georgian, the mechanisms of lexical change, and the influence of language contact on dialect formation. In conclusion, the documentation and systematic analysis of the Klarjian dialect vocabulary are essential for Georgian dialectology, historical linguistics, and the preservation of Georgia's linguistic heritage. Comprehensive fieldwork and the collection of authentic dialectal material will further enhance our understanding of the lexical structure of the dialect and its place within the Georgian linguistic landscape.

CLAIM ADMISSIBILITY FILTERING: A PRE-AGGREGATION GATE FOR EVIDENCE-BEARING KNOWLEDGE GRAPHS

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Edges asserting no claim about their node pair are frequent in LLM-extracted graphs and manufacture false disagreement. We characterize two failure modes and propose a pre-aggregation admissibility gate. Aggregation layers [1] partition evidence edges E(A,B,r) into clusters, reporting study counts, consensus, and disagreement — all functions of E. Two failure modes corrupt it.

Subject attribution drift. The edge is anchored to A, but the span predicates the effect on a different same-type entity. Contrastive spans ("green tea, but not coffee, reduces fasting glucose") yield sign-inverted edges.

Provenance-level drift. The edge reports a cited rather than produced finding, so its paper_id is not the originating study. This double-counts trials and imports citation distortion: 16.9% of medical quotations are inaccurate [2]. Neither is repairable inside clustering, which judges consistency, not membership.

Evidence. Of 47 audited Coffee→Blood glucose edges, 26 were admissible, 11 showed attribution drift, 10 provenance drift. The latter split 5 decreases / 6 no_change, populating opposing clusters and fabricating disagreement.

Solution and contribution. Admissibility becomes an edge property, not a deletion. Rule A: require a source_texts span naming A; route only spans citing multiple same-type entities to an LLM (9/11 rejected model-free, 11/11 with routing). Rule B: label evidence_level from citation markers and rhetorical role [3].

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BEHAVIORAL SOLUTIONS TO GLOBAL WARMING: NUDGING RECYCLING IN YOUNG HOUSEHOLDS

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This study is devoted to the problem of global warming mitigation by investigating how behavioral economics nudges, namely social comparisons and economic incentive, influence recycling motivation among young adults aged 20–30 in Israel. By means of a controlled, four-group empirical experiment, we evaluated the distinct and combined impacts of these behavioral interventions on independent households.

The results demonstrate that while independent social comparison nudges fail to alter behavioral intention, a combined approach integrating both social benchmarks and economic incentives results in robust and statistically significant increase in recycling motivation. The obtained findings suggest that climate policy should move beyond single-method solutions, by prioritizing hybrid and personalized behavioral interventions in order to develop resilient and pro-environmental habits among young people.

NECESSARY AND SUFFICIENT CONDITIONS FOR THE STABILITY OF SOLUTIONS TO A SYSTEM OF LINEAR DIFFERENCE EQUATIONS WITH RANDOM COEFFICIENTS

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A system of linear difference equations with random coefficients is considered (1)

$$X_{k+1} = A(\Im_{k+1}\Im_k)X_k, \qquad (1)$$

where $\Im_k$ – finitely valued Markov chain, taking values $\theta_1,...,\theta_n$ with probabilities $p_j(k) = P\{\Im_k = \theta_j\}$ $(j = 1,...,n)$ satisfying the system of

difference equations $p_j(k+1) = \sum_{s=1}^{n} \pi_{jS} p_S(k) \quad (j = 1,...,n;\ k = 0,1,2,...)$

Theorem 1. For the zero solution of the system of equations (1) to be stable in the mean-square sense, it is necessary and sufficient that the zero solution of the difference equations be stable.

$$D_j(k+1) = \sum_{s=1}^{n} \pi_{jS} A_{jS} D_S(k) A_{jS}^*(k), \qquad A_{jS} = A(\theta_j, \theta_S) \quad (j = s,1,...,n).$$

Introducing Lyapunov functions. The Lyapunov function method can be used to investigate stability. Lyapunov functions are introduced.

$$V_S(k, X) = \sum_{j=k}^{\infty} \left\langle X_j^* H(\Im_j) X_j \,\middle|\, X_K = X, \Im_K = \theta_S \right\rangle \tag{2}$$

By doing so $V_S(k, X) = X^* Q_S(k) X$, we obtained a system of difference equations.

$$Q_S(k) = H(\theta_{S)} + \sum_{j=1}^{n} \pi_{jS} A_{jS}^* Q_j(k+1) A_{jS} \quad (s = 1,...,n) \tag{3}$$

Theorem 2. In order for the zero solutions of the system of difference equations (1) to be stable in the mean square, it is necessary and sufficient that $H_S > 0 \ \ (s = 1,...,n)$ for the system of equations

$$Q_S = H_S + \sum_{j=1}^{n} \pi_{jS} A_{jS}^* Q_j A_{jS} \quad (s = 1,...,n)$$

having a solution $Q_S > 0 \ \ (s = 1,...,n)$.

To describe sudden variations in parameters, probabilistic models based on Markov or semi-Markov processes are widely used, as demonstrated in the stability analysis presented in [1-5].

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THE USE OF NON-MARKOVIAN STOCHASTIC PROCESSES IN ECONOMIC CALCULATIONS

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A theory of non-Markovian stochastic processes for calculating revenues in an economic system, taking into account the fact that revenues and expenses occur at different points in time.

Let an economic system, for example a bank, discretely depend on n $(n = 0,1,2,...)$, and can be arbitrarily divided into m parts $R_1, R_2, ..., R_m$. Let the total amount of funds operate within the system $S(n)$. We assume that the equipment R_k is operational $S_k(n)$ funds. We have the equality

$$S(n) = S_1(n) + S_2(n) + ... + S_m(n). \quad (1)$$

We introduce random variables $p_k(n) = S_k(n)S^{-1}(n)$ $(k = 1,2,...,m)$, which satisfy the relations

$$p_k(n) \geq 0, \quad p_1(n) + p_2(n) + ... + p_m(n) = 1, (n = 0,1,2,...). \quad (2)$$

Let us consider a stochastic probability vector $P(n) = \left(p_1(n)\ p_2(n)...p_m(n)\right)^*$.

We assume that the vectors $P(n)$ $(n = 0,1,2,...)$ form a non-Markovian random process and satisfy a system of difference equations [1].

$$P(n) = \alpha_0 \Pi_0 P(0) + \sum_{k=1}^{N} \alpha_k \Pi_k P(n-k), (n = 0,1,2,...) \quad (3)$$

The conditions met here are:

$$a_k \geq 0 \quad (k = 0,1,...,N), \quad \sum_{k=0}^{N} \alpha_k = 1, \quad P(-k) \in V(k = 0,1,...,N), \quad (4)$$

Π_k $(k = 0,1,...,n)$ – stochastic matrices, i.e. from $P \in V$ it follows that $\Pi_k P \in V$.

Theorem 1. If for the system of difference equations (2) conditions (4) are met, then the system of equations (2) defines a certain stochastic vector $P(n) \in V$ $(n = 0,1,2,...)$.

In the limiting case from Theorem 1. the following theorem can be obtained [4].

Theorem 2. If for a system of differential-difference equations

$$\frac{dP(t)}{dt} = \sum_{k=0}^{N} \alpha_k \left(\Pi_k P(t-\tau_k) - P(t)\right), \qquad (5)$$

the conditions have been met $\alpha_k \geq 0$, $0 \leq \tau_k$, $(k = 0,1,...,N)$; $P(t) \in V$ $t \leq 0$, Π_k $(k = 0,1,...,n)$ – is a stochastic matrix, then the system of equations (4) determines, for $t \geq 0$, a certain stochastic vector $P(t) \in V$ [2].

System of difference equations (3) determines the probabilities of the transfer of funds from one part R_k into another at various points in time [3].

Let us consider the calculation of the economic system's income. Let in part of system R_k at moment n be obtained income (expense) received in proportion $S_k(n)$, that is $\Delta S_k(n) = \beta_k S_k(n)$ $(k = 1,2,...,m)$, where β_k – coefficient depending on the type R_k. At the same time, we find the total income $\Delta S(n) = \sum_{k=1}^{m} \beta_k S_k(n), (n = 0,1,...)$, those $S(n+1) - S(n) = \sum_{k=1}^{m} \beta_k p_k(n) S(n), (n = 0,1,...)$. We finally obtain the equation for the total income $S(n+1) = \left(1 + \sum_{k=1}^{m} \beta_k p_k(n)\right) S(n)$.

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A SOFTWARE APPLICATION FOR COMPUTING THE OPTIMAL CONTROL OF TWO-LINK MANIPULATORS

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In this paper, we discuss an algorithm and a software application for computing the optimal control of the motion of a two-link manipulator performing a transport operation in a horizontal plane (moving a load from an initial position to a final position). The algorithm is based on the Pontryagin maximum principle and the shooting method for solving the corresponding boundary value problem for the Pi-system. Numerical nonlinear programming procedures were used to minimize the resulting residual [1]. The algorithm has been implemented (using the MS Visual Studio C# 2012 software environment) as a software application, some screenshots of which are shown in Fig. 1 – Fig. 4.

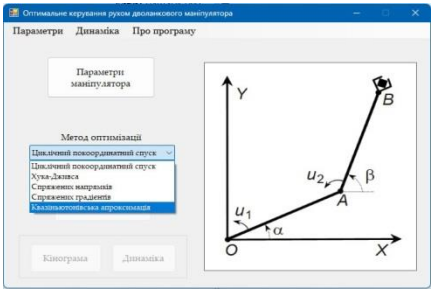

Fig. 1

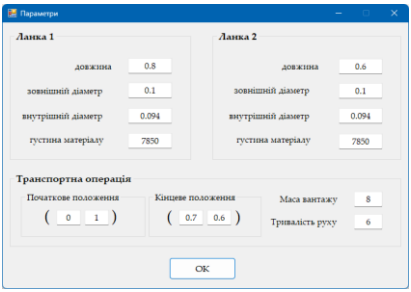

Fig. 2

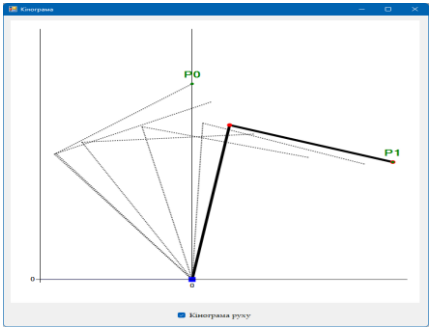

Fig. 3

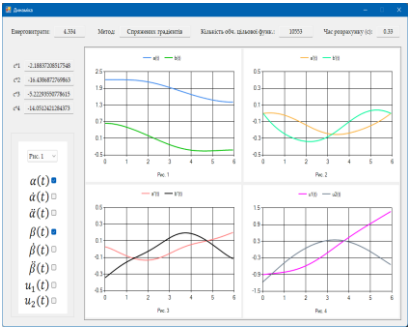

Fig. 4

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SOFTWARE TOOLS FOR PARAMETRIC OPTIMIZATION OF GANTRY ROBOT MOTION

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In this report, we discuss an algorithm and software for the parametric optimization of the motion of a gantry robot with a two-link manipulator [1]. Driven by the control stimuli F, u_1 and u_2, the robot performs a transport operation (in the vertical plane) that moves a payload from an initial state to a final state. The optimization algorithm is based on parametrization the robot's generalized coordinates as the sum of a cubic polynomial and a finite trigonometric series. The polynomial coefficients are determined from the conditions of the transport operation, whereas the series coefficients are found by minimizing an objective function (a control-quality criterion). The algorithm is implemented as a dedicated program written in Java using the Swing library to build the graphical user interface. Selected windows of the program are shown in Fig. 1 (the main window), Fig. 2 (the kinogram of the robot's motion), and Fig. 3 (the plot of the control F).

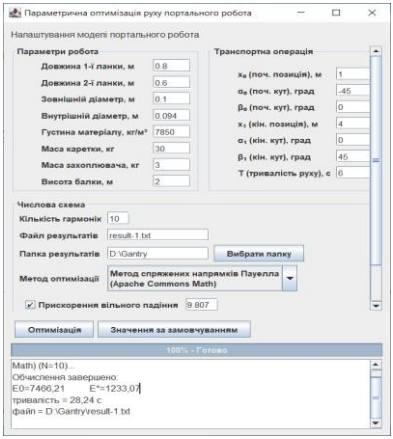

Fig. 1

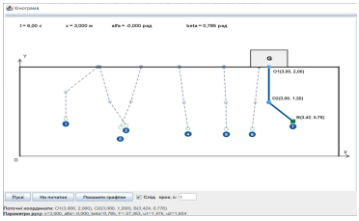

Fig. 2

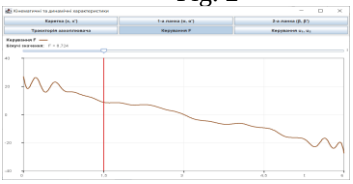

Fig. 3

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TEACHERS DECISION MAKING UNDER UNCERTAINTIES: CHALLENGES, STRATEGIES AND ADAPTATION IN CONTEMPORARY EDUCATION

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Teaching has always involved making decisions but in todays rapidly changing educational environment, teachers are increasingly expected to make important choices without always having clear answers. Changes in educational policies, the growing influence of digital technologies and artificial intelligence. Diverse student needs, and unexpected classroom situations can create uncertainty and make everyday professional; decisions more complex. In such circumstances, teachers need not only professional knowledge but also the ability to adapt, evaluate situations and respond effectively to changing conditions.

The research paper studies how teachers experience and navigate uncertainty in their professional decision making. It focuses on the challenges they face when there is no clear or predictable solution and examines the strategies they use to respond to complex and changing situations. particular attention is paid to the role of professional experience, stress, emotional regulation and digital competence in shaping teachers 'decisions and influencing their ability to adapt.

The study aims to contribute to a deeper understanding of teachers as active decision -makers who constantly navigate uncertainty in their everyday professional lives. It also highlights the importance of creating supportive educational environment when teachers can develop the confidence, flexibility, and resilience needed to make decisions in challenging situations. The findings may provide valuable and professional development, particularly in preparing educators to respond affectively to the growing complexity and unpredictability of contemporary education.

Key words: professional adaptation, teacher resilience, challenges, emotional regulations, decision- making

PROBLEMS UNEMPLOYMENT IN AN ECONOMICALLY LESS ACTIVE REGION

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Unemployment is one of the most important problems in the world. It hinders the socio-economic development of the country, hinders the improvement of the standard of living of the population.

Unemployment has quite negative socio-economic consequences for society, which is adequately reflected in the standard of living of the population and the process of economic development of the country. Therefore, in international practice, serious attention is paid to the assessment of these negative socio-economic consequences. It is relatively easy to assess the economic consequences of unemployment, while the social consequences, which are difficult to distinguish and are complex and cannot be fully evaluated. The negative consequences of unemployment and the degree of its impact on socio-economic development depend on the situation in the country, on the specific parameters of the social situation.

To fight unemployment, we must first analyze its causes. There are many reasons for the emergence of unemployment: the transformation of the economy, the economic downturn, the mistakes made during economic reforms, the introduction of innovations, the high level of corruption, the presence of imperfect institutions, the difficult political and criminogenic situation, etc.

The main indicators of unemployment are: the number of unemployed; duration of unemployment; unemployment rate; the sex-age structure of unemployed; professional-qualified structure of unemployed; sectoral and territorial structure of unemployed.

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RIGIDITY OF MULTI-AGENT FORMATIONS AND A GENERALIZED HERON–CAYLEY–MENGER IDENTITY: FROM AN ALGEBRAIC REALIZABILITY CRITERION TO MULTISTABILITY IN DISTANCE-BASED CONTROL

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In distance-based formation control, the stability of gradient control laws rests on the notion of infinitesimal formation rigidity, and the realizability of a given set of pairwise distances in Euclidean space is classically checked via the sign-definiteness of the Gram matrix recovered from the Cayley–Menger determinant. In this work, for the class of formations admitting a sphere tangent to all inter-agent links ("midsphere" formations), we derive and rigorously prove a closed-form algebraic identity expressing the n-dimensional volume of such a formation through elementary symmetric functions of the agents' tangential lengths. We show that at $n=2$ the identity exactly reproduces Heron's formula, while for $n\geq 3$ it produces a family of previously unstated explicit realizability inequalities generalizing the triangle inequality. We establish that the set of equilibrium formations with fixed aggregate invariants (total link budget and volume) forms a smooth manifold of dimension $n-2$ rather than an isolated point — a fact that directly explains the well-known flip-ambiguity problem in distance-based formation control. A numerical experiment with a gradient control law minimizing aggregate rather than full target invariants confirms the predicted multistability: from different initial conditions the formation converges to geometrically non-congruent equilibria with identical invariants. We further show that the key technical tool of the proof — the Sherman–Morrison–Woodbury lemma for a "diagonal plus rank-2 correction" matrix — coincides with the recursive covariance-update formula of the Kalman filter.

Multi-agent systems increasingly rely on distance-only sensing — range measurements are cheap, robust to occlusion, and require no shared orientation reference, unlike bearing- or position-based schemes. Yet the mathematical object underlying every distance-based control law — the map from pairwise distances back to a physical configuration — is far more permissive than practitioners often assume: a fixed set of

aggregate geometric quantities does not pin down a unique shape, but rather an entire continuum of physically distinct, non-congruent equilibria. This gap between what a controller enforces and what it actually determines is precisely where the classical flip-ambiguity problem in formation control originates, and it has so far been documented empirically rather than derived in closed form. The present work closes that gap: by identifying the exact algebraic condition under which a simplex admits a sphere tangent to all of its edges, we obtain a direct higher-dimensional generalization of Heron's formula, prove it rigorously via the same rank-two matrix update that underlies the Kalman filter, and use it to characterize — not just observe — the equilibrium manifold of a broad class of formation-control laws.

Theorem. Let an n-dimensional formation of $n+1$ agents have pairwise inter-agent distances $d_{ij} = x_i + x_j$ ($x_i > 0$ — the agents' tangential lengths to a common sphere tangent to all links; such a parametrization exists if and only if the sums of distances between opposite pairs of agents are equal). Then the formation's n-dimensional volume V_n satisfies

$$V_n^2 = \frac{2^{n-1}\left[\, 2(n-1)e_{n-1}e_{n+1} - (n-2)e_n^2 \,\right]}{(n!)^2},$$

where $e_k = e_k(x_1, \ldots, x_{n+1})$ is the elementary symmetric function of degree k.

Corollaries. (i) At $n = 2$: $S^2 = e_1 e_3$ — Heron's formula, with the coefficient of e_2^2 vanishing identically since $n - 2 = 0$. (ii) For $n \geq 3$, realizability requires the additional inequality $2(n-1)e_{n-1}e_{n+1} \geq (n-2)e_n^2$, not reducible to coordinate-wise positivity $x_i > 0$ — an explicit counterexample is constructed for $n = 3$. (iii) The set of formations with fixed e_1 (link budget) and V_n (volume) forms an $(n-2)$-dimensional smooth manifold.

The proof reduces the Cayley–Menger determinant to the determinant of the Gram matrix (coinciding with the formation's rigidity matrix), then applies the Sherman–Morrison–Woodbury lemma to a "diagonal plus rank-2 correction" representation of that matrix, with exact cancellation of all terms carrying negative powers of the singled-out coordinate upon passing to the full elementary symmetric functions.

DECISION-MAKING BASED ON A COMBINATION OF THREE METHODS

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The relevance of the study is determined by the fact that modern enterprises operate under uncertainty, limited resources, high competition, and the need for rapid, well-grounded managerial decisions. This is especially important in the defence-industrial sector, where decision-making time is additionally constrained, while technological processes change rapidly, production conditions remain unstable, and errors in resource management may lead to significant losses.

The study proposes combining interval methods of fuzzy logic with the TOPSIS method for multicriteria decision-making in production resource management [1,2]. Four alternatives are considered: increasing production volumes by adding work shifts; partial modernization of production equipment; redistribution of production resources among departments; and attracting a software tool for production planning and control.

The alternatives are evaluated by five criteria: production volume; unit production cost; duration of the production cycle; product quality; stability of delivery deadlines; and the market potential of the product. A software implementation in Python is proposed to automate calculations and reduce the need for repeated manual computation [3].

The proposed approach is expected to support comparison of managerial alternatives under uncertainty and improve the transparency and efficiency of decisions concerning production-resource allocation.

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MODELING COMPUTER NETWORK SECURITY USING ACCESS CONTROL LISTS (ACLS)

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The paper addresses the modeling and improvement of computer network security through the use of Access Control Lists (ACLs). The relevance of the research is determined by the increasing number of threats to modern local networks, including unauthorized access, data leakage, internal attacks, and denial-of-service attacks. The main objective is to develop a hybrid ACL-based access control model, automatically identify logical configuration anomalies, particularly Shadowing and Redundancy, and evaluate the effectiveness of the proposed approach in a simulated network environment.

Access Control Lists are an important component of network security because they allow network traffic to be filtered according to IP addresses, ports, protocols, and other parameters. The study considers the advantages of combining ACL-based filtering with broader access control approaches. In particular, hybrid models can provide a balance between security, flexibility, and usability, making them suitable for complex organizational networks.

The practical part of the research was implemented using Cisco Packet Tracer and the Python programming language. A three-zone network topology was developed, consisting of the INTERNAL, DMZ, and INTERNET zones. A central Cisco 2911 router was used to connect the network segments and perform ACL-based traffic filtering. This topology represents a simplified organizational infrastructure in which internal users, public-facing servers, and external network traffic are separated into different security zones.

To evaluate the proposed approach, a test ACL configuration was created with deliberately introduced logical anomalies. This methodology made it possible to verify whether the developed Python script could correctly identify previously known problems. The script processes the output of the Cisco IOS show access-lists command and analyzes the rules using set-theoretic principles and the Python ipaddress library. Its main functions include parsing IP networks, structuring ACL rules, and comparing rules to identify potential anomalies.

The analysis revealed two principal types of logical errors. The first was a Shadowing anomaly, in which a general permit rule (Rule 10) covers a more specific deny rule (Rule 20). As a result, the intended

restriction is not applied because the router processes ACL rules sequentially from top to bottom. The second was a Redundancy anomaly, where Rule 40 duplicates the effective behavior of another rule, unnecessarily increasing the configuration complexity and consuming processing resources.

Practical verification in the Cisco Packet Tracer environment confirmed the expected behavior of the network. An HTTP connection from the internal network to the DMZ web server was successfully established despite the intended deny rule, confirming the Shadowing anomaly. Additional connectivity testing demonstrated that the remaining access policy operated as expected. The Python script successfully detected both deliberately introduced anomalies and generated a CSV report, which can be used for further analytical processing.

The results demonstrate that automated analysis can improve the reliability of ACL configurations by identifying logical errors that are not necessarily detected through conventional router rule processing. The proposed approach combines practical network simulation with automated software-based analysis, providing both a testing environment and a repeatable method for ACL verification.

In conclusion, the study confirms that ACLs are an effective mechanism for L3–L4 network traffic filtering, while combining ACL-based control with Role-Based Access Control (RBAC) can provide a more flexible solution for complex organizational environments. The developed Python-based anomaly detection method successfully identified the tested Shadowing and Redundancy problems, demonstrating its potential to reduce configuration errors and strengthen network security policies.

Keywords: Access Control List (ACL), network security, Shadowing, Redundancy, hybrid access control, Cisco Packet Tracer, Python.

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HAAR AND CHEBYSHEV WAVELETS FOR SOLVING A BOUNDARY PROBLEM FOR A VARIABLE ORDER PARTIAL PSEUDODIFFERENTIAL EQUATION

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A fractional derivative of order $\alpha(t) \in (0,1)$ replaces the local instantaneous rate of change with a weighted integral over the entire history $[0,t]$:

$$ {}_0\mathcal{D}_t^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\omega)^{-\alpha} x'(\omega) d\omega. \qquad (1)$$

A natural and physically motivated generalization is obtained by replacing the constant α in (1) with a function $\alpha(t)$, which leads to the definition of variable order fractional operators (VOFO). Such models describe systems whose memory properties evolve dynamically. Moreover, VOFO allow the memory exponent itself to vary over time, thereby capturing regime changes in financial markets or changes in material properties that cannot be represented by fixed-order models. Such operators are referred to as pseudodifferential operators (PDOs), and equations containing them are called pseudodifferential equations (PDEs).

Parabolic PDEs involving PDOs with nonsmooth symbols were first introduced by S. Eidelman and Ya. Drin in [1], [2], [3], and [4]. In [5] and [6], PDOs were interpreted as hypersingular integrals.

A subsequent direction of research concerns the solvability of linear variable order fractional partial differential equations (VOFPDEs). In [7], it was shown for the first time that the Haar wavelet collocation method (HWCM) and the Euler-Fourier method are effective methods for obtaining solutions of linear VOFPDEs. For $t > 0$ and $0 < x < \ell$, $u(t,x) \equiv u$ we investigate the solvability of problem (2), (3), (4):

$$ {}_0\mathcal{D}_t^{\alpha} u = u b_1(t) + D_t^1 u b_2(t) + D_t^2 u b_3(t) + a^2 D_x^2 u, \qquad (2)$$

where ${}_0\mathcal{D}_t^{\alpha}$ denotes the variable order FD in the Caputo sense, defined by (1), $b_1(t)$, $b_2(t)$, and $b_3(t)$ are given functions, whereas u and $C(x)$ are unknown functions and

$$\left.\begin{aligned} u(0,x) &= \mu_1 C(x), \quad 0 < x < \ell \\ \mathcal{D}_t^1 u\big|_{t=0} &= \mu_2 C(x), \quad 0 < x < \ell \end{aligned}\right\} \tag{3}$$

$$u(0,x) = u(\ell,x) = 0, \quad t > 0 \tag{4}$$

Using these functions, we obtain an approximate solution u by applying the Haar wavelet collocation method (HWCM).

In [8], we derived, for the first time, solution formulas for classical and nonclassical boundary-value problems for variable order fractional partial differential equations using the HWCM. In [9], two-dimensional Chebyshev wavelets were applied to solve a nonclassical boundary problem for a variable order partial differential equation.

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A COMPARATIVE ANALYSIS OF EPIDEMIOLOGICAL MODELS FOR PREDICTING THE SPREAD OF DISINFORMATION

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Disinformation spreads through digital social networks much like an epidemic: given an observed curve of active spreaders I(t), the task is to estimate the model parameters and forecast it. Eight compartmental models were compared — the classical SI, SIS, SIR, SEIR and the disinformation-oriented SEPNS, SEDIS, modified SEDIS and SEDPNR (Drobiazko, 2026).

For every model the unknown vector θ combines the free transition rates with the initial values of the latent compartments (E_0, D_0). The direct problem is integrated by the LSODA method, and θ is found by minimising $\Phi(\theta) = \Sigma r_i^2$ by a multi-start local search with global differential evolution. Because the RMSE always falls as parameters are added, the models are ranked by AICc, where poorly specified data is ignored (condition number is too high).

On the 1978 English boarding-school influenza sample (N = 763, 14 points) the best models are SEIR by RMSE and SIR by AICc; three of the eight proved to be unidentifiable.

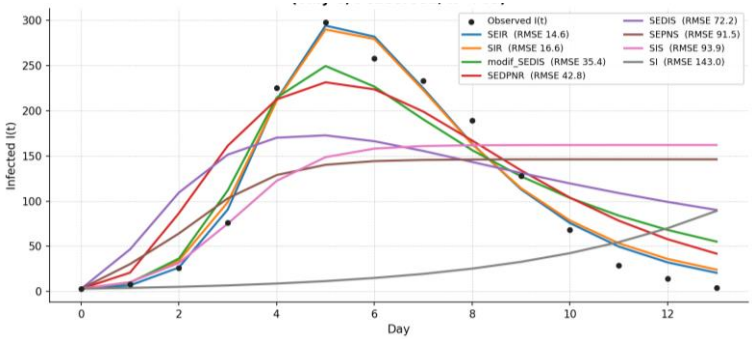

Figure 1 — All models fitted to the influenza I(t) curve

Conclusion. The compact models win: the disinformation-oriented ones lower the RMSE relative to SI/SIS, but on the curve I(t) alone they are rank-deficient, so a smaller RMSE does not indicate a better model. SIR and SEIR are happened to be the best models for these cases.

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HUMAN-AI COLLABORATIVE DECISION MAKING IN MILITARY AND CIVILIAN DOMAINS

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Abstract. The increasing complexity of modern military and civilian systems requires advanced decision-making approaches capable of operating under uncertainty, time constraints, and incomplete information. AI technologies [1,2] provide powerful analytical capabilities; however, human expertise remains essential for interpreting context, ethical reasoning, and responsibility. This paper investigates a collaborative Human-AI decision-making framework and proposes a mathematical model for evaluating the effectiveness of joint decisions. The proposed approach aims to improve the quality, reliability, and resilience of decision support in both military and civilian environments.
Keywords: decision making, artificial intelligence, human-centered AI, military systems, civilian systems, uncertainty, decision support.

The rapid development of artificial intelligence has significantly transformed decision-support processes in numerous domains, including defense, emergency management, healthcare, transportation, and cybersecurity. Modern operational environments are characterized by rapidly changing conditions, incomplete information, and increasing volumes of data that exceed human cognitive capabilities. Although AI systems can process large datasets and identify hidden patterns, they often lack contextual understanding, ethical judgment, and adaptability to unforeseen situations.

Human-AI Collaborative Decision-Making Model.To evaluate the effectiveness of collaborative decision making, we introduce a simplified model that integrates human expertise, AI recommendations, and environmental uncertainty.

$D= \mu H+(1-\mu)A-\propto U-$ where:

- D is the overall decision effectiveness, H denotes the quality of human assessment; A represents the AI-generated recommendation; U characterizes the level of uncertainty; μ is the trust coefficient assigned to the human expert; $\propto$ reflects the impact of uncertainty on decision quality.

The parameter μ determines the balance between human judgment and AI support. In highly dynamic military situations, greater weight may be assigned to experienced operators, whereas in data-intensive civilian applications AI may receive higher importance. The optimal value of μ can be determined through historical data analysis, expert evaluation, or adaptive learning mechanisms. Such adaptation allows the system to dynamically adjust the level of autonomy according to operational conditions [3.4].

Applications in Military and Civilian Domains. [5] In military environments, Human-AI collaboration can support battlefield awareness, threat assessment, logistics planning, and cyber defense. AI algorithms can rapidly analyze intelligence data, while commanders evaluate strategic implications and operational constraints.In civilian domains, collaborative decision-making frameworks can enhance emergency response, disaster management, healthcare diagnostics, and critical infrastructure protection. For example, during natural disasters AI systems may predict risk zones and resource requirements, while emergency managers coordinate evacuation and humanitarian actions. The proposed framework also contributes to improving transparency and trust in AI-assisted decisions.

Conclusions. The proposed mathematical model provides a simple mechanism for evaluating collaborative decision effectiveness and balancing human and artificial intelligence contributions. Future research will focus on adaptive trust mechanisms, explainable AI techniques, and integration with digital twins and cyber-physical systems to support real-time decision making in critical applications.

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ON THE APPLICATION OF MULTILEVEL STRUCTURES OF DIGITAL TWINS IN SEMI-NATURAL MODELING

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Semi-natural modeling of complex products, complex objects and control systems with a multi-level structure is an important tool for the development, research, debugging, testing and operation of such systems and their components in conditions as close to real as possible. These multi-level structures consist of many components, subsystems and systems. Each of them may have its own characteristics and functionality.

Semi-natural modeling combines the advantages of theoretical and experimental modeling. It allows to analyze and test the behavior of systems, conduct experiments and tests that are impossible or undesirable in real conditions. There is a need to use various algorithms and approaches to ensure the reliability and accuracy of modeling and for structural integration of models of different nature. MIL, SIL, PIL, HIL, HITL are key techniques used in the testing and simulation process on each level of the simulation environment. The use of precise digital twins and adaptive and self-learning algorithms accelerates the development cycle of new components and systems, reduce testing costs, and ensures effective testing of non-standard situations, accidents, etc. Adaptation algorithms work on the basis of real measured data and increase the accuracy, flexibility and efficiency of such systems, allow updating the parameters of the model or simulator according to changes in the real object or its environment, automatically adjusting to malfunctions and data loss, etc. Self-learning algorithms allow adjusting digital twins during the simulation process using streaming data from physical sensors, maintaining modularity and updating, detecting and predicting malfunctions, ensuring safety. Important features include support for distributed IoT sensors, fast data preprocessing, compatibility with standard interfaces, flexible scalability, the ability to flexibly configure and debug digital twins, etc. Using digital shadows allows data to be collected without interfering with the actual process, significantly increasing the safety of testing. Integration with AI allows to model complex dynamic processes, simulate emergency scenarios,

flexibly respond to changes and adjust the modeling process, etc. Modern semi-natural simulation is an intelligent, self-correcting environment that can be flexibly reconfigured to user requirements and new tasks.

QUALITATIVE PROPERTIES OF SOLUTIONS TO QUASILINEAR ELLIPTIC EQUATIONS IN GENERALIZED MORREY SPACES WITH VARIABLE EXPONENTS

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This study addresses the qualitative behavior and local-to-global gradient regularity for non-autonomous quasilinear elliptic problems driven by variable-exponent growth structures. Rather than assuming uniform regularity on the domain boundary or continuity in the differential operator, we examine critical qualitative properties when the principal coefficients possess small Bounded Mean Oscillation (BMO) traces and the geometric boundary is minimally smooth in the Reifenberg sense. By utilizing intrinsic maximal operators and localized perturbation lemmas, we establish sharp Calderón–Zygmund type gradient bounds inside generalized Morrey spaces with variable exponents. The analytical framework established herein provides the foundation for modeling elastic membrane equilibria over rough obstacles in smart composites, paving the way for a forthcoming comprehensive analytical study.

Keywords: Quasilinear elliptic equations; qualitative properties; variable exponent Morrey spaces; Reifenberg flat domains; Calderyn–Zygmund theory.

1. Problem Motivation and Mathematical Setup

In contrast to classical L_p-regularity theory, modern applications in heterogeneous material science require analyzing qualitative properties of elliptic operators under non-standard growth conditions. We consider weak solutions $u \in W_0^{1,p(\cdot)}(\Omega)$ to the stationary nonlinear system:

$$-\operatorname{div} \mathcal{B}(x,\nabla u) = \operatorname{div} \mathcal{H}(x) \qquad \text{in} \quad \Omega \subset \mathbb{R}^n \ (n \geq 2), \qquad (1)$$

where Ω is a bounded open domain bounded by a geometrically rough boundary $\partial\Omega$. The continuous variable exponent $p:\bar{\Omega}\to(1,\infty)$ satisfies the standard log-Hölder condition and is bounded by $1<p^{-}\le p(x)\le p^{+}<\infty$. The differential operator $\mathcal{B}:\Omega\times\mathbb{R}^n\to\mathbb{R}^n$ is strictly monotone, measurable in x, and C^1-smooth with respect to $\xi\in\mathbb{R}^n\setminus\{0\}$, satisfying:

$$\langle\mathcal{B}(x,\xi),\xi\rangle\ge\alpha|\xi|^{p(x)},\qquad|\mathcal{B}(x,\xi)|+|\xi||D_{\xi}\mathcal{B}(x,\xi)|\le\beta|\xi|^{p(x)-1},$$

for constants $\alpha,\beta>0$. To capture qualitative spatial behaviors without requiring classical continuity, the spatial heterogeneity of $\mathcal{B}$ is restricted via a small (δ,R_0)-BMO semi-norm:

$$\sup_{0<r\le R_0}\sup_{y\in\Omega}\fint_{\Omega\cap B_r(y)}\left(\sup_{\xi\neq0}\frac{|\mathcal{B}(x,\xi)-\overline{\mathcal{B}}_{B_r(y)}(\xi)|}{|\xi|^{p(x)-1}}\right)^{p(x)}dx\le\delta^{p^{-}}.$$

Simultaneously, the rough boundary $\partial\Omega$ is assumed to satisfy the (δ,R_0)-Reifenberg flatness condition, ensuring uniform local measure density across arbitrarily small geometric scales.

2. Qualitative Regularity in Variable Morrey Spaces

We characterize the qualitative gradient behavior within the generalized variable-exponent Morrey space $\mathcal{M}^{q(\cdot),\Phi}(\Omega)$, equipped with a doubling weight profile $\Phi(x,r)$.

Theorem 1. *Let $q(x)\ge p(x)$ pointwise, and let (Φ_1,Φ_2) be an admissible pair of weights fulfilling the supremal regularity condition:*

$$\sup_{r<s<\sigma}\frac{\operatorname{ess\,inf}_{s<\tau<r}\Phi_1(B_\tau(y))\tau^{\frac{n}{q_1}}}{s^{\frac{n}{q}}}\le K\Phi_2(B_r(y)).$$

Under the general structural and ellipticity constraints of equation (1), there exists a critical threshold $\delta_0=\delta_0(n,p^{\pm},\alpha,\beta)>0$ such that if $\delta\in(0,\delta_0)$, any weak solution u exhibits the qualitative gradient estimate:

$$\|\nabla u\|_{\mathcal{M}^{q(\cdot),\Phi_2}(\Omega)}\le C\left(1+\|\mathcal{H}\|_{\mathcal{M}^{q_1(\cdot),\Phi_1}(\Omega)}^{\frac{1}{p(\cdot)-1}}\right),$$

where the constant $C>0$ is independent of the solution u and the specific roughness profile of $\partial\Omega$.

THE BEHAVIOR OF SOLUTIONS TO NON-STANDARD DEGENERATE PARABOLIC EQUATIONS IN GENERALIZED LORENTZ–MORREY SPACES

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This paper investigates the qualitative behavior and global gradient regularity of weak solutions to quasilinear parabolic systems with non-standard growth conditions. Specifically, we examine divergence-form parabolic operators incorporating variable exponent growth, where the principal nonlinearity exhibits small Bounded Mean Oscillation (BMO) characteristics in the spatial domain and measurable dependencies in time. Assuming the underlying bounded cylinder is sufficiently flat in the sense of Reifenberg, we establish global Calderón–Zygmund type estimates in generalized Lorentz–Morrey spaces. Furthermore, we demonstrate the physical relevance of our analytical findings by modeling time-dependent non-Newtonian fluid transport within heterogeneous porous media.

Keywords: Degenerate parabolic equations; asymptotic behavior; Lorentz–Morrey spaces; Reifenberg flatness; Calderyn–Zygmund theory.

1. Introduction and Mathematical Framework

Let $Q_T = \Omega \times (0,T)$ define a parabolic cylinder over a bounded domain $\Omega \subset \mathbb{R}^n$ ($n \geq 2$) with non-smooth boundary $\partial\Omega$. We analyze the spatial gradient regularity of weak solutions u to the quasilinear Dirichlet problem:

$$\begin{cases} u_t - \operatorname{div} \mathbf{a}(x,t,\nabla u) = \operatorname{div}\left(|F|^{p(x,t)-2} F\right) & \text{in} \quad Q_T, \\ u = 0 & \text{on} \quad \partial_p Q_T. \end{cases} \tag{1}$$

Here, the variable exponent $p : Q_T \to (1,\infty)$ is log-Hölder continuous with $1 < p^- \leq p(x,t) \leq p^+ < \infty$. The vector field $\mathbf{a}$ is measurable in (x,t) and C^1 in ξ, satisfying uniform ellipticity and growth bounds:

$$|\mathbf{a}(x,t,\xi)|+|\xi|\cdot|D_{\xi}\mathbf{a}(x,t,\xi)|\le\Lambda_1|\xi|^{p(x,t)-1},$$
$$D_{\xi}\mathbf{a}(x,t,\xi)\eta\cdot\eta\ge\gamma|\xi|^{p(x,t)-2}|\eta|^2,$$

for constants $\Lambda_1,\gamma>0$. Regarding spatial roughness, we assume Ω is (δ,R_0)-Reifenberg flat and require $\mathbf{a}$ to satisfy a small (δ,R_0)-BMO oscillation condition in x:

$$[\mathbf{a}]_{p(\cdot),R_0}\equiv\sup_{0<r\le R_0}\sup_{y\in\mathbb{R}^n}\fint_{Q_r(y)}\left(\sup_{\xi\neq0}\frac{|\mathbf{a}(x,t,\xi)-\bar{\mathbf{a}}_{Q_r(y)}(t,\xi)|}{|\xi|^{p(x,t)-1}}\right)^{p(x,t)}dxdt\le\delta^{p^-}.$$

2. Main Regularity Theorem

We formulate our estimates within the generalized Lorentz–Morrey space $\mathcal{M}^{q(\cdot),s}_{\varphi}(Q_T)$, governed by an admissible weight function $\varphi(x,r)$ satisfying standard doubling and monotonicity constraints.

Theorem 1 *Let* $p\in(1,\infty)$ *and suppose* (φ_1,φ_2) *constitutes a compatible pair of functional weights satisfying the supremal condition:*

$$\sup_{r<s<\sigma}\frac{\operatorname{ess\,inf}_{s<\tau<r}\varphi_1(Q_\tau(y))\tau^{\frac{n+2}{q_1}}}{s^{\frac{n+2}{q}}}\le C_0\varphi_2(Q_r(y)).$$

Assume that the structural requirements for $\mathbf{a}$ *hold and let* u *be a weak solution to system (1). There exists a sufficiently small threshold* $\delta=\delta(n,p^-,p^+,\Lambda_1,\gamma)>0$ *such that provided* Ω *is* (δ,R_0)*-Reifenberg flat and* $\mathbf{a}$ *satisfies the small* (δ,R_0)*-BMO hypothesis, the spatial gradient obeys the estimate:*

$$\|\nabla u\|_{\mathcal{M}^{q(\cdot),s}_{\varphi_2}(Q_T)}\le C_1\left(1+\left\|\,|F|^{p(\cdot)-1}\right\|_{\mathcal{M}^{q_1(\cdot),s}_{\varphi_1}(Q_T)}\right),$$

where $C_1>0$ *depends solely on structural parameters and* $|Q_T|$.

FUJITA-TYPE CRITICAL BLOW-UP THEOREMS FOR DOUBLY NONLINEAR PARABOLIC EQUATIONS IN GENERALIZED MORREY SPACES

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In this paper, we establish a sharp Fujita-type threshold determining the boundary between global-in-time existence and finite-time blow-up for non-negative weak solutions to doubly nonlinear parabolic equations. We consider quasilinear evolution operators incorporating both slow-diffusion and gradient-dependent absorption terms over unbounded or non-smooth Reifenberg flat domains. Assuming the heterogeneous spatial coefficients satisfy a small Bounded Mean Oscillation (BMO) condition, we prove that the critical Fujita exponent is heavily modulated by the underlying generalized Morrey weight scale. When the reaction exponent falls below this critical geometry-dependent threshold, every non-trivial solution experiences finite-time blow-up regardless of the initial data magnitude. Conversely, supercritical exponents admit global weak solutions for sufficiently small initial configurations.

Keywords: Fujita-type theorem; finite-time blow-up; critical exponent; doubly nonlinear parabolic equations; generalized Morrey spaces.

1. Introduction and Problem Setting

Since the foundational work of H. Fujita (1966) on semilinear heat equations, identifying critical exponents that separate global solvability from finite-time blow-up has been a central theme in qualitative PDE theory. In this work, we investigate the Cauchy-Dirichlet problem for the doubly nonlinear degenerate parabolic equation with a reaction term:

$$\begin{cases} \partial_t \left(|u|^{m-1} u \right) - \operatorname{div} \mathcal{A}(x,t,\nabla u) = \lambda |u|^{q-1} u \quad \text{in } Q_\infty = \Omega \times (0,\infty), \\ u(x,t) = 0 \qquad\qquad \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) \geq 0 \qquad\qquad \text{in } \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 3)$ is a domain bounded by a (δ, R_0)-Reifenberg

flat boundary $\partial\Omega$, $m>0$ represents the porosity parameter, $q>m$ is the reaction exponent, and $\lambda>0$. The differential operator $\mathcal{A}:\Omega\times(0,\infty)\times\mathbb{R}^n\to\mathbb{R}^n$ adheres to p-Laplacian type growth ($p>1$) with measurable time dependency and small (δ,R_0)-BMO spatial oscillations:

$$\langle\mathcal{A}(x,t,\xi),\xi\rangle\geq\nu|\xi|^p,\qquad|\mathcal{A}(x,t,\xi)|\leq\mu|\xi|^{p-1},\qquad[\mathcal{A}]_{p,R_0}\leq\delta^p.$$

Unlike classical Euclidean frameworks where the critical Fujita exponent depends solely on the spatial dimension n, our goal is to show how roughness in $\partial\Omega$ and regularity in generalized Morrey spaces $\mathcal{M}^{s,\varphi}(\Omega)$ alter the asymptotic blow-up threshold.

2. Sharp Fujita-Type Critical Exponent

Let $\varphi(x,r)$ be an admissible generalized Morrey weight function satisfying the doubling condition and controlling the local geometry of Ω. We define the intrinsic geometric scaling parameter κ_φ associated with the Morrey capacity of the domain via:

$$\kappa_\varphi\equiv\inf\left\{\theta>0:\limsup_{r\to\infty}\frac{r^\theta}{\varphi(x_0,r)}<\infty,\quad\forall x_0\in\Omega\right\}.$$

Theorem 1 (Sharp Fujita Threshold). *Assume that* $m(p-1)<p$ *and* $u_0\in\mathcal{M}^{1,\varphi}(\Omega)$ *is a non-negative, non-trivial initial profile. Under the structural and BMO assumptions on* $\mathcal{A}$*, let* q_* *define the critical Fujita exponent given by:*

$$q_*=m+\frac{p(m+1)-m}{n-p+\kappa_\varphi}.$$

Then, the following dichotomous qualitative behavior holds:

1. ***Subcritical and Critical Case (*** $m<q\leq q_*$***):*** *Every non-negative weak solution* u *to system (1) blows up in a finite time* $T^*<\infty$*, meaning* $\lim_{t\to T^{*-}}\|u(\cdot,t)\|_{\mathcal{M}^{s,\varphi}(\Omega)}=\infty$*, regardless of how small the initial norm* $\|u_0\|_{\mathcal{M}^{1,\varphi}(\Omega)}>0$ *is chosen.*

2. ***Supercritical Case (*** $q>q_*$***):*** *There exists a sufficiently small constant* $\varepsilon_0>0$ *such that if* $\|u_0\|_{\mathcal{M}^{1,\varphi}(\Omega)}<\varepsilon_0$*, problem (1) admits a global-in-time weak solution* $u\in L_\infty(0,\infty;\mathcal{M}^{s,\varphi}(\Omega))$.

FORECASTING METHODS IN EDUCATION

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Although recent years have seen growing attention to the whole range of issues related to forecasting and planning in education, researchers' attention has been drawn primarily to quantitative indicators: forecasting the quantitative growth of educational institutions of various types, the numbers of students and pupils enrolled in them, the determination of future demand for qualified specialists in particular fields, and so on. Forecasting constitutes the first and decisive stage of planning in any organization, and in higher education in particular. Projections of student enrollment are of special importance, since they influence an institution's revenue, its staffing needs, its infrastructure and facility planning, and its budgeting [1].

In reality, the pedagogical and didactic problems associated with forecasting the aims, content, methods, means and organizational forms of teaching and upbringing at the various levels of education are still at the stage of formulation and preliminary discussion. In this respect, the elaboration of the theory and methodology of pedagogical forecasting, the search for new and effective methods of prognostic research in pedagogy, their testing and evaluation, and the prompt implementation of forecasting results are of particular relevance.

The information obtained through prognostic research makes it possible not only to identify prospective changes in the organization, structure and content of educational activity, but also to move purposefully in practice towards the projected operating conditions of educational institutions and to introduce the necessary adjustments into the everyday educational process. The paper examines the general principles of pedagogical forecasting, including its aims, content, methods and quality. An appropriate methodology and a set of recommendations for the systematic and consistent implementation of these principles are proposed.

Keywords: Forecasting, planning, education, pedagogical, methodology.

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IDEMPOTENT ELEMENTS OF THE SEMIGROUP $B_X(D)$

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In this paper, we consider complete semigroups of binary relations defined by complete X-semilattice of unions. It is known that the properties of the complete semigroups of unions are closely related to the properties of the semilattice by which this semigroup is defined. Because of this, we fix the diagram of semilattices (We denote this class of semilattices with the symbol $\Sigma_2(X,5)$ and study the properties of the corresponding semigroup (semigroup $B_X(D)$) based on the properties of this diagram.

In particular, since the description of idempotent elements is related to the right-hand units, which in turn is related to the XI-semilattices, we determine the XI-subsemilattice of the semilattice and based on them we make the description of the idempotent elements of the semigroup $B_X(D)$. Using the quantities of reflections, we also obtained the formula for calculating the number of idempotent elements of finite semigroups.

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MACHINE LEARNING AND FUZZY MODELS FOR FORECASTING THE EFFECTIVENESS OF HUMAN-CENTERED IT PROJECTS

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Forecasting the effectiveness of IT projects proceeds under significant uncertainty: requirements evolve dynamically and behavioral factors act non-linearly. A hybrid predictive modeling framework is

therefore proposed, integrated into the decision-support pipeline of Fig. 1 [1].

Fig. 1. Scheme of forecasting the effectiveness of a human-centered IT project (*Source: developed by the author)*

The feature space is constructed from primary project metrics by temporal aggregation, smoothing and lag transformation, complemented by behavioral characteristics of team activity. Non-monotonic dependencies – specifically the effect of optimal cognitive workload – are captured by gradient boosting through additive expansion:

$$F_m(x) = F_{m-1}(x) + \nu h_m(x), \quad \nu \in (0,1] \tag{1}$$

Temporal dynamics of team performance are modeled via recurrent neural networks, while qualitative uncertainty is evaluated by fuzzy rules with Mamdani inference and centroid defuzzification. Each forecast is additively decomposed into feature contributions, so the manager receives not only the predicted values (T_{pred}, P, R) but also areas for corrective intervention.

Empirical verification on software project data indicates that behavioral factors contribute 2.7 times more to effectiveness than purely technical metrics – a finding independently validated by regression and permutation importance analysis. Gradient boosting yields the highest accuracy in predicting duration, while the emotional tone of team communications, assessed via deep learning text analysis [2], serves as an early indicator of productivity decline.

Ultimately, the framework transforms heterogeneous IT project data into quantitatively substantiated, actionable managerial recommendations.

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ON SUFFICIENT OPTIMALITY CONDITIONS IN THE STABILIZATION PROBLEM WITH PHASE CONSTRAINTS

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We investigate the optimal stabilization problem for non-autonomous control systems to construct a feedback control minimizing a cost functional, ensuring global asymptotic stability and compliance with phase constraints. Consider $dx/dt = f(x,u,t)$, $t \ge t_0$, with state $x \in \mathbb{R}^n$ and control $u \in \mathcal{U} \subseteq \mathbb{R}^m$ ($0 \in \mathcal{U}$). The function f is locally Lipschitz, $f(0,0,t) = 0$. Let G_0 and $\Phi(t) \subset \mathbb{R}^n$ ($t \ge t_0$) be compact sets containing the origin in their interiors. The initial state is $x(t_0) = x_0 \in G_0$, and trajectories must satisfy $x(t) \in \Phi(t)$ for $t \ge t_0$. We minimize the functional $\mathcal{I}(u) = \int_{t_0}^{\infty} f_0(x,u,t)\,dt \to \inf$, where $f_0 \ge 0$ is continuous and integrable. Let us introduce the Hamilton-Jacobi-Bellman operator:

$$\mathcal{B}(\mathcal{V},x,u,t) = \partial_t \mathcal{V} + \langle \nabla_x \mathcal{V}, f(x,u,t) \rangle + f_0(x,u,t).$$

Theorem 1. *Let $\mathcal{V}_*(t,x)$ be a positive definite, decrescent, radially unbounded function, and $u_*(t,x)$ be a control such that: 1) $f_0(x,u_*(t,x),t)$ is positive definite; 2) for some $0 < \alpha \le \beta$, $G_0 \subseteq \{x : \mathcal{V}_*(t_0,x) \le \alpha\}$ and $\{x : \mathcal{V}_*(t,x) \le \beta\} \subseteq \Phi(t)$ for $t \ge t_0$; 3) $\mathcal{B}(\mathcal{V}_*,x,u_*(t,x),t) = 0$ for all $x \in \Phi(t), t \ge t_0$; 4) $\mathcal{B}(\mathcal{V}_*,x,u,t) \ge 0$ for all $x \in \Phi(t)$ and any admissible control u. Then $u_*(t,x)$ solves the optimal stabilization problem, and the minimum cost is $\mathcal{I}(u_*) = \mathcal{V}_*(t_0,x_0)$.*

For a linear system $dx/dt = A(t)x + B(t)u$ with a quadratic functional $f_0(x,u,t) = \langle Q(t)x,x \rangle + \langle R(t)u,u \rangle$ ($Q(t) \ge 0$, $R(t) > 0$), assume G_0 and $\Phi(t)$ are convex compacts. Let $c(\Phi(t),\psi) = \max_{x \in \Phi(t)} \langle x,\psi \rangle$ ($\|\psi\| = 1$) be the support function of $\Phi(t)$.

Theorem 2. *Let $P(t)$ be a symmetric matrix satisfying $P(t) \ge \sigma_0 I > 0$ and $\|P(t)\| \le \sigma_1$, which solves the Riccati equation*

$$\dot{P} + PA + A^T P - PBR^{-1}B^T P + Q = 0 .$$

Then the linear feedback control $u_*(t,x) = -R^{-1}(t)B^T(t)P(t)x$ *solves the optimal stabilization problem if* $G_0 \subseteq \{x : \langle P(t_0)x, x\rangle \le r^2\}$, *where the radius* r *satisfies* $r \le \min_{\|\psi\|=1} c(\Phi(t),\psi) / \langle P^{-1}(t)\psi, \psi\rangle^{1/2}$ *for all* $t \ge t_0$. *The optimal cost is* $\mathcal{I}(u_*) = \langle P(t_0)x_0, x_0\rangle$.

The proposed method was successfully verified through numerical simulations applied to the stabilization problem of a multi-mass oscillating system.

IMAGE COMPRESSION USING VARIATIONAL AUTOENCODER AND SEMANTIC SEGMENTATION

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The rapid increase in the amount of visual data has created a need for more efficient image compression methods. Traditional compression techniques process all parts of an image in the same way, without considering that some regions are more important than others. As a result, valuable visual information may lose quality together with less important background details. To solve this, an image compression method that combines semantic segmentation with a variational autoencoder (VAE) can be used. The semantic segmentation model detects the most important regions of an image and preserves them in their original quality, while the remaining areas are compressed using the VAE. This approach reduces the image size while maintaining the quality of the most meaningful content.

The performance of the method depends on the image itself. If the semantically important region occupies a small part of the image, a larger background area can be compressed, resulting in a higher compression ratio but slightly lower overall image quality. In the experiments, the proposed method achieved a compression ratio of about 42%, with PSNR values ranging from 25.11 to 33.63 dB and SSIM values from 0.73 to 0.94. In comparison, using only the VAE produced lower reconstruction quality, with PSNR values between 22.79 and 28.64 dB and SSIM values between 0.54 and 0.81.

The experimental results show that the proposed method can effectively reduce image size while preserving the quality of the most important image regions. By applying lossy compression only to semantically insignificant areas, it achieves a balance between compression efficiency and visual quality. This makes the method suitable for applications where images need to be stored or transmitted efficiently without noticeably affecting the parts that matter most.

ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER ON EXAMPLES IN THE DYNAMICS OF RHEOLOGICAL DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS OF FRACTIONAL TYPES

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We present examples of ordinary and partial differential equations of fractional order on examples of the dynamics of rheological discrete and continuous dynamic systems of fractional type. We will show the derivation of two ordinary differential equations of fractional order, which describe: 1* the dynamics of a rheological Kelvin-Voigt discrete dynamic system - a rheological oscillator of fractional type, with the property of subsequent elasticity; and 2* the dynamics of a rheological Maxwell discrete dynamic system - a rheological creeper with the property of normal stress relaxation. We will show the obtaining of approximate analytical expressions of the self-eigen-gtee and forced modes of dynamics of these two rheological discrete dynamic systems of fractional type.

We will, also, present the derivation of two fractional-order partial differential equations, which describe: 1* the longitudinal dynamics of a rheological Kelvin-Voigt elastoviscous continuous dynamic system - a rheological longitudinal oscillator, of fractional type, in a rod of variable cross-section, with the property of subsequent elasticity; and 2* the longitudinal dynamics of a rheological Maxwell viscoelastic continuous

dynamic system - a rheological variable cross-section of a creeper with the property of normal stress relaxation. We will present the obtaining of approximate analytical expressions for the self-eigen and forced modes of the longitudinal dynamics of these two rheological continuous dynamic systems, of fractional type.

A method for obtaining approximate analytical expressions of the eigen-free and forced modes of dynamics of two rheological discrete and two continuous dynamic systems, bout of fractional type will be presented. The method includes applying Laplace transforms to fractional order equations, expanding into series with respect to a complex parameter, and determining the inverse Laplace transforms of approximate analytical expressions of the eigen-free and forced modes of dynamics of two rheological discrete and continuous dynamic systems.

1* The first model is a basic complex rheological, fractional type, Kelvin-Voigt, elastopiscossic, rheological oscillatory model of longitudinal dynamics of variable cross-section. The normal stress is described as follows, by a differential constitutive relation, of fractional order:

$$\sigma(z,t) = -\left\{ E_0 \frac{\partial w(z,t)}{\partial t} + E_\alpha \mathfrak{D}_t^\alpha \left[\frac{\partial w(z,t)}{\partial t} \right] \right\}, \quad \alpha \in (0.1), t \in (0.b), \qquad (1)$$

where $\mathfrak{D}_t^\alpha[*]$ is the fractional order differential operator, and $\sigma(z,t)$ is the normal stress in the axial direction and $\varepsilon_z(z,t) = \dfrac{\partial w(z,t)}{\partial z}$ is the axial dilation, $w(z,t)$ is the longitudinal displacement. The longitudinal rheological oscillation of material described by a fractional order partial differential equation, in terms of longitudinal displacements of the form (see [1, 2]).

$$\begin{aligned} &\frac{1}{c_0^2} \frac{\partial^2 w(z,t)}{\partial t^2} - \frac{1}{A(z)} \frac{\partial}{\partial z} \left[A(z) \frac{\partial w(z,t)}{\partial z} \right] = \\ &= \frac{c_\alpha^2}{c_0^2 A(z)} \frac{\partial}{\partial z} \left[A(z) \mathfrak{D}_t^\alpha \left[\frac{\partial w(z,t)}{\partial z} \right] \right] + \frac{1}{E_0} q(z,t). \end{aligned} \qquad (2)$$

2* The second model is a basic complex rheological, fractional type, Maxwell, viscoelastic, rheological suite of longitudinal creep dynamics

through tube of variable cross-section. The normal stress is described by the following differential constitutive relation, of fractional order, in the form:

$$\frac{1}{\mathbf{E}}\mathfrak{D}_t^{\alpha}\left[\sigma_z(z,t)\right]+\frac{\sigma_z(z,t)}{\mathbf{E}_{\alpha}}=\left\{\mathfrak{D}_t^{\alpha}\left[\frac{\partial w(z,t)}{\partial z}\right]\right\}\neq const,\ 0<\alpha\leq 1. \quad (3)$$

The longitudinal rheological dynamics of material creep is described by a partial differential equation of the fractional order, in terms of longitudinal normal stresses $\sigma(z,t)$ of the form see [1, 2]):

$$\frac{1}{\mathbf{E}}\mathfrak{D}_t^{\alpha}\left[\frac{\partial^2\sigma_z(z,t)}{\partial t^2}\right]+\frac{\dfrac{\partial^2\sigma_z(z,t)}{\partial t^2}}{\mathbf{E}_{\alpha}}- \quad (4)$$
$$-\frac{\partial}{\partial z}\left\langle\frac{1}{\rho A(z)}\mathfrak{D}_t^{\alpha}\left[\frac{\partial}{\partial z}\left[A(z)\sigma(z,t)\right]\right]\right\rangle=\frac{1}{\rho}\frac{\partial}{\partial z}\left\langle\mathfrak{D}_t^{\alpha}\left[q(z,t)\right]\right\rangle$$

Approximate analytical expressions of the eigen and forced modes, as eigen time functions, of the rheological dynamics of material creep in tube of several cross-sectional curves and for different boundary conditions, the eigen amplitude functions were determined.

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IDENTIFYING A LATENT MARKET FACTOR USING PRINCIPAL COMPONENT ANALYSIS

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Financial markets generate high-dimensional data characterized by strong correlations among asset returns. Principal Component Analysis (PCA) provides a statistical framework for extracting latent factors that capture the common dynamics of correlated financial assets.

PCA was applied to daily log returns of 29 constituent companies of the S&P 500 index. The first principal component was treated as a latent market factor and evaluated through a factor portfolio constructed from its loadings. Its ability to capture common market dynamics was assessed in terms of generalization, robustness, and temporal stability.

The first principal component explained a substantial proportion of the total variance in stock returns. Its loadings were consistently positive across all analysed assets. These findings support interpreting the first principal component as a latent market factor.

A factor portfolio constructed from the component loadings continued to closely track the overall market after being transferred beyond the estimation period. This suggests that the underlying market structure identified by PCA extends beyond the training sample.

During the market shock, the first principal component accounted for an even larger share of the total variance. This demonstrates that periods of elevated market stress amplify the common movement of financial assets. Rolling PCA further showed that the direction of the first principal component remained highly stable over time despite gradual changes in its loadings (Fig. 1). This confirms that the identified market factor remains meaningful over time, although its loadings gradually drift under changing market conditions.

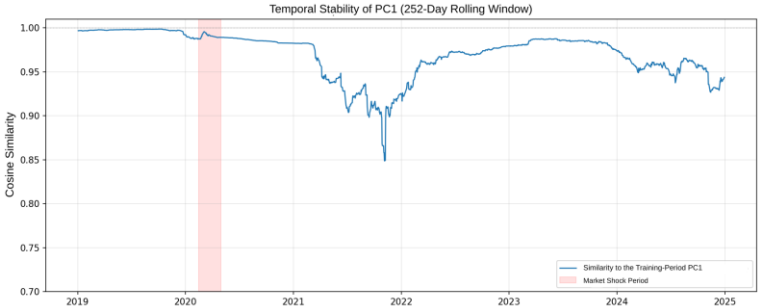

Fig. 1. Temporal stability of PC1

COMPUTATIONAL TECHNOLOGIES BASED ON VORTONS METHOD

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A computational technology for computer simulation based on the vorton method is presented. This version of the computational technology is applied to the test simulation of separated flow around an infinitely thin circular disk of diameter D, oriented orthogonally to the direction of a uniform flow U_∞ of an ideal incompressible fluid (Fig. 1).

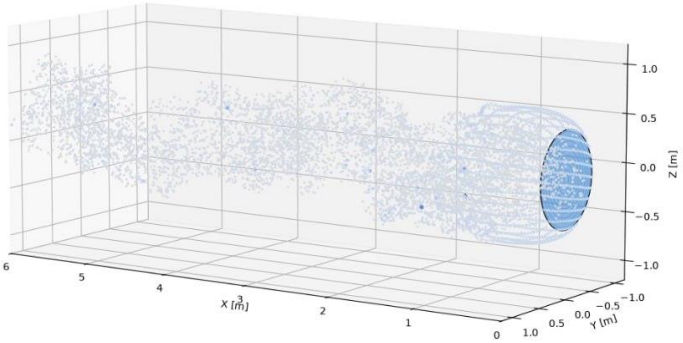

Fig.1

The disk model consists of a system of attached infinitely thin vortex rings, each of which is discretized into a system of vortex segments forming a closed regular polygon.

The no-penetration condition on the disk is satisfied at a system of collocation points located on the disk between the systems of vortex segments forming the infinitely thin vortex rings. At the disk edge, the Kutta–Chaplygin–Zhukovsky condition is imposed.

A wake with circulation is formed downstream of the disk. According to the proposed algorithm, the vortex rings in the wake are decomposed into a system of oriented vortons whose motion simulates the vortex flow, forming a system of circulating flows around the vortons. In the integral sense [1,2], the dynamics of the wake is determined by the dynamics [3] of a system of vortex elements – vortons – which qualitatively corresponds to the dynamics of the model vortex elements represented by a system of infinitely thin vortex rings.

The drag coefficient of the disk C_x reaches its maximum at the beginning of the flow, eventually approaching an approximately

constant mean value $C_x \approx 2.12$.

A comparative analysis of the test simulation results with experimental data and with the results of simulations performed by other authors using different methods showed satisfactory agreement.

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PREDICTING THE RISK OF LOW ACADEMIC PERFORMANCE IN GAMIFIED LEARNING SYSTEMS

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Gamified learning systems promote student engagement in the educational process [1]. Learner interaction data can be analyzed using mathematical models, allowing the identification of insufficient mastery of individual topics and the assessment of the risk of low academic performance [2]. The results of the analysis can be used in decision support systems to generate personalized recommendations and enable timely instructional interventions [2-3]. Mathematical models based on statistical indicators constitute a suitable approach for learning systems with a limited number of new users and a limited amount of accumulated learning activity data.

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LOSS FUNCTIONS IN EXCHANGE RATE FORECASTING

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A forecast is never scored in a vacuum: choosing a loss function is an implicit statement about which statistical functional (mean, median, quantile) a model is entitled to estimate, and whose economic costs that estimate is meant to serve. Two facts, rarely discussed together, anchor this study: not every quantity of interest has a loss function consistent with it, and the model ranked "best" by any given loss function is frequently not the model that actually makes money.

- Accuracy vs. profitability gap. Jamali and Yamani (2019) show that statistical forecast inaccuracy need not imply a lack of trading profitability, and vice versa — undermining the routine practice of ranking models purely by MSE/MAE.
- Elicitability. Per Gneiting (2011), every scoring function is consistent with exactly one statistical functional: MSE with the conditional mean, MAE with the median, pinball loss with a quantile; the Newey–Powell expectile loss offers a jointly elicitable, more tractable alternative to Expected Shortfall.
- The Expected Shortfall problem. VaR is elicitable on its own, but ES is not — no scoring function exists that is minimized by ES alone (Nolde, Ziegel, 2017). ES is only testable jointly with its corresponding quantile (Fissler, Ziegel, 2016; Patton, Ziegel, Chen, 2019). Most applied FX risk studies backtest ES with loss functions that, formally, cannot detect a systematically wrong forecaster.
- Asymmetric loss (LINEX). The sign of the parameter α determines which side of the error is penalized more heavily. Rather than fixing α by assumption, the study proposes estimating it from a hedger's or trader's realized losses — turning the choice of loss function into an empirical question about whose preferences the evaluation represents.
- Volatility and the noisy proxy. Patton (2011) shows that "true" volatility is unobserved and must be proxied by a noisy realized measure; only a restricted class of loss functions (including MSE and QLIKE) preserves the correct model ranking under such noise. Bollerslev, Patton and Quaedvlieg (2018) add that models should react less aggressively to realized measures when those measures are known

to be less reliable — especially relevant for FX given uneven intraday liquidity.

- Diebold–Mariano is not a test of the "correct" model. Patton (2020) shows the comparison is only well posed once both models are explicitly treated as misspecified; the test asks which forecast is closer to the truth under one pre-committed loss function — running it under several loss functions and reporting whichever favors the preferred model is a hidden specification search, not a robustness check.
- Forecast combination. Diebold and Shin (2019) show simple equal-weighted combinations routinely outperform optimally weighted ones. An open question is whether this pattern survives when weights are chosen under an asymmetric, trader-relevant loss instead of MSE (in the spirit of Storti and Wang, 2023, for ES/VaR).

The study aims to quantify: (1) the divergence between accuracy-based and profitability-based rankings of the same FX forecasts; (2) cases where ES forecasts pass conventional backtests yet fail the joint elicitability check; (3) whether the equal-weight combination pattern persists under an asymmetric loss; and (4) an empirically estimated asymmetry parameter α linking realized hedging losses to the shape of the appropriate loss function.

The central shift in perspective: the question is not "MSE vs. MAE," but whether the risk quantity of interest is even statistically testable with the loss function used to score it — and whether the model that wins on that metric is the one a real market participant would actually trade on.

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ON ONE INVERSE BOUNDARY VALUE PROBLEM FOR THE EQUATION OF THE THIRD ORDER DESCRIBING THE PROPAGATION OF LONGITUDINAL WAVES IN A DISPERSIVE MEDIUM WITH NONLOCAL TIME CONDITIONS

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It is known that the practical requirements often lead to the problem of determining the coefficients or the right hand side of the partial differential equations for some known data about its solutions. Such problems are called inverse boundary value problems in mathematical physics.

Let $T>0$ be a fixed constant, and define the domain $D_T=\{(x,t): 0\le x\le 1,\ 0\le t\le T\}$. We consider the one-dimensional inverse problem of determining the unknown set of functions $u(x,t)\in \tilde{C}^{3,2}(D_T)$, $a(t)\in C[0,T]$ and $b(t)\in C[0,T]$ for equation [1]:

$$u_{ttt}(x,t)-u_{txx}(x,t)+u_{tt}(x,t)-\alpha u_{xx}(x,t)= \\ =a(t)u(x,t)+b(t)g(x,t)+f(x,t) \tag{1}$$

under the following conditions:

$$u(x,0)+\delta u(x,T)=\varphi_0(x),\ u_t(x,0)+\delta u_t(x,T)=\varphi_1(x), \\ u_{tt}(x,0)+\delta u_{tt}(x,T)=\varphi_2(x)\quad (0\le x\le 1) \tag{2}$$

$$u(0,t)=u(1,t)=0\quad (0\le t\le T), \tag{3}$$

$$u(x_i,t)=h_i(t),\ \ (i=1,2;\ 0<x_1,x_2<1,\ x_1\ne x_2,\ 0\le t\le T), \tag{4}$$

where $x_i\in(0,1)$ $(i=1,2)$ are known fixed points, $\delta\ge 0$, $0<\alpha<1$ are given numbers, $\varphi_0(x),\varphi_1(x),\varphi_2(x)$, $h_1(t)$, $h_2(t)$, $f(x,t)$, $g(x,t)$ are given functions and

$$\tilde{C}^{(2,3)}(D_T)=\{u(x,t):\ u(x,t),u_{xx}(x,t),u_{txx}(x,t),u_{ttt}(x,t)\in C(D_T)\}.$$

The problem is firstly reduced to the problem that is in a sense quivalent to the original. Then, the Fourier mathod is applied, reducing the problem to solution of a system of integral equations. The existence and uniqueness of the latter equation is proved by the contraction mapping principle, which also yoelds the unique solution of the equivalent problem. Using equivalence, we finally prove the unique existence of a classical solution of the problem under consideration.

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ON A NECESSARY CONDITION IN AN OPTIMAL CONTROL PROBLEM WITH A BOUNDARY FUNCTIONAL FOR A NONLINEAR NONSTATIONARY SCHRÖDINGER EQUATION WITH A SPECIAL GRADIENT TERM

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In this paper, we consider the followıng optimal control problem of minimizing the functional:

$$J_\alpha(v)=\beta_0\left\|\psi(0,.)-y_0\right\|^2_{L_2(0,T)}+\beta_1\left\|\psi(l,.)-y_1\right\|^2_{L_2(0,T)}+\alpha\left\|v-\omega\right\|^2_H \qquad (1)$$

on the set:

$$V=\left\{v=v(t)=\left(v_0(t),v_1(t)\right): v_s\in L_2(0,T), \left|v_s(t)\right|\le b_s,\ s=0,1,\ \overset{0}{\forall} t\in(0,T)\right\}$$

under conditions:

$$i\frac{\partial\psi}{\partial t}+a_0\frac{\partial^2\psi}{\partial x^2}+ia_1(x,t)\frac{\partial\psi}{\partial x}-a(x)\psi+v_0(t)\psi+iv_1(t)\psi+$$

$$+a_2\left|\psi\right|^2\psi=f(x,t),\quad (x,t)\in\Omega, \qquad (2)$$

$$\psi(x,0)=\varphi(x),\ x\in(0,l),\quad \frac{\partial\psi(0,t)}{\partial x}=\frac{\partial\psi(l,t)}{\partial x}=0,\ t\in(0,T), \qquad (3)$$

where $i=\sqrt{-1}$; $a_0>0,\ l>0,\ T>0,\ b_s>0,\quad s=0,1,\quad \alpha\ge 0,\ \beta_0\ge 0,$ $\beta_1\ge 0,\ \beta_0+\beta_1\ne 0$ are given numbers; $\Omega=(0,l)\times(0,T)$; a_2 is a complex number satisfying the conditions:

$$a_2=\operatorname{Re}a_2+i\operatorname{Im}a_2,\ \operatorname{Im}a_2>0,\ \operatorname{Re}a_2<0,\ \operatorname{Im}a_2\ge 2\left|\operatorname{Re}a_2\right|; \qquad (4)$$

$a(x), a_1(x,t)$ are real-valued measurable bounded functions satisfying the conditions:

$$\mu_0\le a(x)\le\mu_1,\quad \left|\frac{da(x)}{dx}\right|\le\mu_2,\quad \left|\frac{d^2a(x)}{dx^2}\right|\le\mu_3,$$

$$\overset{0}{\forall}x\in(0,l),\ \mu_0,\mu_1,\mu_2,\mu_3=const>0; \qquad (5)$$

$$\left|a_1(x,t)\right|\le\mu_4,\ \left|\frac{\partial a_1(x,t)}{\partial x}\right|\le\mu_5,\ \left|\frac{\partial^2 a_1(x,t)}{\partial x^2}\right|\le\mu_6,\quad \overset{0}{\forall}(x,t)\in\Omega,$$

$$a_1(0,t)=a_1(l,t)=0,\quad \mu_4,\mu_5,\mu_6=const>0; \qquad (6)$$

$\varphi(x), f(x,t), y_0(t), y_1(t)$ are complex-valued functions satisfying the conditions:

$$\varphi\in W_2^2(0,l),\quad \frac{d\varphi(0)}{dx}=\frac{d\varphi(l)}{dx}=0; \qquad (7)$$

$$f\in W_2^{2,0}(\Omega),\quad \frac{\partial f(0,t)}{\partial x}=\frac{\partial f(l,t)}{\partial x}=0,\quad t\in(0,T);\ y_0,y_1\in L_2(0,T), \qquad (8)$$

$\omega\in H$ is a given element, $H\equiv L_2(0,T)\times L_2(0,T)$ and the symbol $\overset{0}{\forall}$ means 'for almost all'.

In this paper, we first introduce the adjoint problem to optimal control problem (1)-(4) and study the existence and uniqueness of its solution. We then investigate the differentiability of the functional under consideration and prove a formula for its gradient. Using this formula, we establish a necessary condition in the form of a variational inequality.

The correctness of the formulation of problem (1)-(4) was previously investigated in [1].

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TEACHING DYSTOPIAN LITERATURE AS ANTI-DYSTOPIAN EDUCATION FOSTERING CRITICAL THINKING AND CRISIS RESILIENCE

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The paper examines the teaching of dystopian literature as a form of anti-dystopian education and considers its potential to develop critical thinking, civic responsibility, and resilience in conditions of crisis and uncertainty. Dystopian narratives expose mechanisms of political and ideological control, including propaganda, censorship, manipulation, and enforced conformity.

Studying these mechanisms can help students identify comparable threats in contemporary society and evaluate their consequences for individual autonomy and democratic life. Drawing on George Orwell's *1984*, Ray Bradbury's *Fahrenheit 451*, and Aldous Huxley's *Brave New World*, the paper explores how representations of power, freedom, resistance, and social control can support informed judgment, independent reasoning, and ethical decision-making. It argues that dystopian literature can serve as a valuable resource for civic and democratic education by encouraging students to question dominant narratives, recognize threats to human dignity, and reflect on their own civic agency.

When taught through critical discussion, these texts may help students understand the importance of freedom and democratic values while developing the intellectual and ethical capacity to defend them during periods of crisis and repression.

Keywords: dystopian literature, anti-dystopian education, critical thinking, civic education, crisis resilience.

OPTIMAL DECISION-MAKING ALGORITHMS IN GEORGIAN FOREIGN DIPLOMACY: CAUCASIAN STABILITY AND INTER-ETHNIC TRUST BUILDING

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This paper articulates a distinct mathematical framework using multi-actor strategic game models to formulate diplomatic decision-making operations, separating this study structurally from linear tracking systems

Let us consider a multi-lateral geopolitical mediation and regional conflict resolution problem under high systemic uncertainty. We formalize the diplomatic decision-making infrastructure using a non-cooperative finite game-theoretic framework:

$$\mathbf{\Gamma} = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle,$$

where N represents the set of regional geopolitical actors (including neighboring Caucasus regions).

Let S_i denote the strategy set of actor i, and let $u_i : S \to R$ be the corresponding diplomatic utility payoff function.

Georgia's role is modeled as a strategic mediator optimizing the regional trust vector through a decision-making algorithm that seeks a stable Nash equilibrium profile $(s^* \in S)$ satisfying:

$$u_i(s_i^*, s_{-1}^*) \geq u_i(s_i, s_{-1}^*) \quad \forall s_i \in S_i, \quad \forall_i \in N .$$

Local convergence bounds of the iterative best-response mapping sequences are established using generalized Lipschitz conditions on multi-attribute utility operators. The domain of uniqueness for regional diplomatic stabilization is mathematically proven. Crisis-simulation experiments demonstrate the algorithm's practical efficiency in establishing sustainable regional consensus.

The numerical tracking demonstrated that the equilibrium search process preserves strong computational resilience under external boundary shocks. The computational convergence rates align with our theoretical proofs, verifying this framework as an innovative tool for regional diplomatic decision-making.

ADAPTIVE PID CONTROL VIA ACTOR-CRITIC NEURAL NETWORK FOR CRYPTOCURRENCY VOLATILITY TARGETING

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We formulate portfolio volatility targeting as a closed-loop control problem. Portfolio volatility is defined as $v(t) = |u(t)| \cdot \sigma(t)$, where v^* is the target volatility, $u(t) \in [-1,1]$ is the position size, and $\sigma(t)$ is realized market volatility. Since financial markets are non-stationary, fixed-gain PID controllers may lose effectiveness when volatility regimes change.

We adapt the self-tuning PID architecture of [1] to financial risk management. The control signal is $u = \text{clip}(u_s + u_d, -1, 1)$, where u_s is generated by a fixed PID controller and $u_d = K_p^{\Delta} e_p + K_i^{\Delta} e_i + K_d^{\Delta} e_d$ uses gain corrections generated online by an actor-critic neural network. The tracking error is $e(t) = v^* - v(t)$.

The method was evaluated on hourly BTCUSDT perpetual futures data from Binance for 2019-2025 using five-fold walk-forward validation. A2C-PID was compared with a fixed-gain PID tuned by the Ziegler–Nichols method.

A2C-PID reduced mean maximum drawdown from 10.6% to 3.7% and improved mean total return from -5.5% to 0.03%. Its volatility-tracking RMSE was slightly higher than the baseline (0.557 vs. 0.518), indicating a conservative low-exposure policy that prioritized drawdown reduction over exact volatility tracking. These results show that adaptive PID gain tuning can improve robustness and risk-adjusted performance under non-stationary market conditions.

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AVERAGING IN MINIMAX ESTIMATION FOR PARABOLIC PROBLEMS WITH LINEAR AND NONLINEAR OBSERVATIONS

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Let $\Omega \subset \mathbb{R}^N$ be a bounded domain and $Q_T = \Omega \times (0,T)$. We consider the parabolic problem

$$\frac{\partial y}{\partial t} + A\varphi = f_1(x,t), \quad \varphi\big|_{\partial\Omega} = 0, \quad \varphi\big|_{t=0} = f_0(x), \tag{1}$$

where $A = -\operatorname{div}(a(x)\nabla)$ is a uniformly elliptic operator. The observation has the form

$$y^{\varepsilon}(t) = \int_{\Omega} C(x, \frac{t}{\varepsilon}, \varphi(x,t))\varphi(x,t)dx + f_2(t). \tag{2}$$

The linear observation model corresponds to $C = C(x,\tau)$.

For the linear case, we assume that there exists $\bar{C} \in L^{\infty}(\Omega)$ such that

$$\lim_{s\to\infty} \frac{1}{s} \int_0^s \left| C(x,s) - \bar{C}(x) \right| ds = 0 \quad \text{in} \quad L^2(\Omega). \tag{3}$$

For nonlinear observations, suitable uniform averaging assumptions with respect to the third variable are imposed; in particular, on every bounded set of values of ξ the rapidly oscillating function $C(x,\tau,\xi)$ is approximated, in the corresponding integral sense, by an averaged coefficient $C_0(x)$. The aim is to estimate the functional

$$l(\varphi) = \int_0^T \int_{\Omega} b(x,t)\varphi(x,t)dxdt$$

by linear functionals of the observations

$$\hat{l}(\varphi) = \int_0^T y^{\varepsilon}(t)u(t)dt, \quad u \in L^2(0,T),$$

under uncertain data (f_0, f_1, f_2) belonging to a prescribed bounded set G. The corresponding minimax problem is

$$J_{\varepsilon}(u) = \sup_{(f_0,f_1,f_2)\in G} \left(l(\varphi) - \hat{l}(\varphi) \right)^2 \to \inf \tag{4}$$

Using the averaging method, an averaged minimax problem is constructed. Its optimal estimate provides an approximate minimax

estimate for the original problem with rapidly oscillating observations. In both the linear and nonlinear cases, under the corresponding averaging assumptions, the minimax estimation errors satisfy $\sigma_\varepsilon \to \sigma_0, \quad \varepsilon \to 0,$ where $\sigma_\varepsilon = \inf_u J_\varepsilon(u)$ and σ_0 is the minimax error for the averaged problem. Thus, the rapidly oscillating observation model can be replaced asymptotically by the corresponding averaged one without loss of accuracy in the minimax estimation error.

ADAPTIVE LIPSI METHOD FOR AUTOMATED TESTING LOG ANALYSIS

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Modern automated web application testing systems generate significant amounts of log data about errors and the state of the software environment. Automated analysis of these data enables timely detection of critical events. At the same time, log data is non-stationary: the impact of individual features can change depending on time, load, and the state of the environment [2, 3].

The paper proposes the LIPSI method for automated assessment of log event criticality. The method is based on a logistic model of binary classification, modified by introducing temporal and contextual adaptation of the model weights. Unlike classical logistic regression, the influence of a single feature in LIPSI can vary depending on the time at which the event occurs and the current state of the environment. This allows us to take into account the non-stationarity of log data while maintaining the interpretability of the classification results.

The problem of determining the model parameters is formulated as a stochastic optimization problem. A gradient approach [1] is used for training, in which the basic, temporal, and contextual parameters are sequentially adjusted according to the prediction error. The convergence of the proposed algorithm is investigated using the classical conditions of the Robbins–Monro stochastic approximation [4]. The stability of the model under random perturbations of the input data is also analyzed.

Practical application of LIPSI involves converting a text log message into a numerical feature vector and additionally accounting for the event time and the context of the execution environment. Based on this data,

the model determines the probability that a log event is critical.

Thus, the proposed LIPSI method combines binary classification with adaptation to temporal and contextual changes in the log environment. The method can be used in automated testing systems of web applications for automatic analysis of logs, identification of critical events and further prioritization of detected errors.

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OPTIMAL INVESTMENT DECISION-MAKING UNDER FINANCIAL RISK MATHEMATICAL MODELLING CONSTRAINTS

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Modern corporate financial engineering requires mathematical risk management architectures capable of operating efficiently amid structural capital market fluctuations. Portfolio optimization and capital allocation represent dynamic tracking tasks. This paper introduces an optimal control model designed to govern investment decision-making loops without relying on repetitive linear transformations. Let us consider an optimal asset allocation and systemic risk mitigation problem over a dynamic investment horizon.

Let $X_t \in R^n$ represent the financial portfolio state vector at time (t).

We model the rational decision-making framework through a continuous-time deterministic linear-quadratic optimal control structures:

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \qquad x(0) = x_0,$$

where $u(t) \in R^m$ represents the vector of active investment control

decisions. To maximize returns while systematically minimizing market risks, the decision-making process optimizes the objective Bellman functional:

$$j(u) = \int_0^T [x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)]dt + x^T(T)S_x(T),$$

where $Q(t)$, $R(t)$ and S are positive-definite risk-weight matrices. Local and semi-local convergence parameters of the Riccati differential tracking sequence are analyzed under classical Lipschitz conditions. Numerical simulations validate the algorithm's high robustness in volatile financial environments.

The model was tested using historical financial performance indices. The numerical approximations demonstrate that the Riccati solution matrix preserves asymptotic stability against bounded structural perturbations. The empirical execution tracking mirrors theoretical approximations, validating this control framework as a mathematically sound approach for high-level investment decision-making.

FUZZY DECISION-MAKING IN EDUCATION: OPPORTUNITIES FOR INNOVATION

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Educational decision-making increasingly occurs under conditions of uncertainty, ambiguity, and competing priorities that cannot always be addressed through rigid, binary approaches. This paper examines the implications of fuzzy decision-making for educational reform, with particular attention to Georgia's predominantly structured and standardized education system.

The study employs conceptual and comparative analysis based on relevant academic literature, education policy documents, and selected practices from Finland, Singapore, New Zealand, Canada, the Netherlands, and Norway. The findings indicate that high-performing education systems generally combine "crisp" elements—clearly defined standards, structured curricula, formal policies, and standardized assessments—with "fuzzy" elements such as teacher autonomy, personalized learning, formative assessment, stakeholder participation, and adaptation to local and individual needs. Importantly, a fuzzy environment also creates space for controlled experimentation, enabling

educators and institutions to introduce, test, evaluate, and refine new teaching methods, technologies, curricula, and assessment practices before their wider implementation.

Decision-making model that combines the consistency and predictability of a crisp environment with the flexibility and adaptability of a fuzzy one. The model incorporates a balanced curriculum, personalized and inclusive learning, diversified assessment, teacher empowerment, technology integration, stakeholder engagement, decentralized decision-making, controlled experimentation, and continuous evaluation. The study concludes that fuzzy decision-making should complement rather than replace educational standards. Such a balanced approach could facilitate innovation and promote a more equitable, responsive, student-centred, and effective education system.

MULTI-GPU ALGORITHM FOR SOLVING HYPERBOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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Modeling complex dynamic and wave phenomena with temporal memory — such as wave propagation in viscoelastic media, acoustic/seismic wave attenuation, anomalous dissipation, and structural dynamics of hereditary elastic materials — requires solving hyperbolic integro-differential equations (IDEs). The primary performance bottleneck lies in the non-local Volterra integral term, which exhibits quadratic time complexity $O(N_t^2 \times N_{space})$ due to accumulation over past states, alongside the need to track multiple time levels for second-order time derivative approximations. This work presents a multi-GPU algorithmic framework designed to eliminate memory-bound bottlenecks and communication stalls on heterogeneous supercomputing clusters.

Methodology & Implementation

- **Spatial-Temporal Discretization:** Second-order temporal derivatives and spatial differential operators are approximated via high-order explicit/implicit finite-difference or finite-element schemes (e.g., Leapfrog or explicit Newmark schemes) on 2D/3D block-cyclic partitioned domains Ω_h.

- **Direct P2P Data Exchange:** Inter-GPU halo communications for wave-front boundary exchanges utilize CUDA-Aware MPI combined with NVLink / NVSwitch interconnects, completely bypassing CPU memory staging.
- **Three-Stream Pipelining:** Computations are distributed across concurrent CUDA Streams to overlap internal wave-stencil calculations, boundary data transfers, and numerical quadrature evaluations for the hereditary integral.
- **Memory Management:** Historical solution layers of the dynamic trajectory are managed via a high-speed VRAM circular ring buffer with asynchronous offloading of long-term trajectory data to Host RAM / NVMe.

Key Performance Indicators

- **Communication Overhead:** Reduced to under 5% of the total iteration runtime.
- **Weak/Strong Scaling:** Maintains 85–92% parallel efficiency when scaling up to 32 GPU accelerators.

PEDAGOGICAL DECISION SUPPORT IN AN INNOVATIVE ONLINE EDUCATIONAL ENVIRONMENT

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In modern scientific literature, an innovative online educational environment is considered as a holistic multi-component system that ensures integrated interaction of all participants in the educational process using digital technologies and information and communication infrastructure. Unlike traditional distance learning systems, which mainly performed the function of an electronic repository of educational and methodological materials, the modern online environment combines pedagogical, technological, communication and analytical mechanisms aimed at supporting the individual educational trajectory of students, developing digital interaction and ensuring the continuity of the educational process. Its functioning is based on the principles of openness, interactivity, personalization, adaptability and accessibility.

The process of user interaction with digital resources, a significant array of educational data can form, reflecting the features of educational activity, the intensity of communication, the results of completing

educational tasks, the level of involvement and individual learning trajectories. Each user action - authorization in the system, viewing educational materials, completing practical tasks, taking tests, participating in forums, using digital resources, communicating with a teacher or other students - forms a digital trace that reflects the characteristics of educational behavior. The results of predictive analysis allow to identify student who need additional consulting to assess the level of involvement in learning, to predict the likelihood of academic difficulties, to optimize the study load and to adjust individual educational trajectories.

Digital technologies, educational analytics tools, predictive models and special artificial intelligence tools can perform the function of intellectual information support, providing the teacher with up-to-date data, forecasts and recommendations on possible areas of pedagogical influence. But the final decision remains the result of the teacher's professional judgment, which takes into account analytical indicators, pedagogical experience, the specifics of the academic discipline, individual characteristics of students, ethical aspects and a specific educational context.

TOPOLOGICAL ENSEMBLE FUZZY RISK MODEL FOR DECISION SUPPORT

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Modern anomaly detection methods based on machine learning effectively identify abnormal behaviour in multidimensional data; however, the final decision is usually obtained using a predefined threshold. Therefore, the development of intelligent decision support systems capable of reasoning under uncertainty represents an important scientific challenge.

This paper proposes the design of an intelligent decision support system based on a new Topological Ensemble Fuzzy Risk Model (TEFRM) that combines ensemble machine learning, Topological Data Analysis (TDA) and fuzzy inference into a unified mathematical framework.

The proposed system consists of consecutive stages. The proposed approach introduces an integrated fuzzy risk indicator:

$$Rf = \Phi(S_{IF}, S_{LOF}, S_{SVM}, \beta 1, \rho, \kappa),$$

S_{IF}, S_{LOF}, S_{SVM} - Isolation Forest, Local Outlier Factor and Support Vector Machine; $\beta 1$ - characterizing the topological complexity; ρ - denotes the graph density, while κ - represents the ensemble consistency coefficient defined.

To implement the proposed mathematical model, a new Topological Ensemble Fuzzy Decision Algorithm (TEFDA) is developed. During the first stage, the algorithm calculates anomaly scores using the ensemble of machine learning models. The second stage extracts topological descriptors of the data manifold using Mapper. Next, the obtained parameters are transformed into fuzzy variables through adaptive membership functions. A fuzzy inference engine evaluates the integrated risk indicator according to expert rules that simultaneously consider anomaly intensity, topological complexity and ensemble consistency. Finally, the defuzzified output is converted into an adaptive recommendation describing the required response strategy.

Therefore, the proposed model transforms conventional anomaly detection into a comprehensive intelligent decision support process.

The proposed Topological Ensemble Fuzzy Risk Model provides a new mathematical framework for integrating ensemble learning, topological data analysis and fuzzy reasoning. Replacing crisp threshold classification with an integrated fuzzy risk assessment improves the reliability, adaptability and interpretability of decision making in uncertain environments.

ISSUES REGARDING SHORE PROTECTION ALONG THE BLACK SEA COAST OF ADJARA

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The entire coast of Georgia, and Adjara in particular, constitutes a significant area where the region's recreational potential is actively utilized. At the same time, coastal geosystems are a key element in maintaining the natural biodiversity of the entire region. The Black Sea coast of Georgia spans 310 km, of which 57 km falls within the coastal strip of the Autonomous Republic of Adjara

An examination of the condition of shore protection structures along the Black Sea coast of Adjara, combined with a review of scientific and

technical practices in other maritime nations, reveals a wide variety of employed methods and structural designs. The range of materials used and the methods for modeling shore protection systems are also evolving and expanding. At the same time, the economic aspect of the issue must always be taken into account.

The primary challenge regarding coastal protection along Georgia's Black Sea coast—and in Adjara in particular—is the erosion of beaches and shorelines caused by rising sea levels, storms, and a shortage of river-borne sediment. The construction of dams on rivers and the extraction of gravel from riverbeds have resulted in a situation where the sea removes beach material without depositing anything in its place.

An effective method of shore protection involves creating wide sand-and-pebble beaches in conjunction with submerged breakwaters. Such beaches dissipate wave energy more effectively than any concrete wall. It is preferable to avoid continuous concrete embankments in favor of permeable structures, groynes and breakwaters. Ideally, shore protection structures and methods should be used in combination with one another.

Continuous and systematic monitoring of shore protection systems is essential. Prior to design and construction, it is necessary to implement systems for the mathematical modeling of sediment dynamics—both in the sea and in rivers flowing into it—to simulate processes virtually and assess all potential risks.

A comprehensive shift from the localized repair of individual sections to a unified state strategy for protecting Georgia's entire Black Sea coast.

MARKOV MODEL OF FOOTBALL PASSING REGIMES WITH STOCHASTIC APPROXIMATION REFINEMENT

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A compact continuous-time Markov model represents football possessions as trajectories over five interpretable passing regimes: circulation C, progression P, threat T, shot or terminal attacking action S, and loss L. The approach follows compartment-model logic used in applied dynamical systems [1], but transfers it from population groups to event sequences in football.

Let $X(t)$ belong to $\{C,P,T,S,L\}$. The allowed transition graph is

$$C \to P,L;\ P \to C,T,L;\ T \to P,S,L,$$

where S and L are absorbing states. If $\pi(t)$ is the row vector of state probabilities, the baseline dynamics are defined by the Kolmogorov forward equation

$$\frac{d\pi(t)}{dt} = \pi(t)Q_A \tag{1}$$

and the exact transition operator on an observed interval Δt is

$$\pi(t+\Delta t) = \pi(t)exp(Q_A\ \Delta t) \tag{2}$$

The baseline generator is estimated by the transition count / exposure time rule

$$qi_j = \frac{N_{ij}}{T_i}\ ,\quad q_{ii} = -\sum_{j\neq i} q_{ij} \tag{3}$$

where N_{ij} is the number of observed transitions from state i to state j, and T_i is the total dwell time in state i.

To refine the generator, a stochastic approximation procedure (SAP) is applied to the off-diagonal intensities while preserving the same state space, structural zeros, and absorbing states. The parameter update has the form

$$\theta_{n+1} = \Pi_\Theta\ \left[\theta_n + a_n\ H(\theta_n; X_n, X_{n+1}, \Delta t_n)\right], \tag{4}$$

where the step sequence satisfies the standard stochastic approximation conditions $\sum a_n = \infty$ and $\sum a_n^2 < \infty$ [3, 4].

Two SAP objectives are compared: a one-step transition likelihood $log\left[exp(Q_\theta\ \Delta t)\right]_{ij}$ and a rollout-aware likelihood that propagates $\pi(t+\Delta t) = \pi(t)exp(Q_\theta\ \Delta t)$ over the whole possession trajectory.

On event and spatiotemporal data from seven Bundesliga matches [2], rollout-aware SAP gives the best rollout negative log-likelihood, improving it from 1.562485 to 1.531682, with rollout accuracy increasing from 0.461615 to 0.466957. The remaining calibration error shows underestimation of continued progression *P* and overestimation of terminal loss *L*, suggesting a future discrete-time or semi-Markov companion model.

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DISCREPANCY ANALYSIS IN A SYSTEM OF LOGARITHMIC TREND EQUATIONS FOR UKRAINIAN TAX REVENUE FORECASTING

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Reliable tax revenue forecasting is essential for fiscal planning. Let PN_t denote total tax revenue and T_{kt} denote revenue from a component (VAT, PIT, CIT or excise tax). We estimate three separate trend models:

$$\ln(PN_t) = a_0 + b_o t, \quad \ln(T_{kt}) = a_k + b_k t, \quad s_{kt} = f_k(t), \qquad (1)$$

where $s_{kt} = T_{kt} / PN_t$ is the component share and $f_k(t)$ denotes its fitted trend function (linear). At the forecast horizon t^*, two forecasts are obtained:

$$\hat{T}_k = exp\left(a_k + b_k t^*\right), \quad \hat{T}_k^s = exp\left(a_k + b_k t^*\right) \cdot f_k(t^*). \qquad (2)$$

The discrepancy $\Delta_k = \hat{T}_k^s - \hat{T}_k$ measures disagreement between the two forecasting approaches. Log form:

$$\delta_k = (a_k + b_k t^*) - (a_0 + b_0 t^*) - \ln f_k(t^*). \qquad (3)$$

Data: State Treasury of Ukraine, index.minfin.com.ua 2005–2024 ($N = 20$). For 2025 ($t^* = 21$): PN $\approx$ 2 310 bln UAH (Table 1).

Table 1. Component forecasts and discrepancies for 2025 (mln UAH)

Component	Direct forecast	Share at t*	Via-share forecast	Δk, mln	δk (log)
VAT	890 406	38.6%	890 919	+513	−0.001
PIT	629 963	27.2%	627 707	−2 256	0.004
CIT	165 533	5.6%	129 004	−36 530	0.249
Excise	359 166	14.9%	343 715	−15 451	0.044
Other	289 855	13.8%	318 950	+29 095	−0.096

The CIT forecasts show the largest discrepancy (−22%, $\delta_k = 0.249$): its share fell from 24% to 6%, violating the log-linear trend. VAT is most consistent ($\left|\Delta_k / \hat{T}_k\right| < 0.1\%$, $\delta_k = -0.001$). Excise and Other also require revision ($|\delta_k| > 0.04$). Rule: $|\delta_k| \le 0.05$ endorses; $|\delta_k| > 0.05$ signals structural break.

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ENHANCING THE RESILIENCE OF CRITICAL INFRASTRUCTURE SIGNAL SYNCHRONIZATION SYSTEMS UNDER UNCERTAINTY: MODERN SOLUTIONS AND STRATEGIC PRIORITIES

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The operation of modern mobile telecommunications (LTE Advanced and 5G), Smart Grid power systems, and other high-tech infrastructures depends on the resilience of signal synchronization systems. Time uncertainty in synchronization signals creates significant

risks of service disruption and system failures, which is of particular concern for Ukraine's critical infrastructure under wartime conditions.

To address this challenge, a comprehensive multi-layer resilience architecture is proposed based on echeloned diversification of time sources. The proposed approach integrates enhanced Primary Reference Time Clocks (ePRTCs) employing optical cesium atomic clocks, terrestrial distribution of synchronization signals via fiber-optic communication links and IP/MPLS networks using the Precision Time Protocol (PTP, IEEE 1588), and an intelligent automated monitoring system synchronization signal parameters. Furthermore, the National Space Facilities Control and Test Center is considered as an additional source of precise timing to provide spatial redundancy of time references.

The effectiveness of the developed comprehensive multi-level redundancy architecture incorporating an intelligent automated synchronization signal monitoring system has been validated using the mobile network of the operator *Kyivstar* as a case study. The obtained results confirm compliance with the metrological requirements of ITU-T Recommendations and demonstrate improved resilience of synchronization systems under martial law.

ON THE ANALYSIS OF STOCK PRICE FORMATION MODELS

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As shown in (Kulian, Yunkova, & Korobova, 2022), based on the market model of W. Sharpe, generalized deterministic mathematical models of the formation of the market value of a portfolio of stocks can be written in the form

$$\dot{r}_p = f_p(r_p, x_i, \dot{x}_i, r_i, \dot{r}_i, t)$$

respectively, and are given parametrically. Here r_i is the expected market value of the stock; r_p is the expected market value of the investment portfolio; x_i is a share of the i-type asset in portfolio; t is time.

The problem of optimizing a portfolio of shares from the point of view of its profitability in the static and practical case has the form

$$r_p(T) = \sum_{i=1}^{n} x_i(T) r_i(T) \to \max_{x \in X(t)}, \quad r_p(t_0) = r_{p_0}, \quad t \in [t_0, T]$$

and consists in constructing an investment portfolio with optimal profitability at the final point T. Here $X(t)$ is a bounded closed set of stocks. The right-hand sides of the equations of the system (1) are linear and depend on the parameters. To ensure the possibility of its integration

$$\frac{dr_i}{dt} = \left(\alpha_1 SM_{ind}(t) + \alpha_2 I(t)\right) r_i(t) + \alpha_3 (r_i(t), \rho_{ij}), \quad i, j = \overline{1,n}$$

we will build procedures for their determination.

In different formulations of the problem, the vectors $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ or $\alpha = (\alpha_1, \alpha_2)$ can be considered as the parameter vector which needs to be determined and this problem is addressed in the study.

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A METHOD FOR ASSESSING SYSTEM DEPENDENCIES BETWEEN THE LEVELS OF A COMPLEX PRODUCT

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Identifying system dependencies between the physical and information layers of a complex product is a prerequisite for creating a digital twin and synchronizing it with the actual product. Finding these dependencies also allows for assessing product performance, predicting technical condition, and refining design solutions even before testing is conducted, thereby increasing the efficiency of hardware-in-the-loop modelling of complex technical systems.

A method for assessing system dependencies is proposed for analyzing them. Its principles include: defining the information and physical layers of a product, their main components, functions, and data

exchange channels, and establishing interrelationships. Establishing a set of system dependencies that determine the interactions between the layers of a complex product, specifically functional, information, temporal, control, and feedback dependencies. For each dependency, corresponding criteria and mathematical indicators are formed, which are used to quantitatively evaluate the level of synchronization, the accuracy of information exchange, the efficiency of data transfer, and quality of functioning of the hardware-in-the-loop modeling system. During real-time, semi-naturalistic tests, parameters are simultaneously collected from the physical product and its digital twin.Based on the obtained experimental data, the values of selected indicators are calculated and compared. Based on the analysis results, discrepancies between the physical object and its digital twin are identified, their causes are determined, and adjustments are made to the parameters of the mathematical models and control algorithms. This method provides not only an analysis of individual indicators but also a comprehensive assessment of the performance of the hardware-in-the-loop modeling system for a complex product. The application of this method will enable quantitative assessment of the quality of information exchange, including data transmission delays and losses of information; the level of interaction between a physical object and its digital twin; identification of the most critical system dependencies that negatively impact the accuracy and reliability of modeling. It also creates the basis for the development of an intelligent and dynamic digital twin capable of adapting to changing operating conditions and providing decision support in real time.

MONTE CARLO METHODS FOR THE NUMERICAL ANALYSIS OF EVOLUTIONARY EQUATIONS WITH MULTIVALUED INTERACTION FUNCTIONS

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We study Monte Carlo integration for machine-learning approximations of the evolutionary inclusion

$$\partial u / \partial t - \nu \Delta u + f(u) \ni g(x,t), \quad (x,t) \in Q_T = \Omega \times (0,T),$$

$$\partial u/\partial t - \nu\Delta u + f(u) \ni g(x,t),\ (x,t) \in Q_T = \Omega \times (0,T), \qquad (1)$$

with $u|_{\partial\Omega} = 0$, $u(\cdot,0) = u_0$, where $f : \mathbb{R} \to 2^{\mathbb{R}}$ is a maximal monotone graph. Following the PINN / Deep Galerkin approach, f is replaced by its Pasch–Hausdorff envelope f_k with $Lip(f_k) = 2k$, and u_θ minimizes

$$\mathcal{L}_k(\theta) = E_\mu\left[\left|\frac{\partial u}{\partial t} - \nu\Delta u + f_k(u) - g\right|^2\right] + \lambda_1 E_{\mu_0}\left[|u - u_0|^2\right] + \lambda_2 E_{\mu_\partial}\left[|u|^2\right], \quad (2)$$

the expectations being evaluated over N random points. This step, not the network capacity, is the true bottleneck: the standard estimator satisfies $\sigma^2{}_k(\theta) \geq C \cdot k$, hence the gradient variance grows as $O(k^2/N)$, since the singular force acts only on the strip $\{0 \leq u \leq 1/k\}$, of measure $O(1/k)$. We propose adaptive importance sampling with density

$$q(x,t) \propto 1 + \alpha|\nabla_x u| + \beta \cdot 1_{\{|u| < \varepsilon k\}},\ \varepsilon_k = c/k,\ \beta = O(k), \qquad (3)$$

whose reweighted estimator keeps variance $O(1)$ uniformly in k. With the regularization error $\|u_k^* - u^*\| = O(k^{-1/2})$ this gives

$$E\|u_\theta^{(n)} - u^*\|_{L^2}^2 \leq C_1 k^{-1} + C_2/(nN) + C_3\varepsilon^2_{NN}, \qquad (4)$$

balanced by the choice $k = O(n^{1/2}N^{1/2})$. On a moving-interface benchmark with $k = 200$ the algorithm reaches the loss level 10^{-3} in 2847 ± 312 epochs instead of >20000, lowers the relative L^2 error from 2.1×10^{-1} to 8.3×10^{-3} and the gradient variance from 78.4 to 3.2, at 15% overhead.

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NEW ALGORITHM FOR DETERMINING KEY ECG PARAMETERS ON THE RR-INTERVAL

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An algorithm for the precise determination of waves, intervals, and segments is proposed. The algorithm is multi-step, and at each subsequent iteration of its operation, a gradual refinement of the required ECG parameters is carried out.

The constructed algorithm is based on the authors' previous research dedicated to the precise detection of R-waves [1]. Unlike many other studies aiming to locate P and T waves, the algorithm in this work is applied not to the entire ECG, but to each RR-interval separately, which was precisely identified.

We briefly describe the algorithm. In the first step of the algorithm, the search zones for P, Q, S, T, and U (if present) waves are isolated based on the maximum physiological norms of a healthy person, linked to the duration of the RR interval. In the second step, the coordinates of the P, Q, S, T, and U peaks, as well as the onset and offset points of the P and T waves, are determined on the noisy signal. In the third step, smoothing is performed using an accelerated Savitzky–Golay filter, which incorporates updated values immediately. In the fourth step, a continuous piecewise cubic function is constructed on the smoothed signal using a fixed-length sliding window, where the end of the previous window coincides with the start of the current one. Finally, peak amplitudes, wave durations, as well as intervals and segments are extracted using this analytical function.

The proposed algorithm was tested on publicly available ECG data [2], which contains a standard set of 2-minute ECG recordings used by the scientific community to evaluate automated ECG analysis algorithms. On this data, the algorithm showed very high efficiency.

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METHOD FOR RESAMPLING A CYCLIC ELECTROCARDIOGRAPHIC SIGNAL BASED ON A GREEDY ALGORITHM

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An electrocardiographic signal (ECG signal) belongs to the class of cyclic biomedical signals characterized by variations in heart rhythm and, consequently, by changes in the durations of individual cardiac cycles. Accordingly, such a signal contains a different number of discrete samples within different cycles [1, 2]. Under these conditions, direct matching of morphologically corresponding points of different cardiac cycles becomes complicated, which necessitates preliminary temporal normalization, interpolation, or signal resampling [2–4]. Existing approaches employ various signal-processing techniques, uniform and piecewise-uniform resampling, as well as spline interpolation to transform cardiac cycles into a comparable representation [1–4]. In particular, resampling cardiac cycles to the same number of samples is used to improve the alignment of their morphological structures [2, 3] and to obtain ECG signals of equal length for subsequent comparison of their amplitude characteristics [5]. At the same time, the choice of sampling frequency and interpolation method may affect the results of further ECG signal processing and the estimation of heart rhythm parameters [6].

This study considers a method for ECG signal resampling based on a mathematical model in the form of a cyclic random process with a segmental cyclic structure. To transform cycles of different durations into a consistent discrete structure, a resampling method is proposed that provides the same number of ordered same-phase samples in each cardiac cycle. Unlike approaches in which predominantly geometric or temporal scaling of the entire cardiac cycle or its individual segments is performed [2–5], the proposed method determines sample correspondence by taking into account their relative phase positions and the closeness of ECG signal values. The method is based on a greedy point-matching algorithm employing phase localization, a strict ordering condition for temporal coordinates, and linear interpolation.

The first ECG cycle is taken as the reference, or template, cycle. Let its samples be represented by the sequence

$$P_{1,l}^{ref} = \left(t_{1,l}^{ref}, f(t_{1,l}^{ref})\right), l = 1,\ldots,L_1, \quad (1)$$

where L_1 — the number of samples in the reference (first) cycle. For each subsequent k - th cycle, $k = 2,\ldots,C$, a set of original samples is defined.

$$P_{k,m} = \left(t_{k,m}, f(t_{k,m})\right). \quad (2)$$

The resampling task consists in determining, for each reference sample, the corresponding same-phase sample in the subsequent cycles so that, after processing, the following condition is satisfied.

$$L_1 = L_2 = \ldots = L_C. \quad (3)$$

To prevent the algorithm from switching to another segment of the ECG realization, the relative position of each reference sample within the cycle is determined using a normalized phase coordinate:

$$p_l = \frac{t_{1,l}^{ref} - t_{1,s}}{t_{1,e} - t_{1,s}}, \quad 0 \le p_l \le 1, \quad (4)$$

where $t_{1,s}$ and $t_{1,e}$ - are the initial and final time coordinates of the reference cycle, respectively. In the corresponding k - th cycle, the expected position of the point is determined as

$$t_{k,l}^{exp} = t_{k,s} + p_l(t_{k,e} - t_{k,s}). \quad (5)$$

Thus, if the reference point is located, for example, at 30% of the duration of the first cycle, its corresponding point in the subsequent cycle is initially searched for near 30% of the duration of that cycle. It is the phase coordinate that ensures the structural consistency of the resampling procedure.

A local search interval $I_{k,l}$ is formed around the coordinate $t_{k,l}^{exp}$. For internal points, its boundaries are defined by the midpoints between the neighboring expected coordinates:

$$I_{k,l} = \left[\frac{t_{k,l-1}^{exp} + t_{k,l}^{exp}}{2}, \frac{t_{k,l}^{exp} + t_{k,l+1}^{exp}}{2}\right]. \quad (6)$$

Such a restriction prevents the algorithm from selecting a point with a similar amplitude (value) located in another part of the cardiac cycle and ensures local phase correspondence of the samples.

Within $I_{k,l}$ the absolute difference between the value of each admissible real sample and the corresponding reference value is calculated:

$$\Delta_{k,l,m} = \left| f(t_{k,m}) - f(t_{1,l}^{ref}) \right|.$$

The corresponding point is selected as the one for which this difference is minimal and does not exceed the specified closeness threshold ε. At the same time, the strict ordering condition for the time coordinates imposed by the mathematical model in the form of a cyclic random process is enforced.

$$t_{k,l}^{found} > t_{k,l-1}^{found}, \tag{7}$$

which prevents the permutation of samples and ensures preservation of the temporal structure of the cycle.

If no sufficiently close point of the real ECG signal is found within the specified local interval, linear interpolation between two neighboring samples is applied.

$$\left(t_{k,m}, f(t_{k,m})\right), \quad \left(t_{k,m+1}, f(t_{k,m+1})\right), \tag{8}$$

provided that the reference value lies between their amplitude values. The coordinate of the interpolated point is determined by the following formula:

$$t_{k,l}^{int} = t_{k,m} + \frac{f(t_{1,l}^{ref}) - f(t_{k,m})}{f(t_{k,m+1}) - f(t_{k,m})}(t_{k,m+1} - t_{k,m}). \tag{9}$$

If it is not possible to determine either an admissible point in the real ECG signal or an exact interpolated crossing at the level of the reference value, the nearest admissible point of the piecewise-linear ECG representation is selected.

A specific feature of the proposed algorithm is its greedy nature: at each step, the locally best admissible correspondence is selected, after which the decision is not revised. To control the results, each generated point was assigned a marker: 0 — a real point of the ECG signal or a sufficiently close real point; 1 — a point obtained by interpolation; 2 — the locally nearest admissible point.

As a result of applying the method, the original ECG signal with a uniform sampling step and a different number of samples in individual cycles is transformed into a resampled representation with a variable sampling step, in which each cycle contains the same number of ordered same-phase samples as the reference cycle. Such a representation enables direct matching of corresponding samples from different cycles,

estimation of the discrete rhythm function, and subsequent statistical processing of the cyclic ECG signal.

The method was tested on ECG realizations and demonstrated the possibility of identifying real, interpolated, and locally nearest same-phase samples in the cycles following the reference cycle. Based on the obtained correspondences, the discrete rhythm function was estimated.

Conclusion. The proposed method for resampling a cyclic ECG signal based on a greedy algorithm enables cardiac cycles of different durations to be transformed into a consistent discrete structure with the same number of same-phase samples. The use of a normalized phase coordinate, a local search interval, an amplitude-closeness criterion (closeness of signal values), strict temporal ordering, and linear interpolation makes it possible to preserve the structural correspondence of samples between cycles. The obtained resampled representation can be used for rhythmic structure estimation, construction of the discrete rhythm function, and further statistical analysis of ECG signals.

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A MULTI-CRITERIA FRAMEWORK FOR SCALE–ITERATE–STOP DECISIONS IN DIGITAL PRODUCT PORTFOLIOS UNDER UNCERTAINTY

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Managers of digital product portfolios repeatedly face the same gate decision: scale an initiative, iterate on it, or stop it. It is taken under coupled uncertainties: statistical, from short experiments; structural, from non-stationary acquisition markets and platform policy changes; and epistemic, since several criteria exist only as expert judgements. Prevailing practice collapses this to a single financial threshold such as payback period, which kills slowly maturing assets and over-funds products whose early metrics are noise.

We propose a framework in which each gate is modelled as a choice from the discrete action set {scale, iterate, stop}. A product is described by a criterion vector combining quantitative attributes — the posterior of unit economics, achieved statistical power, retention decay — with qualitative ones, such as compliance exposure or the team's opportunity cost. Quantitative criteria are represented by Bayesian posteriors, qualitative criteria by fuzzy numbers over a linguistic scale, and both are aggregated by a fuzzy TOPSIS-type procedure with weights from pairwise expert comparison.

The sequence of gates is treated as an optimal stopping problem in which the value of iterating includes the option value of the information the next iteration delivers. The loss function is asymmetric: abandoning a viable product and sustaining a non-viable one carry different costs, whose ratio is a portfolio-level parameter. Decision rules are evaluated by Monte Carlo simulation over the posteriors, and robustness as the sensitivity of the ranking to perturbations of the weights.

The framework is illustrated on a portfolio of consumer subscription applications, using retrospective data on gate decisions and their realised outcomes. We show where single-metric and multi-criteria rules diverge, which criteria carry most of the discriminating power, and what this implies for the governance of such decisions.

OPTIMIZATION OF THE SYSTEMS GOVERNED BY LINEAR HYPERBOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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The talk is dedicated to the study of hyperbolic integro-differential equations with partial derivatives.

The main goal is to prove the existence and uniqueness of generalized solutions to the corresponding initial-boundary value problem, as well as to investigate the existence of optimal control for systems described by these models.

The main results regarding the well-posedness of the initial-boundary value problem and the existence of optimal control are obtained using the method of a priori inequalities in negative norms.

In particular, inequalities in negative norms for the integro-differential operator in certain special spaces are obtained.

A theorem concerning the well-posedness, i.e., the existence, uniqueness, and continuous dependence of the generalized solution on the right-hand side of the equation, is presented. Control is exerted through the right-hand side of the equation using a certain special operator. The work examines some examples of such control operators (illustrating various control mechanisms) and the corresponding function spaces. By utilizing proven inequalities and relying on general theorems from the theory of a priori inequalities in negative norms, conditions for the existence of optimal control are established. In particular, restrictions are imposed on the admissible set of controls and the cost criterion in different scenarios.

The talk is based on [1, 2].

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ARTIFICIAL INTELLIGENCE FOR WHEAT YIELD PREDICTION IN UKRAINE

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The agricultural sector of Ukraine is characterized by high-tech processes of growing and processing various crops, including cereals, which make up a significant share of exports and filling the state budget with foreign currency. Winter wheat occupies a special place in these processes.

Automation of the process of growing and processing winter wheat, as one of the main factors ensuring efficiency, requires the development of reliable mathematical models for forecasting the winter wheat harvest and its quality, which creates conditions for planning the stages of effective distribution of finances, which will provide appropriate technological support.

The purpose of the work is to create and test a reliable mathematical model for predicting the yield of winter wheat for the Forest-Steppe zone of Ukraine. It was taken into account that the harvest and its quality are significantly affected by natural factors and growing technologies.

To create and test the reliability of the mathematical model, we used long-term observations over the years, published by the scientific community of Ukraine for various territorial zones of Ukraine. In this case, traditional mathematical statistical methods of processing the results of observations and separate artificial neural models were used, for training which observations over the years were used. The results demonstrated a fairly high degree of convergence.

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A DECISION SUPPORT INFORMATION-ANALYTICAL MODEL FOR DEMOGRAPHIC SECURITY MANAGEMENT

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1. Introduction and Problem Formulation

In the era of global digital transformation and the pervasive integration of artificial intelligence tools, state governance systems require advanced socio-demographic modeling architectures. Traditional demographic asset tracking systems predominantly rely on reactive methodologies, analyzing historical statistical aggregates post-factum. Such approaches introduce subjective, bureaucratic delays and fail to mitigate systemic risks.

This paper introduces a continuous, cyclical information-analytical decision support model designed to transition state governance from a passive observation of demographic crisis to active, result-oriented demographic security management. The framework targets high-frequency migration flow monitoring, eco-migrant re-allocation, and structural repatriation processes. By shifting emphasis from legacy databases to predictive forecasting algorithms, the model enables the targeted prevention of imminent demographic collapse.

2. Hierarchical Matrix Data Pipeline

The core infrastructure of the proposed information model organizes qualitative and quantitative identity parameters through a three-level hierarchical matrix transformation pipeline. This structures raw field questionnaires into computationally viable inputs:

- Initial Identification Matrix (M_I) This matrix acts as the foundational data layer. It profiles primary target objects (emigrants, repatriants, and eco-migrants) by mapping required qualitative vectors, familial backgrounds, professional competencies, and structural needs derived from standard identification questionnaires.
- Electronic Portal Interpretation Matrix (M_P) This operational layer functions within a secure, cloud-based electronic platform. The matrix maps physical identity parameters into dynamic web-objects filled out directly by respondents globally, converting fragmented inputs into standardized relational arrays.

- **Analytical Transformation Matrix** (M_E) Represented at the administrative operator level, this matrix executes the automated extraction and synthesis of the cloud-based web-input into high-density, structured spreadsheet matrices (Excel database formats).

Mathematically, this progressive data refining sequence is characterized by the following deterministic function mapping:

$$M_I \rightarrow M_P \rightarrow M_E$$

3. Algorithmic Processing via Chachanidze's Inference Rules

Once the finalized structured database matrix (M_E) established at the command node, the administrative decision-making engine initiates the automated optimization sequence. Rather than utilizing repetitive linear scaling tools, the spreadsheet database is explicitly processed using Chachanidze's 6 Inference Rules, which are detailed as a separate, independent scientific report within this conference.

By applying first-order predicate calculus and set-theoretic intersection metrics, the algorithm evaluates individual priority coefficients against limited state resource arrays. It automatically populates optimized repatriation registries within the boundaries of personal interests. Crucially, when severe data discrepancies or critical resource misalignments occur, the system triggers Chachanidze's Rule 6, immediately shifting the digital infrastructure into an interactive dialogue mode. This localized feedback loop ensures absolute operational flexibility, allowing administrators to initiate targeted human-in-the-loop negotiations even when initial mathematical intersections fail to register.

4. Numerical Simulations and Conclusion

To validate the resilience of this three-level hierarchical model, numerical simulations were executed mapping various simulated migration pressures and eco-demographic displacement scenarios. The algorithmic transformation pipeline $\left[M_I \rightarrow M_P \rightarrow M_E\right]$ maintained structural data integrity and zero packet loss under high-volume input noise. The computational results confirm that the framework provides a highly efficient, mathematically sound tool for automated demographic quality management, risk mitigation, and sovereign security engineering under uncertainty.

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COMPETENCE-BASED PREPARATION OF AI AND IOT BUSINESS LEADERS FOR DECISION-MAKING UNDER UNCERTAINTY: THE TRANSLEADER FRAMEWORK

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The rapid development of Artificial Intelligence (AI) and the Internet of Things (IoT) is transforming contemporary organizations into complex, data-driven and highly interconnected socio-technical systems. Decision-making in such environments is increasingly performed under conditions of incomplete information, uncertainty, dynamically changing data, cybersecurity risks, and competing technological and business objectives. Consequently, the preparation of future IT business leaders requires not only advanced technical knowledge but also the ability to evaluate uncertain situations, make responsible decisions, coordinate multidisciplinary teams, and adapt organizational strategies to rapidly changing technological environments.

This contribution presents the **learning requirement framework developed within the Erasmus+ TransLeader project, "The transferable training model – the best choice for training IT business leaders" (2023-2-PL01-KA220-HED-000179445)**. The project develops a transferable and industry-oriented training model integrating AI and IoT education with leadership and organizational competencies. Particular attention is paid to creating a framework compatible with European educational practices while supporting the modernization and international alignment of IT education in Ukraine.

The proposed framework organizes learning requirements for IT business leaders into four interconnected competence groups: **(1) technical competencies, (2) leadership and organizational competencies, (3) professional dispositions, and (4) self-management competencies**. Technical requirements include AI and machine learning, data management, software engineering, IoT systems, network and security management, and systems analysis and performance optimization. These competencies provide future leaders with the technological background required to understand data-driven systems and evaluate alternative technological solutions.

From the perspective of decision-making under uncertainty, particular importance is assigned to leadership competencies involving **problem-solving and decision-making, change management, visionary leadership, conflict management, innovation and creativity, and enterprise management**. They are complemented by professional dispositions such as ethical responsibility, critical self-reflection, responsiveness, resilience, inclusiveness, and accountability. Self-directed learning, technical adaptability, flexibility, and courage in decision-making under uncertainty constitute an additional layer of the framework. Together, these competence groups reflect the fact that decisions concerning AI and IoT systems cannot be based solely on technical performance indicators; they must simultaneously consider uncertainty, organizational objectives, security, ethical implications, stakeholder expectations, and available resources.

The framework is intended to support a transition from conventional knowledge-oriented curricula toward **competence-based and scenario-oriented learning**. AI/IoT learning scenarios can reproduce realistic decision situations in which students analyze incomplete or changing information, compare alternative technological solutions, assess risks, justify decisions, and evaluate their consequences. Examples include selecting machine-learning models under uncertain data quality, balancing accuracy and computational cost, managing cybersecurity risks in IoT infrastructures, or making business decisions based on imperfect predictions generated by intelligent systems. Such scenarios integrate mathematical and computational reasoning with leadership practice and responsible decision-making.

The TransLeader approach therefore positions **decision-making under uncertainty as a transversal competence connecting AI, IoT, business management and leadership education**. The resulting learning requirement framework provides a basis for designing transferable curricula, adaptive learning scenarios, project-based activities and assessment methods that prepare future IT business leaders to operate effectively in uncertain and rapidly evolving digital environments. The proposed approach may also contribute to broader discussions on how uncertainty, artificial intelligence and decision-support methods can be systematically incorporated into higher education for future technology leaders.

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FUZZY SET OF ROUTES IN TRANSPORTATION PROBLEM

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The classical transport problem (TP) minimizes the cost of transportation from a set $I = \{1, 2, ..., m\}$ of points of departure to a set $J = \{1, 2, ..., n\}$ of points of consumption. We assign the pair $(i, j) \in I \times J$ of indices to each communication route between the i th supplier and the j th consumer. Assume that a decision maker cannot clearly state which routes from the set $S = I \times J$, can actually be used, but can only specify a membership function $s(i, j)$, $(i, j) \in S$ of a fuzzy set $\tilde{S}$ of indices of communication routes that, in decision maker's opinion, should be used. Then a transportation problem with the fuzzy set $\tilde{S}$ of communication routes arises:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min,$$

$$\sum_{j\in J} x_{ij} \le a_i, \;\; i \in I; \; \sum_{i\in I} x_{ij} \ge b_j, \;\; j \in J; \; x_{ij} \ge 0, \;\; i \in I, j \in J;$$

$$x_{ij} > 0, \; ((i, j), s(i, j)) \in \tilde{S}.$$

According to [1], this report discusses the costs and corresponding amount of transporting product along a fuzzy set of feasible routes. We prove that the total transportation cost has the form of a type-2 fuzzy set on the real line. We investigate properties of this set and explain how to construct it with the help of the decomposition approach. It is shown that a type-2 fuzzy set representing the total transportation cost can be decomposed according to the secondary membership grades into a finite collection of embedded fuzzy numbers. Each of them is a total transportation cost of the corresponding fuzzy TP with a crisp set of feasible routes. This set is the corresponding cut of the original fuzzy set of feasible routes.

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LINGUISTIC MARKERS OF UNCERTAINTY IN CONTEMPORARY FRENCH: MODALITY, PREDICTION, AND DECISION-MAKING

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In the contemporary communicative environment, decision-making processes are increasingly carried out under conditions of uncertainty, incomplete information, and the assessment of future events. This circumstance has a significant impact on the linguistic system, as language represents not only a mechanism for the transmission of information but also a fundamental means of expressing human cognitive, evaluative, and predictive activity. It is through language that possible scenarios are constructed, degrees of probability are determined, and decision-making processes are shaped communicatively.

The present study is devoted to the analysis of linguistic markers expressing uncertainty in contemporary French, with particular attention given to modal verbs and constructions conveying probability. The primary focus of the research is on linguistic units such as *pouvoir*, *devoir*, *sembler*, and *paraître*, as well as lexical-syntactic constructions including *peut-être*, *probablement*, *sans doute*, and *il est possible que*. These linguistic means express the speaker's attitude toward the reliability of information, possible developments, and anticipated outcomes.

The aim of the study is to determine the functional and pragmatic roles played by uncertainty markers in contemporary French discourse and to examine how they contribute to the linguistic modelling of decision-making and prediction processes. The research is based on contemporary theoretical approaches to modality, epistemic evaluation, pragmatics, and discourse analysis.

The analysis demonstrates that uncertainty markers in French do not function merely as grammatical or lexical units. Rather, they operate as significant communicative strategies that enable speakers to express hypotheses, assess the quality of information, mitigate categorical statements, and present possible scenarios of future developments.

The study reveals that epistemic modality in contemporary French constitutes a linguistic mechanism through which individuals interpret reality, evaluate possible outcomes, and predict future events. Thus, the investigation of linguistic markers of uncertainty extends beyond the boundaries of purely grammatical analysis and is closely connected with the cognitive, communicative, and social functions of language.

INTENT-CENTRIC AI AGENTS FOR DECISION-MAKING UNDER UNCERTAINTY

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Modern digital ecosystems remain app-centric: users manually select applications, compare providers, interpret interfaces, authorize transactions and execute fragmented tasks. This paper introduces the Intent-Centric Operating Environment (ICOE) as an original conceptual architecture for AI-assisted decision-making under uncertainty, especially relevant for mobile ecosystems and smart AI devices. In ICOE, the user defines a goal; a personal AI agent interprets intent, queries a machine-readable Global Skill Registry, ranks operators by price, time, reliability, privacy and trust, and executes selected actions through standardized APIs in isolated environments. Fig. 1 illustrates a mobile/smart-device scenario: the user requests a taxi, music or restaurant booking; the agent selects an operator through the registry and executes the action. Thus decision-making shifts from manual interface navigation to automated service discovery and multi-criteria selection. Main uncertainties include intent ambiguity, incomplete provider data, changing user preferences and execution-security risks. The architecture combines semantic analysis, contextual preference modeling, registry-based service discovery, risk filtering and feedback adaptation. ICOE is positioned not as a voice assistant, but as a mediator between the user and a dynamic market of digital services. The contribution is the introduction of ICOE as a framework for reducing cognitive load and improving decision quality; future work includes formal intent representation, trust scoring and optimization models.

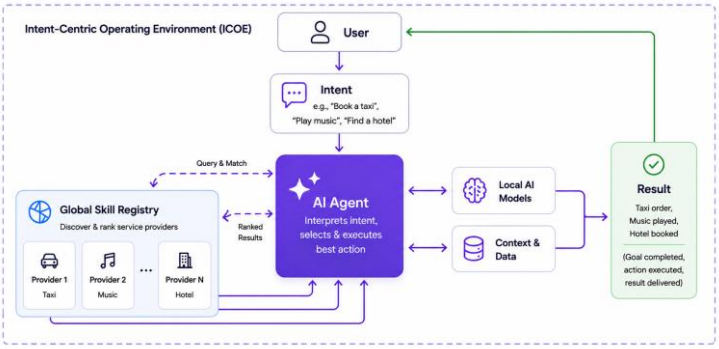

Fig. 1. ICOE flow from mobile/smart-device intent to AI-mediated service selection and execution.

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OPTIMIZING DATABASE RETRIEVAL USING AGGREGATED AND PROBABILISTIC QUERY CHARACTERISTICS

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The expanding volume of data in information systems demands new approaches to relational database retrieval. Traditional exact matching or LIKE-based searches are simple to implement but scale poorly and fail to reflect document relevance. Consequently, query optimization is vital, directly determining data access time, recall, and decision-support quality.

While PostgreSQL offers built-in full-text search, its standard evaluation relies heavily on lexical features. It overlooks crucial factors like a priori thematic relevance, temporal value, popularity, and short-query ambiguity.

The probabilistic approach to information retrieval addresses this by estimating relevance based on document features, typically combining term frequency, inverse document frequency, and length normalization. Moreover, contemporary query optimization research indicates that distribution estimation accuracy directly dictates execution plan selection. This creates a need for an interpretable, reproducible method that operates within relational database management systems without relying on proprietary industrial data.

This study addresses this gap at the intersection of retrieval algorithms, data mining, and scalable system design.

This paper develops and evaluates search query optimization tools using an aggregated probabilistic ranking function in PostgreSQL. Tested on a benchmark dataset, the approach is analyzed alongside system scalability under an increasing text corpus volume.

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SYSTEM ANALYSIS AND DECISION MAKING FOR USAGE OF RENEWABLE SOURCES OF ENERGY IN AGRICULTURAL ENTERPRISES

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Under the conditions of regular and irregular power outages in Ukraine, farms, especially those engaged in livestock farming, suffer greatly in the winter, when uninterrupted heating for animals and electricity for the functioning of the farm are needed. It becomes extremely important to transition to at least partial self-sufficiency of farms in electricity and heat energy from renewable energy sources. In particular, it is natural for livestock farms to use biogas as an energy source. But regular collection of biogas fuel and its conversion into electricity requires, in turn, energy costs. Thus, the task of balancing the complex power system of a farm enterprise arises, to ensure both generation and consumption of energy. Farmers do not have the opportunity and appropriate qualifications to independently solve this problem in the optimal way. We propose to develop a decision support system based on real data and modeling, which would allow farmers to optimally supply their farms with energy in the context of an energy crisis. The goal of the current report is to demonstrate the results of system analysis of energy needs of farms and application of decision-making methods for the use of biogas and other renewable energy sources. Cognitive map [1] is suggested as a basic model for further development of a control strategy in this complex dynamic system with uncertainties.

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A HIERARCHICAL DECISION SUPPORT SYSTEM FOR GEOGRAPHIC LOCALIZATION OF UKRAINIAN DIALECT WORDS

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The processes of language standardization and urbanization contribute to the gradual disappearance of regional linguistic features; therefore, the digital preservation of dialect heritage is an important research task. This study uses data collected through the Interactive Map of Ukrainian Dialects platform (dialectmap.org), developed by the authors. The automatic localization of dialect words is complicated by the spatial fuzziness of dialect boundaries, the heterogeneity of crowdsourced data, orthographic and phonetic variation, and a substantial imbalance in the regional representation of the collected material. These factors create a high level of uncertainty in the decision-making process.

To address this problem, a hierarchical decision support system is proposed for determining the geographic origin of a dialect word at the levels of region, district, and settlement. The system is based on a hybrid CharBiLSTM + k-NN architecture [1]. A dual-channel CharBiLSTM processes the orthographic form and phonetic transcription of a word and generates a probability distribution over the regions. When the model confidence is insufficient, the decision is automatically marked as uncertain and referred to an expert for verification. The k-NN algorithm then sequentially refines the predicted district and settlement within a filtered search space.

The proposed approach mitigates the impact of data imbalance and provides an interpretable decision-making process. The step-by-step localization procedure makes effective use of dominant linguistic clusters and improves settlement-level accuracy compared with direct district-level classification. Future research will focus on expanding the corpus, applying transformer-based models, and moving from discrete classification to the prediction of precise geographic coordinates of linguistic phenomena.

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MULTIVARIATE STATISTICAL ANALYSIS OF STUDENTS' ACADEMIC CHARACTERISTICS: CLUSTER SEGMENTATION AND FACTOR ANALYSIS

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In the contemporary educational environment, statistical analysis of students' academic data represents an important instrument for evaluating educational processes, identifying student profiles, and supporting evidence-based decision-making. The aim of our study is to identify patterns and relationships within students' academic data through the application of multivariate statistical analysis methods. The empirical basis of the study consists of a dataset of 120 students, including scholarship amount, academic year, and specialization. Statistical data processing was performed using the STATISTICA 12 software package.

At the first stage of the study, descriptive statistical methods were applied to assess measures of central tendency, variation, and distributional shape. Subsequently, hierarchical cluster analysis was conducted to identify groups of students with similar characteristics, while factor analysis was used to investigate the common latent structure among the analyzed variables. The cluster analysis revealed distinct homogeneous groups of students based on the combined consideration of academic year and scholarship characteristics. The results of the factor analysis indicated the presence of common variance among the analyzed variables.

The findings confirm that the combined application of multivariate statistical methods enables the identification and more in-depth interpretation of structural patterns within student data. This approach can be applied to educational data analysis, student segmentation, and evidence-based academic management.

Keywords: multivariate statistical analysis; students' academic data; descriptive statistics; cluster analysis; factor analysis; student segmentation; STATISTICA 12.

ON THE CONVERGENCE OF A DIFFERENCE METHOD FOR SOLVING AN OPTIMAL CONTROL PROBLEM FOR A LINEAR SCHRÖDINGER EQUATION WITH A SPECIAL GRADIENT TERM AND A COMPLEX TIME-DEPENDENT POTENTIAL

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In this paper, we consider the following optimal control problem of minimizing the functional:

$$J(v)=\left\|\psi(.,T)-y\right\|^2_{L_2(0,l)} \tag{1}$$

on the set:

$$V\equiv\left\{v=(v_0,v_1): v_s\in L_2(0,T), \left|v_0(t)\right|\le b_0, 0\le v_1(t)\le b_1, \overset{0}{\forall} t\in(0,T)\right\}$$

under conditions:

$$i\frac{\partial\psi}{\partial t}+a_0\frac{\partial^2\psi}{\partial x^2}+ia_1(x,t)\frac{\partial\psi}{\partial x}-a(x)\psi+v_0(t)\psi+iv_1(t)\psi= \\ =f(x,t),(x,t)\in\Omega, \tag{2}$$

$$\psi(x,0)=\varphi(x),\ x\in(0,l),\ \psi(0,t)=\psi(l,t)=0,\ \forall t\in(0,T), \tag{3}$$

where $i=\sqrt{-1}$; $a_0>0, l>0, T>0$, $b_s>0$, $s=0,1$ are given numbers; $\Omega=(0,l)\times(0,T)$; $a(x), a_1(x,t)$ are real-valued measurable bounded functions satisfying the conditions:

$$0\le a(x)\le\mu_1, \left|\frac{da(x)}{dx}\right|\le\mu_2, \left|\frac{d^2a(x)}{dx^2}\right|\le\mu_3, \overset{0}{\forall} x\in(0,l), \\ \mu_1,\mu_2,\mu_3=sabit>0; \tag{4}$$

$$0\le a_1(x,t)\le\mu_4, \left|\frac{\partial a_1(x,t)}{\partial x}\right|\le\mu_5, \left|\frac{\partial^2 a_1(x,t)}{\partial x^2}\right|\le\mu_6, \overset{0}{\forall}(x,t)\in\Omega, \\ \mu_4,\mu_5,\mu_6=sabit>0; \tag{5}$$

$\varphi(x), f(x,t), y(x)$ are complex-valued functions satisfying the conditions:

$$\varphi \in \overset{0}{W}{}_2^2(0,l),\ f \in \overset{0}{W}{}_2^{2,0}(\Omega),\ y \in \overset{0}{W}{}_2^1(0,l). \qquad (6)$$

For the optimal control problem (1)-(3), the convergence of the difference method for solving it is studied. First, the optimal control problem is discretized and an estimate for the solution of the corresponding difference scheme is established for each discrete control. Based on this, an error estimate for the difference scheme is proved. Next, using this estimate, the convergence of the difference approximations with respect to the functional is proved.

It should be noted that similar questions regarding optimal control problems for linear and nonlinear Schrödinger equations without a special gradient term have previously been studied, for example, in [1-3].

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INVERSE PROBLEMS FOR STOKES EQUATIONS UNDER UNCERTAINTY

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Let Ω be a bounded region in R^n space with piecewise-smooth boundary Γ, and let $f(x)$ be some vector function $f(x) = (f_1(x),...., f_n(x))^T$, whose components are square-integrated, $x \in \Omega$. Let us also assume that the vector function

$\varphi(x)=(\varphi_1(x),...,\varphi_n(x))^T$ and the scalar function $p(x)$ are generalized solutions of the Stokes equation

$$-\gamma\Delta\varphi+\operatorname{grad} p=f(x),\ x\in\Omega\ (\gamma>0), \qquad (1)$$

$$\operatorname{div}\varphi=0,\ x\in\Omega,\ \varphi\big|_{\Gamma}=0.$$

It is known [1], that there is a unique function from the Sobolev space $H_0^1(\Omega)$ and the function $p(x)\in L_2(\Omega)$, which are the generalized solution of equation (1).

Its studied the case, when the function $f(x)$ is unknown and scalar functions are given

$$y_k(t)=\int_{\Omega}\left(h_k(t,x),\varphi(x)\right)_{R^n}dx+v_k(t),\ k=\overline{1,N},t\in(0,T),$$

where $h_k(t,x)$ is known vector functions such that $\int_0^T\int_{\Omega}\left|h_k(t,x)\right|^2dtdx<\infty$, $v_k(t)$ is a sequence of unknown functions from space $L_2(0,T)$.

Let the functions $v_k(t),k=\overline{1,N}$ belong to bounded sets of $L_2(0,T)$, spaces, which we denote by $V_k,k=\overline{1,N}$, and the vector function $f(x)$ belongs to a bounded set F of the $L_2(\Omega)$ space. Denote by G_y the set

$$G_y=\left\{f:y_k(\cdot)-\int_{\Omega}\left(h_k(\cdot,x),\varphi(x)\right)_{R^n}dx\in V_k,k=\overline{1,N}\right\}\cap F,$$

and through G_y^- and G_y^+ are convex bounded closed G_y sets, which are the lower and upper approximations of the set in the sense that $G_y^-\subseteq G_y\subseteq G_y^+$.

Let us denote by B a linear continuous operator acting from the space $L_2(\Omega)$ into the H - Hilbert space . The upper and lower guaranteed estimates of the element Bf are the quantities $(Bf)_-$ and $(Bf)_+$, which are found from the conditions

$$\min_f\max_{f_1\in G_y^+}\left\|Bf-Bf_1\right\|_H=\max_{f_1\in G_y^+}\left\|(Bf)_+-Bf_1\right\|_H=\sigma_+,$$

$$\min_{f} \max_{f_1 \in G_y^-} \left\| Bf - Bf_1 \right\|_H = \max_{f_1 \in G_y^-} \left\| (Bf)_- - Bf_1 \right\|_H = \sigma_-.$$

The quantities σ_+ and σ_- are called the upper and lower errors of the guaranteed estimates.

The problems of calculating the upper and lower estimates of vectors Bf and their errors for sets of a special type are investigated.

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ESTIMATION OF STATIONARY PARAMETERS IN FAKE INFORMATION PROPAGATION MODELS UNDER UNCERTAINTY

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The propagation of disinformation in social networks is temporally and spatially heterogeneous because of the structure of connections among users and their distance from the source of a message. The similarity of this process to an epidemic makes it possible to use compartmental models [1, 2], while the spatial coordinate can be interpreted as a cybernetic or social distance in the network [3]. The purpose of this study is to estimate unknown parameters of a spatial-diffusion SIRC model for fake information propagation using observations with deterministic errors.

The information community is divided into four groups: $S(t,x)$ denotes susceptible users; $I(t,x)$ denotes active spreaders of fake information; $R(t,x)$ denotes users who have stopped spreading it; and $C(t,x)$ denotes spreaders of refutations and fact-checking information.

The dynamics of these states are described by the following system of nonlinear parabolic equations:

$$\begin{cases} \dfrac{\partial S}{\partial t} = D_S \nabla^2 S - \beta SI - \alpha SC, \\ \dfrac{\partial I}{\partial t} = D_I \nabla^2 I + \beta SI - \gamma I - \delta IC, \\ \dfrac{\partial R}{\partial t} = D_R \nabla^2 R + \gamma I + \eta C, \\ \dfrac{\partial C}{\partial t} = D_C \nabla^2 C + \alpha SC + \delta IC - \eta C. \end{cases}$$

The vector of unknown parameters is $\theta = (D_S, D_I, D_R, D_C, \alpha, \beta, \gamma, \delta, \eta)^T$, $\theta_i \geq 0$, $i = \overline{1,9}$.

The coefficients D_S, D_I, D_R, D_C characterize the spatial diffusion of the information states. The parameters α and β determine the transition of susceptible users to fact-checkers and spreaders of fake information, respectively; γ describes the natural exit from group I; δ characterizes the interaction between spreaders and fact-checkers; and η describes the exit from group C. Homogeneous Neumann conditions are imposed on the boundaries of the spatial domain.

The continuous model is discretized in time and space. Second-order spatial derivatives are approximated by central differences, while time derivatives are reconstructed at the grid nodes. After grouping the terms, the observation system at time t_k takes the form

$$Y_k = \rho_k \theta + \Xi_k, \ k = \overline{1,N},$$

where Y_k contains the time derivatives of the state functions, ρ_k is formed from the values of S, I, R, C and their finite-difference Laplacians, and Ξ_k describes the errors. Thus, the identification problem is reduced to a finite-dimensional estimation problem for the parameter vector.

Prior information about the parameters is specified by a vector $\theta^{(0)}$ and a positive-definite weighting matrix. For constant parameters, the estimate is obtained by minimizing a quadratic functional that combines the model residual with the deviation from the prior vector. This leads to a system of linear algebraic equations. The proposed approach makes it

possible to incorporate prior information and limit the influence of errors.

The computational experiment was conducted on the time interval $[0,30]$ and the spatial domain $[0,1]$. The uniform grid contained $N_t = 80$ time layers and $N_x = 30$ spatial nodes. For the four model components, discretization produces

$$N_{eq} = 4N_t N_x = 4 \cdot 80 \cdot 30 = 9600$$

scalar equations. The initial source of fake information was represented by a localized profile at the center of the domain: the proportion of spreaders was 0.03, while the proportion of fact-checkers was 0.02.

Errors were introduced only into the vector of time derivatives. At each node, the following perturbation was used:

$$Y_{kj}^{\varepsilon} = Y_{kj} + \sigma_{\xi} \left| Y_{kj} \right| \xi_{kj}, \quad \xi_{kj}^{S}, \xi_{kj}^{I}, \xi_{kj}^{R}, \xi_{kj}^{C} \in [-1,1],$$

where the absolute value is taken componentwise and $\sigma_{\xi} = 0.5\%$ or 1%. The prior vector $\theta^{(0)} = 0.9\theta^{*}$ had a systematic deviation of 10%. The parameters were estimated according to the criterion

$$\hat{\theta}_{\lambda} = \arg\min_{\theta} \left\{ \left\| Y^{\varepsilon} - \rho\theta \right\|^2 + \lambda \left\| \theta - \theta^{(0)} \right\|^2 \right\},$$

and the parameter λ was selected adaptively using the validation residual. This reduces the influence of inaccurate prior information.

Three scenarios were considered: C1 was the baseline scenario; C2 used increased values of α and δ; and C3 used an increased value of D_C. Recovery quality was evaluated only for parameters with nonzero true values:

$$\varepsilon_i = \frac{\left| \hat{\theta}_i - \theta_i^{*} \right|}{\left| \theta_i^{*} \right|} \cdot 100\%, \quad i \in I_0, \quad I_0 = \{ i \in \overline{1,9} : \theta_i^{*} \neq 0 \},$$

$$\bar{\varepsilon} = \frac{1}{|I_0|} \sum_{i \in I_0} \varepsilon_i, \quad \varepsilon_{\max} = \max_{i \in I_0} \varepsilon_i$$

The main results for the more difficult perturbation level $\sigma_{\xi} = 1\%$ are presented in Table 1. For $\sigma_{\xi} = 0.5\%$, the mean errors in scenarios C1–C3 were 0.054643%, 0.025135%, and 0.122144%, respectively, while the maximum errors were 0.235864%, 0.067085%, and 0.742872%.

Table 1. Errors for $\sigma_{\xi} = 1\%$

Scenario	λ^*	$\bar{\varepsilon}$, %	$\varepsilon_{\max}$, %
C1	0	0.277761	1.637854
C2	10^{-8}	0.124028	0.726983
C3	0	0.178147	1.103873

The adaptive algorithm selected a zero or very small value of λ^*. Therefore, when the observations are sufficiently informative, the estimate is determined primarily by the data, and the inaccurate prior vector does not cause a bias of approximately 10%. At the perturbation level of 1%, the mean error ranged from 0.124028% to 0.277761%, and the maximum error did not exceed 1.637854%. The best mean accuracy was obtained in scenario C2, while the largest deviations were observed for the parameter δ.

Thus, the finite-difference approximation reduces the identification of the spatial-diffusion SIRC model parameters to systems of linear algebraic equations. Adaptive use of the prior parameter vector, together with normalization and physically admissible constraints, ensures robust parameter recovery under deterministic perturbations of the system right-hand side. The estimated parameters can be used to forecast the propagation of fake information and analyze fact-checking measures. Further research will focus on validating the approach using real-world data.

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RECOGNITION OF PULSE SIGNALS UNDER UNCERTAINTIES

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Let the point $r_0 = (x_0, y_0, z_0)$ contain a source emitting a pulsed signal that propagates in space with a constant velocity c.
At points $r_k, k = \overline{1,m}$ at time $t_j, j = \overline{1,N}$ we measure the magnitude of this signal with errors η_{kj}. If at the moment t the signal magnitude was equal to $s(t, r_0, \theta_0)$ at the point r_0, then we will have the relation

$$y_{kj} = \frac{s(t_k - \tau_j, r_0, \theta_0)}{|r_j - r_0|} + \eta_{kj}, \quad k = \overline{1,m},\ j = \overline{1,N}, \qquad (1)$$

at the points $r_k, k = \overline{1,m}$, where θ_0 is unknown parameter vector from a finite-dimensional space, $\tau_j = \frac{|r_j - r_0|}{c}$.

The problem of pulse signal recognition is to select a subset of pulse signals $s(t, r_0, \theta_j), j = \overline{1,p}$ that, under certain constraints on the errors η_{kj} in a certain sense satisfies equation (1).

We investigate the case where the errors η_{kj} are deterministic and the case where the errors are random. For deterministic errors of the form $|\eta_{kj}| \le \bar{\eta}_{kj}$, where $\bar{\eta}_{kj}$ are some known numbers, we formulated a criterion for selecting signals from the set $s(t, r_0, \theta_j), j = \overline{1,p}$.

In the case where the errors η_{kj} are stochastic with zero mean and unknown second-order moments, the recognition of pulse signals is performed using Fourier transform estimation. For this purpose, a set is given to which the Fourier transforms of the signal $s(t, r_0, \theta_0)$ belong and guaranteed linear estimates of the short-time Fourier transforms are found, which are then compared with the short-time transforms of the signals $s(t, r_0, \theta_j), j = \overline{1,p}$.

Calculations are performed for the case when the pulsed signal is rectangular with frequency filling and for signals that have a Gaussian distribution.

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ENSEMBLE MACHINE LEARNING FOR COMPUTING SOLUTION TUPLES OF CUBIC MATRIX EQUATIONS

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We consider the cubic matrix equation $X^3 + A_2X^2 + A_1X + A_0 = O$, $A_i, X \in \mathbb{R}^{p\times p}$, which has up to $\binom{np}{p}$ isolated solutions [1] (84 for $n = p = 3$). Matrix branched continued fractions [2] represent a solution as the iterative process

$$X_{(t)} = q_0 + p_1(q_1E + X_{(t-1)})^{-1} + p_2(q_2E + X_{(t-1)})^{-1},$$

where E is the identity matrix, $q_0 = (q_1 + q_2)E - A_2$, and p_1, p_2 are analytic in q_1, q_2, A_i. Every successful pair gives a solution tuple (q_1, q_2, X), but manual selection of (q_1, q_2) reveals only a few of them.

We reformulate this search as binary classification: $y = 1$ if the process converges with residual $r = \left\| X^3 + A_2X^2 + A_1X + A_0 \right\|_F \le 10^{-6}$, and $y = 0$ otherwise; 12 features derived from (q_1, q_2) are used. The five-phase algorithm consists of random exploration of $[-80,80]^2$, training of an ensemble of gradient boosting, random forest and a multilayer perceptron, AI-guided selection of candidates, local search and two rounds of adaptive refinement with uncertainty sampling.

For the canonical and non-canonical examples of [2] with $p = 3$ the method found 6 unique tuples each instead of the 4 obtained manually ($+50\%$); all solutions were verified by substitution. The validation rate of the AI-guided search is 99.5% and 96.3% against 11.6% and 16.1% for random sampling. Over six random seeds the coefficient of variation is below 1% (Wilcoxon test, $p = 0.016$).

Conclusion. The proposed algorithm finds 50% more solution tuples of cubic matrix equations than manual selection of the parameters and uses the computational budget 6–8 times more efficiently, which makes it suitable for a systematic study of solution sets of matrix polynomial equations.

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ASYMPTOTIC ANALYSIS OF A STOCHASTIC MODEL OF THE TECHNOLOGICAL LABOR MARKET USING DIFFUSION APPROXIMATION

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We consider a stochastic model of the technological labor market that describes the interaction between the number of workers and the level of automation. A deterministic Lotka-Volterra type system has the following form

$$\begin{cases} \dfrac{dN(t)}{dt} = aN(t) - bN(t)A(t) - cN^2(t), \\ \dfrac{dA(t)}{dt} = -dA(t) + eN(t)A(t), \ t > 0. \end{cases} \tag{1}$$

To account for rapid random changes, we apply the diffusion approximation. Under the balance condition, $\int_E \pi(dx) C(u;x) \equiv 0$. Let us consider the normalized system:

$$\varepsilon^{-1} C\left(u^{\varepsilon}(t), x\left(\frac{t}{\varepsilon^2}\right)\right), \quad t \geq 0 \tag{2}$$

where $u^{\varepsilon}(t)$ – the state vector of the system, $x\left(\frac{t}{\varepsilon^2}\right)$ – a fast ergodic Markov process, $C(u;x)$ – the velocity. Under ergodic conditions $\varepsilon \to 0$ the stochastic system weakly converges to the limiting diffusion process, the generator has the form:

$$L\varphi(u) = b(u)\varphi'(u) + \frac{1}{2} B(u)\varphi''(u) \tag{3}$$

where $b(u)$ – drift parameter, $B(u)$ – diffusion coefficient. The limit process is given by a stochastic differential equation:

$$d\zeta^0(t) = b\left(\zeta^0(t)\right)dt + \sigma\left(\zeta^0(t)\right)dW(t) \tag{4}$$

where $\sigma^2(u) = B(u)$, $W(t)$ – Wiener process.

The diffusion approximation allows us to describe random fluctuations in the number of workers and the level of automation caused by rapid changes in technological regimes under conditions of uncertainty.

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DIFFUSION APPROXIMATION OF A STOCHASTIC MODEL OF RADIOPHYSICAL PROCESSES IN UAV MICROCONTROLLER SYSTEMS

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We consider a stochastic model of radiophysical processes in UAV microcontroller power supply systems under rapid electromagnetic interference and propulsion loads. The normalized system has the following form:

$$\frac{du^{\varepsilon}(t)}{dt} = \varepsilon^{-1} C\left(u^{\varepsilon}(t), x(t/\varepsilon^2)\right), \quad t \geq 0 \tag{1}$$

where $u^{\varepsilon}(t)$ is the state vector of the system, $x(t/\varepsilon^2)$ is an ergodic Markov process, and $\varepsilon > 0$ is a small parameter. Under the balance condition $E_{\pi} C(u;x) \equiv 0$ and ergodic conditions, the stochastic process weakly converges to the limit diffusion process. The generator has the form:

$$L\varphi(u) = b(u)\varphi'(u) + \frac{1}{2} B(u)\varphi''(u), \qquad (2)$$

where $b(u)$ is the drift parameter and $B(u)$ is the diffusion coefficient. The limit process for supply voltage deviations is given by an Ornstein-Uhlenbeck stochastic differential equation:

$$d\zeta_0(t) = -\alpha\zeta_0(t)dt + \sigma dW(t), \qquad (3)$$

where $\alpha > 0$, $\sigma = \sqrt{B(u)}$, and $W(t)$ is a Wiener process.

The diffusion approximation allows us to describe random fluctuations in power supply voltage caused by rapid ESC switching and motor load changes under conditions of uncertainty.

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ALEXANDER–SPANIER COHOMOLOGY AS A ROBUST DECISION CRITERION NEAR A HOPF BIFURCATION UNDER PARAMETRIC UNCERTAINTY

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Parametric uncertainty may change the qualitative long-term behavior of a dynamical system. We consider the parameterized Hopf normal form on the phase space $X = \mathbb{C}$:

$$\dot{z} = (\lambda + i)z - |z|^2 z, \ \lambda \in U = [-1,1].$$

Let the corresponding flow be denoted by

$$S_t^{\lambda}(z_0) = z(t;\lambda,z_0).$$

Using polar coordinates $z = re^{i\theta}$, the system becomes

$$\dot{r} = r(\lambda - r^2),\ \dot{\theta} = 1.$$

Consequently, the compact asymptotically stable set associated with the long-term regime is

$$A_\lambda = \begin{cases} \{0\}, & \lambda \le 0, \\ \{z \in \mathbb{C} : |z| = \sqrt{\lambda}\}, & \lambda > 0. \end{cases}$$

For $\lambda > 0$, the set A_λ is a stable periodic orbit with basin of attraction $\mathbb{C} \setminus \{0\}$. Since a point and a circle are compact metric spaces, their Alexander–Spanier cohomology groups agree with the corresponding classical cohomology groups. Therefore,

$$H^1_{\mathrm{AS}}(A_\lambda;\mathbb{Z}) \cong \begin{cases} 0, & \lambda \le 0, \\ \mathbb{Z}, & \lambda > 0. \end{cases}$$

The corresponding cohomological dimension is

$$\dim_{\mathbb{Z}} A_\lambda = \begin{cases} 0, & \lambda \le 0, \\ 1, & \lambda > 0. \end{cases}$$

Suppose that a decision-maker selects a nominal parameter c, while the actual parameter satisfies

$$\lambda \in I_\varepsilon(c) = [c-\varepsilon, c+\varepsilon] \subseteq U,\ \varepsilon > 0.$$

A decision is called strictly robust if the cohomological signature remains unchanged throughout $I_\varepsilon(c)$ and this interval has positive distance from the bifurcation value $\lambda = 0$. It follows that $c < -\varepsilon$ guarantees a strictly robust equilibrium regime, whereas $c > \varepsilon$ guarantees a strictly robust periodic regime. The band $|c| \le \varepsilon$ is critical because the uncertainty interval crosses or touches the bifurcation threshold and has no positive topological safety margin. Thus, the Alexander–Spanier cohomological signature provides an explicit robust decision criterion under bounded parametric uncertainty.

Keywords: Alexander–Spanier cohomology; Hopf bifurcation; parametric uncertainty; robust decision; attractor.

CONTROLLED MARKOV PROCESSES WITH ABSORPTION FOR MAXIMIZING PRODUCTION OUTPUT BEFORE BANKRUPTCY

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Production systems operating under uncertainty may be modeled by controlled Markov processes with absorption, where the absorbing state corresponds to enterprise bankruptcy. The objective is to maximize the expected discounted production output accumulated before the absorption moment.

A mathematical model describing the evolution of financial capital and production resources is proposed. The Bellman optimality equation is derived, existence of an optimal stationary policy is discussed, and monotonicity of the value function together with the threshold structure of the optimal production policy is established. Value iteration is presented as a numerical approach for computing the optimal strategy.

The proposed framework provides a rigorous mathematical basis for production planning under bankruptcy risk and can be extended to more general stochastic manufacturing systems.

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SOLVING A STRUCTURALLY FUZZY FLEXIBLE JOB SHOP SCHEDULING PROBLEM: A REAL-WORLD CASE OF A SERVICE COMPANY

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Maintenance scheduling for service companies requires deciding in parallel which jobs to accept and how to allocate resources under uncertainty. Addressing gaps identified in our survey [1] regarding utilization of structural fuzziness and applications on real-world datasets, this paper develops an algorithmic framework for solving a bi-

objective structurally fuzzy flexible JSSP model with order acceptance [2] and compares the performance of four multi-objective metaheuristics on a real dataset.

Structural fuzziness represents uncertainty in the structure of the optimization problem itself, where the eligibility of machines for job processing is fuzzy rather than crisp. The problem is transformed using β- decomposition into a finite family of structurally crisp subproblems. Multi-objective metaheuristics – NSGA-II, MOPSO, MOACO, and MOABC – are implemented and compared.

We evaluate the framework on a dataset from Zeppelin Ukraine LLC (157 maintenance jobs, 17 mobile workshops). Results show that MOACO generates the richest Pareto front, while NSGA-II exhibits the most robust performance. Despite variations in Pareto quality and computational efficiency, all four metaheuristics generate compromise schedules differing by less than 0.8% in expected revenue, demonstrating the robustness of the approach.

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ANALYSIS OF MATHEMATICAL MODELS OF TAXATION AND INFLATION

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Mathematical modeling of taxation and inflation processes is one of the priority problems in modern economic science. The purpose of this qualification thesis is to analyze current mathematical modeling methods for taxation—namely, the Lorenz curve with the Gini coefficient, the Laffer curve based on Saez's formula, and the cost-reducing tax mechanism model with a price-cap regulation for

monopolies—as well as inflation models: the Phillips curve, Cagan's hyperinflation model, and the inflation targeting model.

A software suite was developed in Python to simulate and visualize the examined models. The analysis revealed: for the Phillips curve, government intervention is efficient only up to the natural level of the labor market; for Cagan's hyperinflation model, economic agents' expectations are a significant driver of hyperinflation; for the inflation targeting model, the dependencies of the loss function under various values of key indicators; for the Lorenz curve with the Gini coefficient, the patterns of how the income concentration parameter influences the development of inequality in income tax distribution; for the Laffer curve based on Saez's formula, the possibilities of applying the Pareto parameter to determine the share of the population with an income at or above the average, as well as the role of elasticity in shaping the share of taxable income when the tax rate changes, and the combined effect of these factors on the optimal tax rate; and for the cost-reducing tax mechanism model with a price-cap regulation, how this mechanism alters the monopolist's incentives in favor of production efficiency. The developed software is recommended for use in the educational process, scientific and analytical research, and forecasting the outcomes of tax and monetary reforms.

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ARCHITECTURE OF A COGNITIVE DIGITAL TWIN FOR A UAV SWARM[1]

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A cognitive digital twin (CDT) of a UAV swarm is an intelligent digital model that synchronizes with the physical swarm in real time and provides forecasting, adaptation and decision support under uncertainty Unlike traditional digital twins, the CDT incorporates cognitive mechanisms for situation analysis, knowledge acquisition, event prediction, and self-adaptation [1,2]. The CDT operates in a closed-loop "physical system ↔ digital model" cycle, receiving telemetry from the actual swarm and returning optimized control decisions. The CDT architecture is based on the following principles: modularity; scalability; service-oriented interaction; cognitive adaptation; continuous synchronization of the physical and digital environments; explainability of decisions made; and resilience to failures of individual modules. The proposed architecture is based on the concept of a hybrid control system that integrates a ground control center (CCU) with a CDT, a swarm of autonomous UAVs, and a radio communication network. The CDT of the ground CCU functions as a strategic analytical core that integrates modeling, planning, and post-mission analysis tasks to support adaptive and intelligent UAV control. On-board AI agents implement real-time decision-making based on local sensor data interactions with neighboring drones and the CDT, which simplifies the integration of new components; provides increased swarm resilience to uncertainty, individual agent failures, and dynamic changes in the external environment.

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A CONCEPTUAL FRAMEWORK FOR THE COGNITIVE BEHAVIOR OF INTELLIGENT AGENTS IN A UAV SWARM[2]

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The cognitive autonomous behavior of intelligent agents in a UAV swarm is enabled by the agents' ability, under decentralized real-time control, to process information about their environment, predict situations, exchange data, reallocate and modify tasks in the event that a particular UAV fails, and make decisions independently based on swarm intelligence [1]. The cognitive behavior of UAV swarm agents is implemented as a cycle: perception → interpretation → prediction → planning → action → evaluation → learning. The cycle repeats, ensuring: knowledge accumulation; improved decision quality; and adaptation to environmental changes, which underlies the generalized architecture of the UAV swarm. The generalized architecture accounts for the fact that each agent (drone) in the swarm functions as a separate intelligent unit with its own mission, logistics, power supply, predictive modeling that assesses obstacles or threats in real time, and consideration of the trajectories of neighboring agents. In this context, an agent's cognitive behavior is interpreted as purposeful, adaptive, and explainable activity. Thus, in complex swarm systems, the concept of constructing an architecture for agents' cognitive behavior combines adaptability and explainability**,** making it suitable for critical systems

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with high reliability requirements; provides a methodological foundation for constructing formal models, ensuring the swarm's resilience to uncertainty and incomplete information, multifactorial and dynamic influences, conflicting goals and resource constraints, and the need for autonomous decision-making in real time.

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ARCHITECTURE OF THE INTEGRATED "DT + HIL" SYSTEM

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The report presents a comprehensive and formally structured architecture of an integrated system Digital Twin (DT) and Hardware-in-the-Loop (HIL) system, designed to support real-time cyber-physical interaction, multiphysics modeling, industrial communication, and hybrid machine-learning-enhanced computation. The architecture is developed as a multilayer framework that ensures modularity, scalability, robustness, and compatibility with heterogeneous hardware and software components. It reflects the growing need for unified methodologies capable of bridging high-fidelity digital models with real industrial equipment under strict timing constraints.

The proposed architecture consists of six interconnected layers: the physical layer, data layer, integration layer, modeling layer, control layer, and visualization layer. Each layer is responsible for a specific set of functions and interfaces, while the integration layer acts as the central communication hub that synchronizes data flows, manages industrial protocols (OPC UA, MQTT, CAN, EtherCAT), and maintains temporal consistency between digital and physical processes. The physical layer incorporates sensors, actuators, PLCs, microcontrollers, and real-time hardware platforms, forming the foundation of the cyber-physical loop.

The modeling layer supports multiphysics ODE/PDE models, stochastic processes, discrete-continuous dynamics, and hybrid ML-enhanced models, enabling accurate and computationally efficient

simulation. The control layer integrates classical and advanced algorithms, including PID, LQR, MPC, and reinforcement learning, ensuring flexible and adaptive system behavior. The visualization layer provides SCADA dashboards, 3D environments, and analytical interfaces for operator interaction, monitoring, and diagnostics.

Special emphasis is placed on real-time synchronization, delay compensation, buffering strategies, and stability mechanisms, which are essential for reliable HIL operation. The architecture is designed to accommodate future extensions, including distributed HIL systems, deep hybrid models, and digital factory integration.

THE IMPACT OF STOCHASTIC UNCERTAINTY AND TIME-DELAY IN QUALITATIVE ANALYSIS OF "DIRECT" CONTROL SYSTEMS

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The scalar stochastic system of "direct" closed-loop control described by the Ito differential equation with constant time-delay, in which the uncertain nonlinear inverse control function satisfies the linear sector condition and the Lipschitz condition, is considered. The report considers the issue of qualitative analysis of such systems:

$$dx(t) = \left(a_0 x(t) + a_\tau x(t-\tau) + bf\left(\sigma(t)\right)\right)dt + \left(g_0 x(t) + g_\tau x(t-\tau)\right)dw(t)$$

$$\sigma(t) = cx(t).$$

Here $a_0, a_\tau, g_0, g_\tau, b, c \in \mathbb{R}$ are constant coefficients ($c \neq 0$), $\tau > 0$ is the delay, $w(t)$ is a standard Wiener process, and $\sigma(t)$ is the output signal fed into the nonlinearity f.

$$0 \le f(\sigma)\sigma \le k\sigma^2, \quad f(0) = 0, \quad k - const$$

The research method chosen was the second Lyapunov method using Lyapunov functions of Lurie-Postnikov type with additional Razumikhin's condition.

Sufficient conditions for absolute asymptotic stability in the mean square are obtained. The conditions are algebraic in nature, depend only on the coefficients of the system and the constant of the linear sector span and do not contain the value of the delay, so stability is guaranteed for any deviation of the argument.

In addition, in the process of proving, an exponential estimate of the decay rate of the second moments of the solution was established.

The key idea of this study is to show the transparency and constructiveness of the proposed approach when considering scalar systems with the aim of its further extension to multidimensional control systems.

FACTORS FACILITATING KNOWLEDGE TRANSFER IN PRIMARY SCHOOL AGE

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Introduction and Relevance of the Research. In the contemporary educational field, the quality of teaching is determined not only by the theoretical knowledge accumulated by a student, but also by the ability to apply this knowledge in a multifaceted, practical, and functional manner. In primary grades, the transfer of accumulated knowledge constitutes a pedagogical and psychological challenge, which implies the relocation of rules, ideas, and skills mastered in one context into another situation. According to Lev Vygotsky and Jean Piaget, the thinking of a child at this age is in the concrete-operational phase. They often find it difficult to generalize learned subject matter and to perceive abstract connections between different subjects or real-world phenomena. In primary grades, knowledge transfer cannot be realized without a teacher acting as a purposeful guide. The relevance of this research is precisely driven by the circumstance that the problem of mechanical acquisition of theoretical material remains acute in school practice, which hinders the development of functional transfer in primary school-age students. "Knowledge is effective when it can be adopted by students in new situations, i.e., transfer of learning" (Bransford, Brown, & Cocking, 2000).

Research Objective. The main objective of this paper is to explore strategies that facilitate the transfer of knowledge acquired at a primary school age into practice, along with their systematic application, the active engagement of the student, and the development of critical-analytical and creative skills regarding facts.

Methodology. To achieve the set goals and objectives, quantitative, qualitative, observational, and surveying methodologies are utilized in

the research. To construct the theoretical framework, we critically reviewed relevant scientific literature from psychology, pedagogy, cognitive science, and contemporary didactics. The empirical part of the study relies on qualitative approaches: we conducted focus group research and carried out in-depth interviews with primary stage teachers. We observed the instructional process in several public and private schools within the Batumi and Khelvachauri municipalities. For the purpose of data observation, we analyzed samples of complex assignments performed by students, which assisted us in evaluating the real effects of various research approaches.

Results and Findings. As a result of the conducted research, three leading pedagogical strategies emerged, which substantially and reliably improve the rate of knowledge transfer in primary grades. Specifically:

1. **Contextualized and phenomenon-based teaching:** The delivery and processing of theoretical, abstract material through examples drawn from the student's daily experience, which must be given a systematic character. This approach helps the student achieve "near transfer."
2. **Integration of interdisciplinary projects:** Dismantling subject barriers and establishing purposeful connections during Georgian language, art, and natural science lessons.
3. **Activation of metacognitive reflection:** Fostering students' thinking—children explain and justify from the very beginning the path through which they reached a specific conclusion. This approach was identified as a significant component. In such cases, knowledge becomes more solid, flexible, and transferable.

Conclusion. Transfer is a process inherent to knowledge, which at the primary stage requires the flexibility of learning material and continuous, mental facilitation by the educator. The development of transfer at a primary school age is encouraged by active learning; implementing the mechanism for its translation and application into practice is the duty of the teacher, which serves as a guarantee for long-term academic and life success.

Keywords. Knowledge transfer, primary school age, interdisciplinary teaching, pedagogical strategies.

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ANALYSİS OF THE UKRAINIAN MUSIC MARKET USING DATA MINING METHODS

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The rapid growth of music streaming platforms has generated large volumes of data that provide new opportunities for understanding the factors influencing the popularity of music releases. This study presents a data-driven analysis of the Ukrainian music market using modern statistical methods and machine learning techniques to identify the key determinants of track success and develop predictive models for music popularity. The research is based on a dataset containing approximately 386 000 Ukrainian music tracks and their associated characteristics.

During the data preprocessing stage, new indicators describing track performance were introduced, including listening intensity and track virality. Statistical analysis was performed to investigate the influence of various musical and contextual factors on commercial success. The significance of the observed differences was evaluated using the Mann–Whitney U test, while the Benjamini–Hochberg procedure was applied to control the false discovery rate in multiple comparisons.

To predict track success, clustering techniques were combined with the CatBoost machine learning algorithm. The study included feature selection, clustering of musical compositions, model training, and comprehensive performance evaluation. The proposed approach demonstrated high predictive accuracy, achieving a classification accuracy of 95.1% and a Macro F1-score of 0.9158.

The obtained results demonstrate that the integration of statistical analysis with machine learning methods provides an effective framework for analyzing the Ukrainian music market and forecasting

the potential success of new releases. The proposed methodology can support artists, producers, and music industry stakeholders in making data-driven decisions regarding music promotion and release strategies.

OBJECT CLASSIFICATION WITH OVERLAPPING CLASS REGIONS AND LOW INTER-CLASS SEPARABILITY BASED ON IMPRECISE DATA

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Classification of natural and anthropogenic objects from remote sensing data faces two concurrent sources of uncertainty: finite measurement accuracy of object attributes and low inter-class separability. Standard classifiers relying on exact feature values fail to handle both simultaneously.

We present a supervised classification method combining interval representation with the Dempster–Shafer Theory of Evidence (DST). Measurement error δ is modelled explicitly as an interval for each attribute:

$$\xi_k = [x_k - \delta, x_k + \delta],\ k = 1, \ldots, K\ .$$

Class domains are interval numbers from training data; feature scales are intervalized into single- and multi-class zones. Basic probability assignments (BPAs) are formed from overlap lengths, discounted by per-attribute reliability α_k and fused via Dempster's rule:

$$m_{1\oplus 2}(A) = \frac{\Sigma_{B\cap C=A}\, m_1(B)\cdot m_2(C)}{1-K}\ ,$$

where K is the conflict mass. The final class is chosen by maximum pignistic probability:

$$Bet\ P(H_l) = \frac{\Sigma_{A\ni Hl}\, m(A)}{|A|}\ .$$

A numerical example with three overlapping classes shows that even when two attributes output mixed support, the third and the higher-reliability attribute becomes decisive after discounting and fusion.

The method is applicable to land cover and soil type classification from aerospace imagery, providing a robust, interpretable alternative to point-estimate classifiers for decision making under uncertainty.

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TOPOLOGICAL CLASSIFICATION OF PRO-HAMILTONIAN FLOWS ON THE GIRL'S SURFACE

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The study of dynamical systems on singular and stratified spaces extends the classical qualitative theory from smooth manifolds to spaces with self-intersections, where essentially new dynamical phenomena arise. Among such spaces, the Girl's surface is a classical transversal immersion of the projective plane into three-dimensional Euclidean space. Its stratified structure makes it a natural object for investigating Hamiltonian-type dynamics beyond the category of smooth surfaces.

We investigate simple pro-Hamiltonian flows on the Girl's surface. The considered flows have no singular points on one-dimensional strata, while the restriction of the flow to each two-dimensional stratum is generated by a simple Morse Hamiltonian function. We assume that every connected component of the intersection of a neighborhood of the triple point with a two-dimensional stratum contains either one or two hyperbolic sectors.

We describe the local structure of the flow near the triple point and prove that every simple flow without interior saddle points belongs to one of five possible local configurations. To classify global flow structures, we construct distinguishing graphs based on Reeb graphs and prove that they are complete invariants of topological equivalence. We also establish realization and classification theorems, providing necessary and sufficient conditions for the topological equivalence of flows in terms of graph isomorphisms preserving orientations, labels, and prescribed edge orderings. In addition, we prove the structural stability of simple pro-Hamiltonian flows without singular points on one-dimensional strata.

Finally, we derive recursive formulas for computing the number of topologically inequivalent flows with a prescribed number of separatrices. The obtained recurrence relations make it possible to enumerate topological types algorithmically. The proposed approach reduces the classification and enumeration problems on the Girl's surface to the corresponding problems for Hamiltonian flows on disks, providing effective combinatorial tools for the analysis of dynamical systems on singular spaces. These methods may be extended to more general classes of stratified surfaces with self-intersections.

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ON CONTROL OF WIRELESS CHARGING AND LANDING PLATFORMS FOR UAV SWARMS VIA RETRIAL QUEUES

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In this paper, we consider a two-dimensional continuous-time Markov process $Q(t) = (Q_0(t), Q_1(t))$, $t \geq 0$ in the phase space $S = \{0,1,...,c\} \times Z_+$, where $Q_0(t)$ is the number of busy wireless charging pads at time t, and $Q_1(t)$ represents the number of drones waiting in the holding area (orbit) at time t. The process describes the service dynamics of an automated multi-channel charging station for a UAV swarm under a limited retrial policy caused by battery constraints, where the arrival rate of incoming drones depends on the number of waiting UAVs. The system is characterized by the following parameters: λ_j is the mission return arrival rate, μ is the charging rate, and ν is the retrial rate from orbit.

For this system, we consider the situation when the arrival rate is controlled according to a threshold strategy to prevent airborne battery drain and airspace overcrowding. In this strategy, if at some moment of time the total number of waiting drones in orbit does not exceed the threshold H, the system operates in the standard regime with arrival

rate λ_1. If the number of waiting drones exceeds H, the system switches to a restricted mode with a reduced arrival rate λ_2 ($\lambda_2 < \lambda_1$) by slowing down inbound flights or holding back new departures.

According to this strategy, a multi-criteria optimization problem for the energy-efficient operation of the UAV charging infrastructure is formulated and solved, with the quality functional (accounting for waiting energy consumption, pad idle costs, and emergency landing penalties) expressed in terms of the system's stationary probabilities [1, 2].

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THE ARCHITECTURAL PHENOMENON OF THE DEREYURT MOSQUE

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The eastern part of historical Chaneti (Lazistan) constitutes a unique cultural landscape, where the distinctive character of Colchian material culture took shape over centuries. Despite the religious and political changes during the late feudal period, the local population—the Laz—preserved the construction craftsmanship inherited from their ancestors. One of the most prominent examples of this is the **Dereyurt Mosque (Dereyurt Köyü Camii)**, which clearly demonstrates the vitality of Georgian architectural genetics.

The study substantiates that despite the building's Islamic cultic purpose, its structural foundation—the **"Chanti" (interlocking log)** nail-less construction technique—is a direct continuation of Colchian wooden architecture. Based on typological and iconographic analysis, the article traces parallels between the ornamental system of the Dereyurt Mosque (rosettes, geometric braids) and the traditional architecture of southwestern Georgia (Adjara, Meskheti).

The research establishes that the Dereyurt Mosque is an example of

the Georgian architectural school built and preserved by Laz masters, where authentic construction culture proved resilient to religious-political transformations. This work makes a significant contribution to the study of the material culture of historical Chaneti as an integral part of the unified Georgian cultural space.

Methodology

The study utilizes an **interdisciplinary approach** that combines art history and ethnological methods. Specifically:

- **Comparative Analysis:** The structural elements of the Dereyurt Mosque are compared with the wooden structures of Adjara and Samtskhe-Javakheti. The analysis focuses on the identity of the "Chanti" (interlocking) masonry.
- **Architectural Morphology:** This method was used to determine the building's correspondence with local, Laz construction craftsmanship.
- **Visual Documentation and Iconography:** These methods were also employed during the research process.

Research Findings

The study established the following facts:

1. Technological Authenticity: It was determined that the Dereyurt Mosque is built using the principle of so-called "dry" wooden construction, which is typical of Colchian architecture. The building's corner joints (the so-called "tail") and the interconnection of planks replicate the construction of the Laz *Oda* (house) and *Nalia* (granary). This indicates that the builders were local Laz masters and not architects invited from the imperial center.

2. Ornamental Reception: A certain part of the ornaments in the mosque's interior has no analogues in Ottoman art, but has clear parallels in Georgian folk cultic and domestic architecture.

The Role of Local Craftsmen / "Ustas": It was revealed that the cultic buildings of eastern Chaneti form a unified architectural subgroup. This points to a strong school of Laz *Ustas* (masters) who maintained the autonomy of construction craftsmanship from the imperial style.

FUZZY LINEAR REGRESSION FOR FUZZY-STRUCTURED DATA

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Let $N=\{1,\ldots,n\}$ be the set of indices of fuzzy data observations. For each $i\in N$, let us associate a vector of independent variables $x_i=(x_{i0},x_{i1},\ldots,x_{ip})^T\in\mathbb{R}^{p+1}$, $x_{io}=1$ and a fuzzy dependent variable $\tilde{y}_i=(y_i,e_i)_T$ represented by a symmetric triangular fuzzy number with center y_i and width $e_i\geq 0$. The report considers the fuzzy linear regression model

$$\tilde{Y}_i=\tilde{A}_0x_{i0}+\tilde{A}_1x_{i1}+\ldots+\tilde{A}_px_{ip},\ i\in N,$$

where $\tilde{A}_j=(a_j,c_j)_T$, $j=0,\ldots,p$ are symmetric triangular fuzzy parameters; $a_j\in\mathbb{R}$ is the center, whereas $c_j\geq 0$ is the width of the corresponding parameter $\tilde{A}_j$, $j=0,\ldots,p$. Unlike classical regression, deviations between the observed and estimated values are interpreted not only as random errors but also as a manifestation of the fuzziness of the system under investigation [1]. The predicted value is given by $\tilde{Y}_i=(a^Tx_i,c^T|x_i|)^T$ with $a=(a_0,\ldots,a_p)^T, c=(c_0,\ldots,c_p)^T$, and $|x_i|$ is the vector of the absolute values of the components of x_i, $i\in N$. The problem of estimating the fuzzy regression parameters is given by

$$\min_{a,c}J(c)=\sum_{j=0}^{p}c_j,\quad c_j\geq 0,\quad j=0,\ldots,p,$$

$$a^Tx_i+(1-H)c^T|x_i|\geq y_i+(1-H)e_i,$$

$$a^Tx_i-(1-H)c^T|x_i|\leq y_i-(1-H)e_i,\ i\in N,$$

where $H\in[0,1]$ is a prescribed level of consistency of the model parameters.

The report investigates the properties of the fuzzy linear regression and a method for constructing it in the case where fuzzy data samples possess a fuzzy structure specified by their corresponding degrees of membership in a certain fuzzy set. The fuzzy structure of the data means that the data have different degrees of truthfulness, quality, or confidence. This is usually caused by incomplete confidence in information sources, as well as insufficient information regarding the

relevance, objectivity, integrity, and other characteristics of the data.

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PROPERTIES OF SOLUTIONS OF EPIDEMIOLOGICAL DYNAMICAL SYSTEMS WITH IMPULSE PERTURBATIONS

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We study epidemiological dynamical systems with impulse perturbations of the view:

$$\begin{cases} dS(t) = S\left\{\left(r(t) - b(t)S - \dfrac{\alpha(t)P}{S + a(t)P} - \dfrac{\lambda(t)I}{S+I}\right)dt + d\eta_1(t)\right\}, \\ dI(t) = I\left\{\left(\dfrac{\lambda(t)S}{S+I} - \mu(t)\right)dt + d\eta_2(t)\right\}, \\ dP(t) = P\left\{\left(\dfrac{\beta(t)S}{S + a(t)P} - \gamma(t)\right)dt + d\eta_3(t)\right\}. \end{cases} \quad (1)$$

We take study [1] as a basis to prove the existence of global positive bounded solutions under some conditions to the parameters of the system. Instead of Wiener process [1], we discuss a process with independent increments $\eta(t)$ as a perturbation. $\eta_i(t)$ are independent copies of $\eta(t)$, which are set through their generators.

Assumption. For any $t \geq 0$, $r(t), b(t), \alpha(t), a(t), \lambda(t), \mu(t), \gamma(t), \beta(t)$ are bounded positive and locally Lipschitz functions. Predictable characteristics of the semimartingale $\eta(t)$ are also bounded.

Theorem 1. Under the previous assumption, there is a single global solution $(S(t), I(t), P(t))$ of the system of equations (1) for any initial data $(S(0), I(0), P(0)) \in R3_+$ and the solution will remain in $R3_+$ with probability 1. Namely, $(S(t), I(t), P(t)) \in \mathrm{R3}_+$ for all $t \geq 0$ almost surely.

Theorem 2. Suppose that the Assumption holds true and measure of jumps of the process with independent increments is finite. The solution $(S(t), I(t), P(t))$ of a system of stochastic differential equations (1) with non-negative initial conditions has the following property for any

$p > 0$: there is a positive constant $K_1(p)$, such that

$$\overline{\lim_{t\to\infty}} \mathbf{E}(S^p(t)) \le K_1(p)$$

and there are positive constants K_2, K_3 such that

$$\overline{\lim_{t\to\infty}} \mathbf{E}(I(t)) \le K_2, \quad \overline{\lim_{t\to\infty}} \mathbf{E}(P(t)) \le K_3.$$

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SCALE-INVARIANT SHORT-TERM LOAD FORECASTING UNDER STRUCTURAL REGIME BREAKS

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The full-scale war in Ukraine caused a structural regime break in the United Energy System of Ukraine: the base load level decreased sharply and the statistical properties of consumption changed. Models trained on absolute load values can therefore lose accuracy because their internal representation remains tied to the pre-war operating point.

We evaluate whether a scale-invariant representation improves short-term load forecasting across regimes without retraining. Hourly data for 2014 – 2022 were divided into Pre-War, COVID, and War periods. LSTM and Transformer models were trained only on Pre-War data and tested on all three regimes. Two settings were compared: absolute values with global normalization and relative load changes with local normalization of 72-hour input windows. The forecast horizon was one hour.

The difference was most pronounced in the War period. With absolute values, LSTM produced MAPE above 60%, while Transformer achieved 11.5%. After transition to the scale-invariant representation, MAPE decreased to 4.3% for LSTM and 3.2% for Transformer. Thus, both architectures benefited substantially from relative representation, while Transformer retained the highest forecasting accuracy.

Conclusion The results show that the robustness of short-term load forecasting under structural regime breaks depends not only on model architecture but also on data representation. Forecasting relative changes with local normalization reduces dependence on the absolute system

scale and preserves useful short-term dynamics across substantially different operating regimes.

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A NEW RESULTS ON THE OPERATOR EXTRAPOLATION ALGORITHM

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The operator extrapolation algorithm is a very valuable alternative to the well-known Korpelevich iterative extragradient method for solving variational inequalities, since it requires only one operator call and one metric projection at each step [1, 2]. We present a theoretical analysis of an operator extrapolation algorithm for approximate solving pseudo-monotone variational inequalities in real Hilbert spaces. A variant of the operator extrapolation method with an inaccurate implementation of the step is also considered. It is assumed that the computational error sequence is summable. Additionally, we investigate the algorithm's adaptive stepsize strategy, which arises when it is impossible to calculate the Lipschitz continuity constant of a operator correctly. We prove a weak convergence theorems under some appropriate conditions.

There are a few potential directions for our future research: study of algorithms in a stochastic setting [3, 4], due to its relevance for the training of generative adversarial networks; obtaining the rate of convergence of these algorithms in the case of, for example, strongly pseudo-monotone operators; further study of variants of algorithms for pseudo-monotone problems in uniformly convex and uniformly smooth Banach spaces.

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A VARIANCE REDUCTION STOCHASTIC EXTRAGRADIENT METHOD WITH RANDOMIZED FEASIBILITY UPDATES

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In this talk we consider a variational inequality problem. However, in many large-scale applications the operator is not available exactly but only through a stochastic oracle. Also, a practically important yet challenging case arises when the the feasible set itself has a complex structure.

We propose and analyze a stochastic extragradient algorithm with variance reduction and randomized feasibility updates for solving monotone variational inequality problems whose constraint set is the intersection of a simple bounding set and a (possibly infinite) family of convex functional level sets.

The method integrates the loopless SVRG-EG algorithm of Alacaoglu and Malitsky [1] with the randomized Polyak-type feasibility steps approach of Chakraborty and Nedic [2, 3], requiring only projections onto simlpe set.

Under standard monotonicity and L-Lipschitz-in-mean oracle assumptions, we establish a descent inequality for a Lyapunov function that couples the optimality gap with a feasibility tracking term.

From this inequality we derive:

1) almost-sure convergence of the iterates to the feasible set and almost-sure convergence of the distance to every solution;
2) O(1/K) convergence rates for expected feasibility error and the weak expected dual gap.

Numerical experiments on a matrix game with quadratic constraints confirm the theoretical rates.

A natural direction for future work is to analyze our algorithm under strong monotonicity assumption or in a non-Euclidean setup.

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METHODS FOR SOLVING MULTICRITERIA OPTIMIZATION PROBLEMS ON COMBINATORIAL CONFIGURATIONS WITH FUZZY DATA

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A vector optimization problem $Z(F, \tilde{X})$ with fuzzy criteria and a fuzzy feasible domain is considered. The feasible set is a fuzzy Euclidean combinatorial configuration of permutations $\tilde{X} = \{(x, \mu(x)): x \in P_n^k(A) \cap D\}$, where $P_n^k(A)$ is the set of permutations of a multiset $A = (a_1 \le \ldots \le a_n)$ with k distinct elements, $D = \{x: Bx \le \beta\}$, and $\mu: P_n^k(A) \to [0, 1]$. The linear criteria $f_k(x) = \langle c_k, x \rangle$, $k = 1, \ldots, s$, are treated as fuzzy goals in the sense of Bellman–Zadeh [1] with membership functions $\mu_k = (\langle c_k, x \rangle - d_k)/b_k$, where d_k is a threshold and b_k a tolerance. The problem is formalized in terms of fuzzy multisets, which allows the fuzzy permutation polytope $\Pi_n^k(A) = \text{conv}\, P_n^k(A)$ to be introduced and the Edgeworth–Pareto principle to be extended to it. Efficiency in the senses of Smale, Pareto and Slater is defined for the fuzzy case and the chain $Sm \subseteq P \subseteq Sl$ is proved. All efficient solutions lie at vertices of $\Pi_n^k(A)$, which reduces the problem to linear programming preserving the combinatorial structure [2, 3].

Three methods are justified. The **guaranteed result method** maximizes the least degree of goal attainment and leads to the z-problem: $z \to \max$, $\langle c_k, x \rangle - b_k z \ge d_k)$, $x \in \Pi_n^k(A)$, $Bx \le \beta$, $0 \le z \le 1$. The **method of sequential concessions** ranks the criteria and solves s linear problems with the constraints $\mu_j(x) \ge \mu_j^* - \Delta_j$. The **ideal point method** stays in this linear class for $p=1$ and $p = \infty$, coinciding for $p = \infty$ with the z-problem under $d_k := f_k^{\min}$, $b_k := f_k^* - f_k^{\min}$. Membership in $P_n^k(A)$ is

checked by partial sums; otherwise a cut is added. For $n = k = 3$, $A = (1, 2, 3)$, $s = 2$, $d_1 = d_2 = 10$, $b_1 = b_2 = 4$ all three algorithms give $x^* = (1, 2, 3)$, $z^* = 0.75$, $\mu_1^* = \mu_2^* = 0.75$. The methods generate different efficient solutions by varying d_k, b_k and Δ_k.

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L^P CONVERGENCE FOR A STOCHASTIC OPTIMAL CONTROL PROBLEM IN THE AVERAGING SCHEMA

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We investigate the L^p convergence for a stochastic optimal control problem with Markov switching in the averaging schema. The system's evolution under diffusion noise and random mode switching is modeled by combining a standard Wiener process with a uniformly ergodic Markov process $x(t)$.

The stochastic evolutionary system is governed by the equation:

$$dy^{\varepsilon}(t) = C\left(y^{\varepsilon}(t), x(t/\varepsilon)\right)dt + a\left(t, u^{\varepsilon}(t)\right)dW(t),$$

where $C(y,x)$ captures the deterministic trend, and $a(t,u)$ quantifies the system's sensitivity to noise.

The control function $u^{\varepsilon}(t)$ is determined using a stochastic optimization procedure:

$$du^{\varepsilon}(t) = \alpha(t)\nabla_{\beta(t)} J\left(y^{\varepsilon}(t), u^{\varepsilon}(t), x(t/\varepsilon)\right)dt,$$

where $J(y,u,x)$ is a quality criterion and $\nabla_{\beta(t)} J$ is the finite-difference gradient approximation.

Under assumptions of global Lipschitz continuity, linear growth of the coefficients, and appropriate conditions on the learning rate $\alpha(t)$

and finite-difference step $\beta(t)$, we can compare the perturbed two-component process $(y^{\varepsilon}(t), u^{\varepsilon}(t))$ with the solution $(\hat{y}(t), \hat{u}(t))$ of the corresponding averaged system.

For every $p \geq 2$ and finite horizon $T > 0$, we establish the L^p convergence estimate, where C_T and K_T are positive constants independent of ε:

$$\mathbb{E}\left(\sup_{0 \leq t \leq T} \left(\left\| y^{\varepsilon}(t) - \hat{y}(t) \right\|^p + \left\| u^{\varepsilon}(t) - \hat{u}(t) \right\|^p \right) \right) \leq C_T \varepsilon^p e^{K_T T}$$

This L^p bound justifies the use of the averaged system as an approximation, allowing for a significant simplification of the optimal control synthesis procedure by operating on smooth deterministic averages of the original stochastic coefficients rather than analyzing the complex fast-switching system directly.

TWO-STEP COMBINED METHODS FOR SOLVING NONLINEAR EQUATIONS WITH A DECOMPOSITION OF THE OPERATOR

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Let B_0 and B_1 be Banach spaces, and let D be a nonempty convex subset of B_0. We consider the nonlinear equation

$$H(x) = F(x) + G(x) = 0, \tag{1}$$

where $F : D \subset B_0 \to B_1$ is a Fréchet-differentiable operator and $G : B_0 \to B_1$ is a continuous, non-differentiable operator. To numerically solve (1), we apply the two-step methods

$$x_{n+1} = x_n - A_n^{-1} H(x_n), \quad y_{n+1} = x_{n+1} - A_n^{-1} H(x_{n+1}), \tag{2}$$

where

$$A_n = F'\left(\frac{x_n + y_n}{2} \right) + G(2y_n - x_n; x_n), \tag{3}$$

or

$$A_n = F'\left(\frac{x_n + y_n}{2} \right) + G(2x_n - y_n; y_n), \tag{4}$$

$n = 0,1,2,\ldots$. Here $G(u;v)$ is the first order divided difference of the operator G at the points u and v. The decomposition allows the derivative of F and the divided difference of G to be combined in a single operator. These iterative processes are combined two-step modifications of the Newton and Kurchatov methods [1, 2]. We study the local and the semi-local convergence of the methods (2)–(3) and (2)–(4). The convergence conditions, the convergence balls and the error estimates for the generated sequences are determined. The semi-local convergence is based on majorizing scalar sequences. Moreover, the domain of uniqueness of the solution is established. Numerical experiments confirm the theoretical finding.

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ON THE GAUSS-NEWTON-BROYDEN METHOD FOR SOLVING NONLINEAR LEAST SQUARES PROBLEM

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Let us consider the nonlinear least squares problem [1, 2]. It is necessary to find $x^* \in D \subseteq \mathbb{R}^n$ such that

$$h(x^*) = \min_{x \in D} h(x), \quad h(x) = \frac{1}{2} H(x)^T H(x). \qquad (1)$$

Here $H : D \subseteq \mathbb{R}^n \to \mathbb{R}^m$ is a nonlinear vector-valued function, $m \geq n$, and it can be represented in the form $H = F + G$ [3]. It is assumed that F is a differentiable function, whereas G is a continuous function and can be nondifferentiable on D or represented by a complicated analytical expression. Nonlinear least squares problems arise in a wide range of scientific and engineering applications, including the solution of overdetermined systems of equations and parameter estimation for physical processes based on measurement data.

For numerical solving of the problem (1) we propose the Gauss-Newton-Broyden method

$$A_k = F'(x_k) + B_k,$$
$$x_{k+1} = x_k - \left(A_k^T A_k\right)^{-1} A_k^T H(x_k),\ k \geq 0, \tag{2}$$

where B_0 is a given matrix and

$$B_{k+1} = B_k + \frac{(y_k - B_k s_k)s_k^T}{s_k^T s_k}, \quad y_k = G(x_{k+1}) - G(x_k), \quad s_k = x_{k+1} - x_k.$$

We provide a local convergence analysis of the method (2) for zero-residual problems under classical Lipschitz conditions. A numerical study of the proposed method on problems with with zero and nonzero residuals, and nonlinear complementarity problems was also conducted.

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PARAMETER IDENTIFICATION PROBLEMS IN DYNAMIC NEURAL NETWORKS UNDER UNCERTAINTY

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Let the dynamics of the neural network be described by the equation

$$\frac{dx_i}{dt} = -\gamma_i x_i(t) + \psi_i((W(x(t) + b, e_i),$$
$$x_i(0) = x_{0i}, \tag{1}$$

where γ_i is a positive parameter, W is the coefficient matrix $(W_{ij})_{i,j=\overline{1,n}}$, b is a vector with components b_i, $i = \overline{1,n}$, ψ_i is the activation function.

Suppose that at the points t_k, $k=\overline{1,N}$ the values $x_i(t)$, and the values of their derivatives $\frac{dx_i}{dt}$ are known with errors η_k. It is also known that the errors η_k lie within the bounds $\eta_k^- \le \eta_k \le \eta_k^+$, where η_k^- and η_k^+ are known numbers. The problem we study is to determine the set of parameters W, that satisfy the system of inequalities $\alpha_{ik} \le \psi_i(Wx(t_k)+b, e_i) \le \beta_{ik}$, $i=\overline{1,n}$, $k=\overline{1,N}$, and, among them, find optimal parameters in a certain sense, where in the corresponding inequalities $\alpha_{ik} = \eta_{ik}^- + \gamma_i x_i(t_k)$, $\beta_{ik} = \eta_{ik}^+ + \gamma_i x_i(t_k)$.

The paper shows that this set can be approximated by inner approximating sets contained in the prescribed set and outer approximating sets containing this set, for which the optimal values $\hat{W}$ and $\hat{b}$ of the parameters can be found according to the specified criteria.

For equation (1) conditions for the stability of the solution are given in the form of constraints on the coefficients γ, W, b and these conditions $\gamma_i, \hat{W}, \hat{b}$ are verified for specific examples.

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NATURAL VS. SPECIFIC TYPING RHYTHM IN AUTHENTICATION INTERFACE CONCEALMENT: ADVANTAGES AND TRADE-OFFS

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This study extends the concept of Authentication Interface Concealment (AIC) [1], a multi-layered alternative to flawed "Security through Obscurity" approaches [2]. While Level 1 conceals the interface using obfuscated scripts, Level 2 validates access using keystroke dynamics. In behavioral authentication, two modes can be applied: Natural Typing Rhythm (NTR) and Specific Typing Rhythm (STR).

The primary advantage of NTR over STR is high usability. NTR evaluates a user's unique, habitual typing rhythm during standard input, eliminating the need to memorize artificial rhythmic codes (as required in STR). This provides seamless protection against keyloggers and shoulder surfing.

However, NTR has limitations compared to STR. Because NTR relies on natural typing habits, it is more sensitive to physiological factors (e.g., fatigue or emotional state), potentially increasing false rejections. In contrast, STR acts as a deterministic "rhythmic key," making it easier to analyze. Despite these trade-offs, practical implementation demonstrates the viability of NTR for implicit user verification within the AIC framework [3].

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DOMINANCE RELATION IN MULTIPLE-EXPERT MODAL SYSTEMS OF PARTIAL PREDICATES

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In this work, we investigate the dominance relation properties in Multiple-Expert Modal Systems (MEMS) of partial single-valued quasiary predicates (*P*-predicates), which were proposed in [1]. Specifically, we consider MEMS with both separate and unrestricted dominance. The basic dominance condition (DS) is defined as follows: if $e \gg g$, then $T(\Phi^e) \subseteq T(\Phi^g)$ and $F(\Phi^e) \subseteq F(\Phi^g)$. Here, $e \gg g$ indicates that expert e dominates expert g, while the predicates Φ^e and Φ^g are the values of the formula Φ for these experts, respectively.

In MEMS with separate dominance, when DS is satisfied, we have: $e \gg g$ and $\Phi^e(d) = T \Rightarrow \Phi^g(d) = T$; $e \gg g$ and $\Phi^e(d) = F \Rightarrow \Phi^g(d) = F$; $e \gg g$ and $\Phi^e(d) \uparrow \Rightarrow$ the possible values are: $\Phi^g(d) = T$, $\Phi^g(d) = F, \Phi^g(d) \uparrow$.

Thus, the DS condition perfectly captures the essence of dominance.

Theorem 1. The basic compositions of the MEMS language preserve the DS condition.

Consequently, the DS condition holds for all formulas of the language, proving its *soundness*.

Theorem 2. Under the DS condition in MEMS of *P*-predicates with unrestricted dominance, the following distribution of truth values between dominants and immediate subdominants is possible:

$T\ F\ \perp$	$T\ \ F$	$T\ \ T$	$F\ \ F$	$T\ \perp$	$F\ \perp$	$\perp\ \perp$
\ | /	\ /	\ /	\ /	\ /	\ /	\ /
$\varnothing$	$\varnothing$	T	F	T	F	$T, F, \perp$

We investigated modifications of the DS that allow for the combination of T and F for dominants of a single subdominant. However, such modifications are invalid because they are not preserved under the $\vee$. Thus, for MEMS of *P*-predicates, the proposed DS condition is *sound* and entirely *adequate*. The obtained results can be used to develop multi-agent artificial intelligence systems capable of effective action coordination and decision-making.

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SEMANTIC PROPERTIES OF MULTIPLE-EXPERT MODAL SYSTEMS OF QUASIARY PREDICATES

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Modal logics of the epistemic type are used in artificial intelligence and computer science to model multi-agent systems for action coordination and decision-making. In [1], we introduced Multiple-Expert Modal Systems (MEMS) based on the first-order logics of partial predicates. To coordinate decisions in multiple-expert systems, a dominance relation is introduced on the set of intelligent agents (experts). Unlike existing systems, MEMS require the preservation of both truth and falsity values, rather than truth alone. To avoid inconsistent situations, we proposed MEMS with *separate* dominance, which implies that an expert can be directly dominated by only one other expert.

This work investigates the semantic properties of pure first-order MEMS of partial single-valued quasiary predicates, or *P*-predicates.

MEMS is an object $\boldsymbol{M} = (Exp, \succ, St, \triangleright, A, I)$, where: Exp is a set of intelligent experts (agents), $\succ \subseteq Exp \times Exp$ is a relation of immediate dominance, $\triangleright \subseteq Exp \times St \times St$ is an accessibility relation for experts between possible situations (states), A is a set of basic data, $I : Fm \times Exp \times St \to Pr$ is is an interpretation mapping of formulas; Fm is a set of formulas of the MEMS language, Pr is a set of quasiary predicates of the form ${}^{V}A \to \{T, F\}$.

The languages of such MEMS are described, and the formula interpretation mapping is defined. The presence of dominance and accessibility relations determines the peculiarities of the interpretation of modalized formulas. We introduce the relations of *IR*-, *T*-, *F*-, and *TF*-set-consequence on the set of formulas specified by expert and state names in MEMS. Non-modal properties of these relations are analogous to the corresponding properties of similar relations in traditional first-order logics of quasiary predicates. We have established MEMS-specific properties of modality elimination, as well as the guarantee conditions for each of the *IR*-, *T*-, *F*-, and *TF*-set-consequence relations. On this semantic basis, we plan to construct sequent calculi for MEMS.

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DEVELOPMENT OF A SCENARIO GENERATION AND VALIDATION FRAMEWORK FOR LARGE LANGUAGE MODEL-BASED DECISION-MAKING MODULES IN AUTONOMOUS DELIVERY SYSTEMS

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Autonomous delivery robots traditionally rely on deterministic modules for localization, mapping, navigation etc. These modules are effective for low-level tasks, but they are not sufficient for high-level planning in dynamic and partially observable environments. Recent surveys show that large language models (LLMs) are increasingly studied as a subsystem for high-level decision making tasks in such robotics systems [1].

The validation of robots' modules is commonly performed using robotic simulators (Gazebo, NVIDIA Isaac Sim etc), which accurately reproduce robot dynamics and sensor behavior. However, these simulators are designed primarily for testing physical interaction with the environment rather than evaluating high-level decision making tasks performed by LLM modules.

This work proposes a system for scenario generation and validation of LLM-based decision-making agents that are integrated into autonomous delivery systems. The system consists of two independent modules. The scenario generation module first generates delivery missions from a set of environment rules. These missions are then provided to the second module, an LLM agent, which autonomously performs high-level planning and task execution by iteratively reasoning and invoking simulated environment functions to obtain the information required for decision making. Within the proposed system, the LLM agent is implemented as a ROS 2 node. The node invokes functions of the simulated environment that return observations such as battery level, robot position, and obstacle information.

The proposed system provides a computationally efficient approach for testing high-level planning capabilities in autonomous delivery robots and can help to integrate LLM agents into delivery systems in future.

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METHOD FOR SOLVING INTEGRAL EQUATIONS BASED ON THE MATHEMATICS OF FUNCTIONAL INTERVALS

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To solve integral equations of the form

$$y(x) = f(x) + \lambda \int_a^x k(x, s, y(s))\, ds\,, \qquad (1)$$

based on the mathematics of functional intervals, a two-way iterative method is proposed. It is based on the conclusions of the following theorem.

Theorem. *Let: 1) the function be* continuously *differentiable on the interval* $[a, b]$, *and the kernel* $k(x, s, y(s))$ *be integrable,*

2) the points b_i, $(i = 0, 1, \cdots, n)$, *where* $b_0 = a, b_n = b$, *are chosen so that on each interval* $[b_i, b_{i+1}]$, *the functions* $\underline{g}(x) = \underline{k}\, x + \underline{m}$, $\overline{g}(x) = \overline{k}\, x + \overline{m}$, *where the constants* $\underline{k}$, $\overline{k}$, $\underline{m}$, $\overline{m}$, $\underline{\Delta} \ge 0$, $\overline{\Delta} \ge 0$, $y_a = y(a)$, $y'_a = y'(a)$, *are known such that,* $\underline{k} = y'_a - \underline{\Delta}$, $\overline{k} = y'_a + \overline{\Delta}$, $\underline{m} = y_a + \underline{k} \cdot a$, $\overline{m} = y_a + \overline{k} \cdot a$, *ensure the fulfillment of the condition* $\underline{g}(x) \le y'(x) \le \overline{g}(x)$.

Then: 1) the integral equation *(1) on each interval* $[b_i, b_{i+1}]$ *is equivalent to the original problem* $y'(x) = f'(x) + \lambda\ k(x, x, y(x))$, $y(b_i) = f(b_i)$, *2) the square parabolas* $\overline{p}_{1a}(x) = 0.5\,\overline{k}\,(x^2 - a^2) + \overline{m}\,(x - a) + y_a$, $\underline{p}_{1a}(x) = 0.5\,\underline{k}\,(x^2 - a^2) + \underline{m}\,(x - a) + y_a$, $\overline{p}_{2a}(x) = 0.5\,\overline{k}\,(x^2 - s_i^2) + \overline{m}\,(x - s_i) + y_{s_i}$, $\underline{p}_{2a}(x) = 0.5\,\underline{k}\,(x^2 - s_i^2) + \underline{m}\,(x - s_i) + y_{s_i}$, *where* $s_i = (b_i + b_{i+1})/2$, *form a parabolic parallelogram [1], which is guaranteed to contain a solution to equation (1).*

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ABOUT ANOTHER RECURRENT FORM OF REPRESENTATION FOR PARAMETER IDENTIFICATION BY THE METHOD OF LEAST SQUARES OF RELATIVE ERRORS UNDER NON-CLASSICAL ASSUMPTIONS FOR SYSTEMS UNDER UNCERTAINTIES

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Successful construction of a mathematical model for an object in most cases depends on the requirements imposed on its accuracy. The latter is in most cases based on the analysis of the absolute errors of the model. But in this work, the main attention is paid to the analysis of the relative errors of the mathematical model. This is relevant when building mathematical models in such fields as finance, economics, chemistry, physics, etc. Therefore, the method of least squares of relative errors is often used as the main method for parameter estimation of a mathematical model in these areas.

The estimate of unknown parameters for the basic regression model by this method in the most general case, even with a possible violation of the classical assumption that guarantees its uniqueness, was obtained earlier. An explicit form of representation for the set of all these estimates was proposed by using the Moore-Penrose matrix pseudo-inversion operator. In this paper, another explicit representation form for the set of all these estimates is obtained, which simplifies its use because it requires pseudo-inversion only of symmetric matrices.

For practical use, the estimate with the least Euclidean norm can be chosen as the unique estimate over the entire set of these estimates. Recurrent forms of representation for this unique estimate of the least squares method of relative errors allow us to avoid the need not only to use the Moore-Penrose matrix pseudo-inversion operator, but also matrix inversion in the usual sense.

Then the second recurrent form of representation for the above-mentioned estimate, which was proposed in the paper, may be useful. A characteristic feature of the last recurrent representation is a common recurrent for recalculating precisely the unique parameter estimate.

Knowledge the current values of the weighted residual sum of squares of relative errors will be useful for assessing the quality of the constructed mathematical model. To recalculate this quantity, it is proposed to use an appropriate recurrent algorithm, which also does not require the use of either the Moore-Penrose pseudo-inversion operator or matrix inversion in the usual sense. The use of this pair of recurrent

algorithms can successfully solve the problem of unknown parameter estimation for a mathematical model of the object under study.

MIXTURES OF SUBORDINATORS FOR RISK MEASUREMENT IN OPTION PRICING

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Classical diffusion models do not always provide an adequate description of financial markets, especially during periods of low market activity and irregular price movements. Subdiffusive models can account for such behavior by introducing a random time change into the asset price dynamics. However, existing frameworks predominantly rely on a single subordinator (e.g., Gamma or Inverse Gaussian), limiting their flexibility in modeling complex market regimes.

To address this gap, this paper introduces a novel subdiffusive option pricing model governed by a mixture of subordinators. Mixtures of two tempered stable subordinators S_{mix} are considered as a L´evy process with the Laplace transform:

$$E\left(e^{-uS_{mix}(t)}\right) = e^{-t\left(c_1\omega_1(u)+c_2\omega_2(u)\right)}; \quad c_1 + c_2 = 1; \quad c_1, c_2 > 0;$$

where $\omega_1(u)$ is the Laplace exponent of the first subordinator, and $\omega_2(u)$ is the Laplace exponent of the second subordinator.

The Value-at-Risk evaluating procedure for the proposed model with mixtures of subordinators was discussed and we show that this procedure is based on the Fractional Fokker-Planck equation (FFPE).

The study aims to show that mixtures of subordinators can provide a more flexible framework for financial risk assessment and improve the description of market dynamics compared with standard diffusion models.

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PROPERTIES OF NETWORK EQUILIBRIUMS

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Investigating the influence of network topology on the set of equilibriums is an important problem in the game theory [1]. We consider a non-cooperative game $G = \langle X_i, u_i : i \in N \rangle$, where X_i, $i \in N$, are the sets of players` strategies, u_i, $i \in N$, are the payoff functions, and $N = \{1,...,n\}$ is the set of players. Let the communicative capabilities of the players be determined by a network, which is defined on N by the adjacency matrix $A = \{a_{ij}\}_{i,\, j \in N}$. A situation $\hat{x} = (\hat{x}_i)_{i \in N}$ is called a network equilibrium of the game G, if the inequalities

$$\min_{x_j \in X_j,\, j \in \bar{N}_i(A)} u_i(\hat{x}_i, (\hat{x}_j)_{j \in N_i(A)}, (x_j)_{j \in \bar{N}_i(A)}) \geq$$

$$\min_{x_j \in X_j,\, j \in \bar{N}_i(A)} u_i(x_i, (\hat{x}_j)_{j \in N_i(A)}, (x_j)_{j \in \bar{N}_i(A)}),$$

$\forall x_i \in X_i$, $i \in N$ hold, where $N_i(A) \subseteq N$ is the set of neighbors of the player $i \in N$, with whom player $i \in N$ interacts, and $\bar{N}_i(A) = N \setminus N_i(A)$ is the set of other players. Important properties of the network equilibriums are the tendencies of both the existence of equilibriums and the change of payoff in these equilibriums upon narrowing the communicative capabilities. The following theorem holds true.

Theorem 1. Let the situation $\hat{x}$ be a network equilibrium of the game G with the model of players` interaction given by the adjacency matrix $A = \{a_{ij}\}_{i,\, j \in N}$. Then, if the game is finite or the sets $X_i$, $i \in N$, of players` strategies are compact, and the payoff functions u_i, $i \in N$, are continuous, then there always exists such a submatrix $A' < A$, that under the model of players` interaction given by the adjacency matrix A', the set $NE(A')$ of network equilibria is not empty, and any equilibrium $\bar{x} \in NE(A')$ from this set gives the player $i \in N$ a payoff $u_i(\bar{x})$, which does not increase, that is $u_i(\bar{x}) \leq u_i(\hat{x})$.

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DIGITAL TWINS IN HIL - SIMULATION OF AIRCRAFT STRUCTURES

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The integration of digital twin technology with Hardware-in-the-loop modeling opens up new opportunities for improving the operational efficiency of aircraft structures. The combination of real hardware components with a digital model of the structure makes it possible to simulate its operating conditions, investigate the response to various types of loads, and assess the current technical condition without the need for large-scale and expensive full-scale tests.

One of the promising areas of development in the maintenance of aviation systems is taking into account the concepts of Condition-Based Maintenance (CBM) and Predictive Maintenance (PdM) when creating a digital twin of an aircraft design. At the stage of developing a digital twin, it is planned to integrate sensors and data acquisition systems that provide information about loads, deformations, vibrations, temperature, and other structural parameters during operation. Within the CBM concept, the digital twin should provide continuous analysis of the received data, assessment of the current technical condition, and detection of deviations indicating the occurrence or accumulation of damage. To implement the PdM concept, structural degradation models, prediction algorithms, and machine learning methods are integrated into the digital twin. The results of modeling and forecasting are used to determine the optimal time for maintenance, repair, or replacement of individual structural elements. Thus, when creating a digital twin, not only the geometric and physical characteristics of the structure are taken into account, but also models of its technical condition, damageability and life cycle. The combination of digital twins, semi-real-time modeling, and CBM and PdM concepts enables the transition from routine maintenance to intelligent condition management. In this case, the time of reaching the limit state is determined and the optimal time for maintenance or repair is calculated, preventing emergency failures and unreasonable costs based on the analysis of data on the current state of the structure, operating history and the nature of the loads. This improves the reliability of the aircraft, reduces the number of unplanned failures, optimizes operating costs, and increases the efficiency of managing the technical condition and residual life of the aircraft structure.

A COMPARATIVE ANALYSIS OF THE GENETIC ALGORITHM AND THE *m*-STEP SECANT METHOD

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Two families are compared: the m-step secant iteration and the genetic algorithm, a stochastic population search. Let X, Y be Banach spaces, $D \subseteq X$ open convex domain. Let us consider the operator equation

$$F(x) = 0, \quad F : D \subseteq X \to Y, \tag{1}$$

and find $x^* \in D$ satisfying (1), where $F \in C^1(D)$ is Fréchet differentiable and $F'(x^*)$ invertible, $\beta = \left\| [F'(x^*)]^{-1} \right\|$.

For Y a Hilbert space the genetic algorithm minimizes instead

$$\Phi(\mathrm{x}) = 1/2 \left\| F(x) \right\|_Y^2, \quad F(x^*) = 0 \Leftrightarrow x^* \in \arg\min_{x \in D} \Phi(x).$$

Each outer iteration of m-step secant method builds A_i once and takes m inner steps:

$$\begin{cases} A_i = F(x_{i-1}; x_i), \quad \phi_i^{(0)} = x_i, \\ \phi_i^{(j+1)} = \phi_i^{(j)} - A_i^{-1} F(\phi_i^{(j)}), \quad j = 0, \ldots, m-1, \\ x_{i+1} = \phi_i^{(m)}. \end{cases} \tag{2}$$

Here $F(x; y) \in \mathcal{L}(X, Y)$ is the divided-difference operator. If on ball $B(x^*, r)$ the divided difference satisfies Lipschitz condition

$$\left\| F(x; y) - F(u; v) \right\| \le L \left(\left\| x - u \right\| + \left\| y - v \right\| \right),$$

then there exists $\rho > 0$ such that for distinct $x_{-1}, x_0 \in B(x^*, \rho)$ method (2) converges to x^* with order p_m, where p_m is the positive root of $p^2 = p + m$ [1] and equal to $p_m = \left(1 + \sqrt{1 + 4m} \right) / 2$, in particular $p_1 \approx 1.618$, $p_2 = 2$, $p_{10} \approx 3.702$.

The genetic algorithm evolves a population P_g of N real-coded individuals [2] with selection S, crossover C, mutation M and elitism:

$$P_{g+1} = E_e \left(P_g \right) \uplus \left(M \circ C \circ S \right) \left(P_g \right), \quad \left| P_{g+1} \right| = N,$$

where E_e keeps the $\lfloor eN \rfloor$ best individuals by Φ. For the genetic algorithm no such estimate exists: elitist variants reach the global minimum of Φ in the limit, without a rate [2].

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FROM STANDARD OPERATORS TO UNINORMS: ASSESSING DECISION RELIABILITY ACROSS DISTINCT FUZZY PETRI NET FORMALISMS

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Traditional Artificial Intelligence models often struggle with reasoning under uncertain or incomplete information, particularly within complex systems that require simultaneous modeling of concurrency and synchronization. While Fuzzy Petri Nets (FPNs) effectively combine classical Petri net theory with fuzzy logic to address these limitations, a significant research gap remains due to the lack of standardized benchmarking frameworks for different FPN classes.

To resolve this issue, this paper introduces a systematic multi-criteria experimental framework designed to evaluate and benchmark decision quality within an exemplary Decision Support System (DSS) for Train Traffic Control. The study rigorously evaluates three distinct FPN formalisms that share a baseline net structure but differ fundamentally in their mathematical operators and binding functions: FP-net (Model A, utilizing standard Zadeh and Goguen t-norms and s-norms), GFP-net (Model B, implementing flexible families of Hamacher operators), and UP-net (Model C, employing highly generalized conjunctive and disjunctive uninorms with a configurable neutral element $e \in (0,1)$).

The experimental methodology evaluates a real-world railway conflict scenario governed by specific production rules ($r1$ – $r4$). The framework utilizes two distinct types of synthetic input data split across four datasets to test the models under diverse statistical profiles: regular datasets (Test1 and Test3, containing discrete value sequences drawn

from [0, 0.5, 1] and [0, 0.25, …, 1]) and random datasets (Test2 and Test4, featuring continuous values sampled from the unit interval [0,1]), scaling up to a sample size of $N = 3125$. Furthermore, a comprehensive Pearson correlation analysis was performed on the data from Test3 and Test4 to mathematically map the inter-model relationships and behavioral compliance between models A, B, and C.

The results demonstrate that GFP-net consistently outperforms both FP-net and UP-net, yielding the highest counts of unambiguous (*Uam*) and reliable (*Rel*) decisions under both fixed ($Thr = 0.1$) and dynamic thresholds. Computational efficiency remains excellent, with execution times being uniform and negligible (≤ 0.14s) across all test profiles. Additionally, sensitivity analysis of the UP-net indicates that its decision quality is inversely proportional to the value of e; lower values (~ 0.1) enhance reliability, whereas higher values (~ 0.9) trigger a critical mass of transition activation failures ($Amz = 3125$).

In conclusion, extending operator flexibility through GFP-nets significantly enhances decision reliability in fuzzy environments. Future work will investigate extreme value operator triplets and evaluate structural robustness against distinct fuzzification and data preprocessing techniques.

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AI-BASED MULTIMODAL SUPPLY CHAIN MANAGEMENT IN AN UNPREDICTABLE ENVIRONMENT

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Modern global logistics systems are characterized by a high level of uncertainty, caused by geopolitical changes, disruptions in transport corridors, climatic factors, and changes in freight flows. These challenges are particularly significant for Georgia, as part of the transport corridor connecting Europe and Asia, where the Black Sea ports play an important role in the organization of multimodal transportation.

Our research covers issues related to the integration of artificial intelligence technologies into supply chain management systems, with a particular emphasis on dynamic and unpredictable business environments.

The aim of the research is to develop an AI-based intelligent management mechanism that ensures the improvement of port logistics operations, freight flow forecasting, and adaptive response to disruptions.

The report provides a comparative analysis of traditional and AI-based approaches, identifying key challenges (data quality, technology integration, organizational readiness) and opportunities (early identification of risks, accurate demand forecasting, adaptive chain reconfiguration). The study draws on existing literature and practical examples to demonstrate how AI can transform the supply chain management paradigm, moving from static planning to dynamic, self-adaptive systems.

The proposed approach is based on the Microsoft Azure Cloud platform, Azure Machine Learning models, and Power Platform components. Real-time data processing makes it possible to assess port workload, forecast container flows, and identify logistics risks. Power Apps and Power BI provide process visualization and support for operational decision-making.

The research results demonstrate that the integration of artificial intelligence and cloud services increases the efficiency of multimodal transportation management, improves resource utilization, and contributes to the sustainable operation of the port logistics system.

MANAGEMENT OF HYDRODYNAMIC RISKS IN MARINE ENGINEERING THROUGH THE SYNTHESIS OF BERNOULLI'S LAW AND CFD SIMULATIONS

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In modern marine engineering, safe vessel navigation fundamentally relies on the physical principles of fluid dynamics and digital modelling. The objective of this paper is to conduct a systematic analysis of critical hydrodynamic risks—such as the Squat Effect, Bank Effect, and propeller cavitation—in shallow or restricted zones through the integrated application of Bernoulli's equation and Computational Fluid Dynamics (CFD).

Bernoulli's fundamental equation $(P + \frac{\rho v^2}{2} + \rho g h = const)$ defines the inverse proportionality between static pressure (P) and flow velocity (v) during a vessel's movement. When navigating through narrow channels, the restriction of the space beneath the vessel's hull forces the water flow to accelerate, which is followed by a sharp drop in static pressure beneath the keel.

As theoretical data and simulation tables demonstrate, with an increase in speed (for example, from 4 knots to 14 knots), the Under Keel Clearance (UKC) decreases from 2.5 meters to a critical 0.05 meters, while the static pressure drops from 95 kPa to 25 kPa. This process creates a powerful "suction" force that pulls the vessel toward the bottom and increases the risk of grounding with the ground.

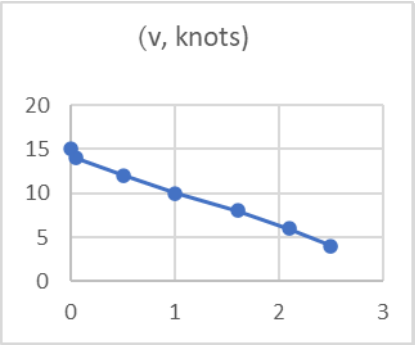

Fig. 1.

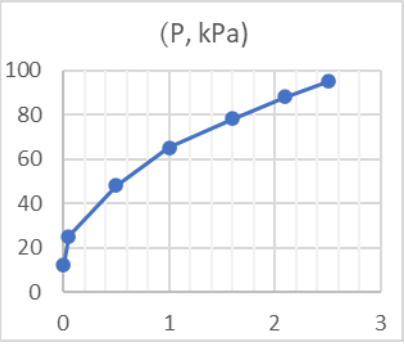

Fig. 2.

Unlike classical analytical methods, CFD technologies make it possible to accurately determine critical speed thresholds individually, taking into account the parameters of each specific vessel and channel. Accordingly, the synthesis of physical laws and digital modeling represents an indispensable tool for forecasting navigational risks and optimizing maritime safety.

ALGEBRAIC CHARACTERIZATION OF COMMUTATIVE TOPOLOGICAL GROUPS WITH THE SEPARATION PROPERTY

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A commutative topological group in which every compact subset is contained in a subgroup generated by a single element of the group is called a topological group of rank one. Such groups are of only four types, two of which the quasicyclic group $C(p^{\infty})$ and the group Q_p of all p-adic numbers are groups possessing the divisibility property.

The paper presents an algebraic characterization of divisible topological groups. It is proved that if G is a weakly locally compact divisible group, then G has the following form:

$$G = \sum_i T_i \dotplus \sum_i T_i' \dotplus \sum_i Q_i \dotplus \sum_i Q_i' ,$$

where T_i and T'_i are groups of type $C(p^{\infty})$ groups, and Q_i and Q'_i are groups of type Q_p.

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PIXEL-AWARE EVALUATION OF CAMERA LAYOUTS

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Conventional camera placement methods evaluate layouts using visibility or (k)-coverage metrics [1]. Although recent studies mainly improve optimization algorithms [2], the underlying evaluation criteria remain largely unchanged. This work introduces a pixel-aware metric incorporating observation uncertainty caused by finite camera angular resolution.

For each camera, corresponding pixel sectors observed by neighboring cameras define an uncertainty region U. The reliable observation region is evaluated as

$$R = V_2 - U, \tag{1}$$

where V_2 denotes the region observed by at least two cameras. Thus, the proposed metric combines multi-camera visibility with observation reliability.

Fig. 1 compares two layouts with nearly identical two-camera coverage of 38013.53 and 38144.02 area units (<1% difference). However, their reliable observation regions equal 10037.10 and 15786.31 area units, respectively, corresponding to +57.3%.

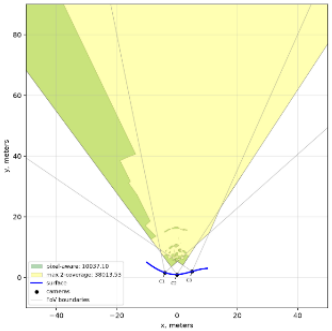

Fig. 1 – Layout A

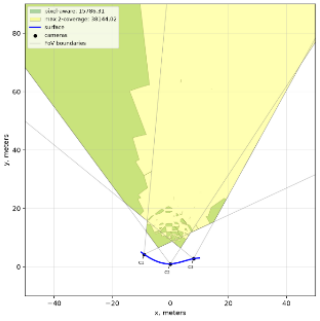

Fig. 1 – Layout B

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DIGITAL TWINS FOR HIL-MODELING OF MOBILE MEDICAL AND DIAGNOSTIC COMPLEXES

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This report presents an expanded review of the current state of the subject area, covering key concepts, technologies, and scientific approaches related to Digital Twins (DT), Cyber-Physical Systems (CPS), Hardware-in-the-Loop (HIL) simulation, multiphysics modeling, and hybrid machine-learning-enhanced modeling techniques. The review demonstrates that these technologies have become fundamental components of modern industrial automation, digital transformation, and the Industry 4.0 paradigm, enabling high-fidelity modeling, prediction, and integration of physical and digital system components in the development and creation of modern mobile medical and diagnostic complexes

Digital Twins are examined as dynamic digital replicas of physical objects that are continuously updated based on streaming data, control algorithms, and predictive models. They enable virtual experimentation, parameter optimization, behavioral analysis under uncertainty, and safe testing of control algorithms without risking physical equipment. HIL technology, in turn, provides the capability to connect real hardware to a simulation environment, making it possible to test controllers, validate algorithms, and explore fault or emergency modes in real time.

Special attention is given to multiphysics models that describe interactions among mechanical, electrical, thermal, hydrodynamic, and other processes within complex systems. The section also examines hybrid models that combine physical equations with machine learning, providing enhanced accuracy and simulation speed. Industrial integration protocols (OPC UA, MQTT, CAN, EtherCAT) are analyzed as essential mechanisms for reliable data exchange between digital and physical components.

The literature review shows that despite significant progress, the integration of DT and HIL remains insufficiently formalized, and existing approaches do not fully account for real-time constraints, the complexity of hybrid models, or the limitations of industrial communication protocols. A theoretical foundation for further research has been formulated, identifying key challenges, trends, and development directions within the subject area.

TINYVLA: COMPACT VISION-LANGUAGE-ACTION MODEL FOR GARMENT MANIPULATION

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The development of intelligent robotic systems capable of performing complex manipulation tasks with deformable objects, such as garments, is often constrained by the high computational complexity of existing large-scale models. This study presents TinyVLA, a compact vision-language-action (VLA) architecture designed to reduce the computational demands of the original SmolVLA [1] model while preserving its capacity to generate continuous robot actions from multi-view visual observations, natural-language instructions, and robot proprioceptive states.

The proposed TinyVLA architecture introduces a compressed design combining an 8-block Vision-Language Model (VLM) backbone with an 8-block action expert. A key scientific novelty of this architecture is that each action-expert block integrates both self-attention and cross-attention mechanisms. This structure preserves temporal interactions among action tokens while maintaining direct conditioning on multimodal VLM representations. The approach was evaluated using the garment subset of the LeHome dataset, which includes 1000 manipulation episodes across four categories: long pants, short pants, long tops, and short tops.

Experimental results demonstrate that TinyVLA successfully reduces the model size from approximately 450 million to 225 million parameters, representing a twofold reduction. Despite this significant compression, the final model achieved an average action-prediction loss competitive with the original SmolVLA baseline. These findings confirm that efficient compression of VLA models is possible without substantial degradation in action-generation performance, paving the way for the deployment of lightweight robotic policies on affordable and resource-constrained robotic platforms.

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MATHEMATICAL MODELLING OF DEMOGRAPHIC POTENTIAL OF A COUNTRY: LESLIE MATRIX, ARIMA AND LSTM ENSEMBLE APPROACH

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Demographic modeling under conditions of high socio-economic uncertainty requires robust hybrid analytical frameworks capable of capturing both structural population mechanics and nonlinear stochastic trends. Traditional pure time-series extrapolation methods often fail during abrupt structural transitions, underscoring the necessity of structural-stochastic integration. This study develops an integrated decision support system (DSS) combining a deterministic 17-cohort Leslie matrix model, an autoregressive ARIMA model, and a Long Short-Term Memory (LSTM) neural network into a dynamically weighted ensemble.

The structural core models age-structured cohort transitions via the discrete equation $n(t+1) = L \cdot n(t) + m(t)$, where L is the 17×17 Leslie matrix and $m(t)$ represents net migration vectors. Spectral analysis resolves the characteristic equation $det(L - \lambda I) = 0$ to evaluate asymptotic structural growth via the dominant eigenvalue λ_1. Ensemble weighting is dynamically computed using the inverse Root Mean Square Error ($w_i \propto 1/RMSE_i$) derived from retrospective backtesting on historical data (2017–2021). Global sensitivity analysis is executed via the Morris screening method using the SALib library to quantify parameter impacts across input factor spaces.

Retrospective validation established the superiority of the Leslie matrix model (MAPE = 1.47%, R2 = 0.6932). Spectral evaluations yielded $\lambda_1 = 0.9226$ for pessimistic, $\lambda_1 = 0.9469$ for baseline, and $\lambda_1 = 0.9871$ for optimistic scenarios, indicating a long-term structural contraction trend of approximately 5% per 5-year step ($\lambda_1 \approx 0.95$). Sensitivity analysis identified the Total Fertility Rate (TFR) and 20–45 age migration cohorts as primary control leverage points. The developed DSS provides quantitative scenario evaluation capabilities to support national strategic planning.

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PROACTIVE OPERATIONAL RISK MANAGEMENT VIA A MULTIFACTORIAL INTEGRATED ARCHITECTURE

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In the modern era, rapid digital transformation and growing technological reliance make predictable operational risk management essential. Traditional practices rely predominantly on historical statistical analysis, such as past risk events, incident counts, and financial loss metrics. While this historical baseline is foundational, it primarily reflects realized risks, offering limited early threat identification, trend analysis, and forward-looking forecasting.

This limitation directly jeopardizes Operational Resilience. Traditional frameworks focus strictly on risk prevention, leaving organizations ill-prepared for inevitable IT incidents, cyberattacks, and third-party risks. Consequently, during disruptions, organizations struggle to operate within their Impact Tolerances, leading to protracted recovery times and severe financial or reputational losses.

Addressing this requires expanding traditional predictive models through a multifactorial architecture. This framework integrates historical data with real-time operational risk governance telemetry. Specifically, it aggregates data generated from Risk and Control Self-Assessments (RCSA), Key Risk Indicators (KRI), scenario analyses, stress testing, Root Cause Analysis (RCA), and Business Continuity Management (BCM) exercises to evaluate both the likelihood of threats and internal control efficiency.

An effective solution lies in establishing a technologically integrated architecture driven by a dynamic decision-making framework. Powered by automated Data Aggregation, Predictive Analytics, real-time streaming, and Machine Learning (ML), this unified ecosystem continuously monitors the risk landscape. Ultimately, this multifactorial model transforms operational risk management into a proactive enabler of Operational Resilience, equipping organizations to maintain critical services and ensure continuity during times of systemic shock.

Note: The research, views, and conclusions expressed in this thesis reflect solely those of the authors and do not necessarily represent the official position of the National Bank of Georgia.

EVALUATION OF THE RESULT OF THE OPTIMAL SOLUTION OUTPUT IN ARTIFICIAL INTELLIGENCE PROBLEMS

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When solving combinatorial optimization problems, as a rule, the result is evaluated using correlation methods or the structure of the input information is analyzed. Different approaches affect the accuracy of the result and the speed of the algorithm. For problems with a clear structure of the input data, mathematical models are developed quite thoroughly and the global solution obtained by by complete search for the simulated objective function coincides with the purpose of the study. Methods based on recognizing the structure of the input information, in comparison with correlation methods, the global solution for large dimensions of the problem is often found polynomially. This is due to the fact that the combinatorial configuration (the argument of the objective function) is built in the process of recognizing the elements of the set of input data without searching through the options, due to which a functional connection is established between them. In addition, for some artificial intelligence problems in which a situation of uncertainty arises, such approaches may be the only possible ones for their solution.

In problems with fuzzy input data, which are related to artificial intelligence problems, in addition to the number of operations spent on finding a global solution, it is necessary to take into account the similarity measures, which play a major role here and the choice of which largely determines the solution itself. The global solution obtained by the simulated objective function in these problems does not always coincide with the research objective, that is, a situation of uncertainty arises. for which: a) there is a global solution; b) there is an admissible solution; c) there is no solution that would coincide with the research objective.

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DIGITAL TWIN AND HIL INTEGRATION METHODOLOGY

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The rapid development of modern technologies—particularly Digital Twins (DT), Cyber-Physical Systems (CPS), and Hardware-in-the-Loop (HIL) simulation—is fundamentally transforming approaches to the design, analysis, and operation of complex engineering systems. In industry, energy, transportation, aviation, robotics, and other domains, there is a growing demand for high-fidelity models capable not only of reproducing the behavior of physical systems but also of interacting with real hardware in real time. This creates the need for new methodologies that combine mathematical modeling, machine learning, industrial communication protocols, control algorithms, and simulation tools.

A Digital Twin in its modern interpretation is not merely a model but a **dynamic, evolving digital replica of a physical object**, continuously updated based on sensor data, telemetry, and predictive algorithms. DTs enable virtual experimentation, parameter optimization, failure prediction, control algorithm testing, and behavioral analysis under uncertainty. HIL technology, in turn, provides the ability to connect real hardware to a simulation environment, allowing engineers to test controllers, validate algorithms, and study system behavior without risking damage to physical components.

Despite significant progress in DT and HIL technologies, their **integration** remains a complex scientific and engineering challenge. The difficulties arise from model heterogeneity, differing time scales, real-time constraints, industrial protocol limitations, multiphysics interactions, and the need to ensure simulation stability and accuracy. Moreover, modern systems often include stochastic processes, discrete-continuous transitions, logical conditions, nonlinear interactions, and hybrid models combining physical equations with machine-learning components.

This study proposes a **multilayer cyber-physical architecture** comprising a physical layer, data layer, integration layer, modeling module, control module, and visualization module. Such a structure ensures modularity, scalability, flexibility, and extensibility. Additionally, a mathematical framework is developed, covering multiphysics models, stochastic processes, discrete-continuous descriptions, operator structures, and hybrid ML-enhanced models.

ON STATIONARY DISTRIBUTION FOR MIGRATION PROCESSES OF A SPECIAL TYPE

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An important branch of queueing theory is the theory of retrial queuing systems (see. [1]). They are used in the design of computer networks, in the analysis of stochastic information processing networks and modern communication systems, etc.

Retrial systems differ in the strategies of primary and secondary calls, as well as in the number of servers. The multidimensional nature of the models makes it possible to carry out a systematic analysis of stochastic networks and to investigate their integral performance characteristics. As the models become more complex, new mathematical problems arise, in particular, the development of methods for evaluating system performance measures under stationary operating conditions.

In this paper, a class of two-dimensional migration processes is introduced, which can be used to model the service process for various retrial systems. The arrival and service mechanisms of requests are determined by the controlling parameters of the migration. The main objective of the paper is to derive closed-form expressions for the stationary probabilities in terms of the parameters of the service process model.

We are dealing with a retrial queue of M/M/m/N type with intensity λ of the input Poisson flow of the primary calls, and ν is a parameter of the repeated calls. If there is a free server at the time of a call arrival, the call occupies one of the available servers, and its service time is exponentially distributed with parameter μ. Explicit formulas for calculating the stationary probabilities were obtained in vector-matrix form. These formulas are convenient for numerical.

In the case of a single server (m=1), the vector-matrix representation of the stationary probabilities is significantly simplified. In conclusion, we note that simple formulas, similar to those obtained for the M/M/1/N - system with retrials, can be derived for other types of migrations, determined by the choice of the dependence of local characteristics on the phase point of the selected domain.

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RESULTS OF OPTIMAL CONTROL OF UNSTEADY THERMOELECTRIC COOLING PROCESS IN THE MODE OF MINIMUM POWER CONSUMPTION

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The process of unsteady thermoelectric cooling is an example of an object with distributed parameters, which is described by a system of nonlinear differential equations of parabolic type with non-standard boundary conditions. The corresponding equations are given in [1] and approaches to solving the problem of finding the optimal time dependence of the current to achieve the lowest cooling temperature within a given time are described. These approaches are applied to optimize the unsteady thermoelectric cooling process in the minimum power consumption mode.

Under certain assumptions, thermal processes in thermoelectric legs are described by the unsteady heat conduction equation:

$$c(T)\frac{\partial T}{\partial t} = \kappa(T)\frac{\partial^2 T}{\partial x^2} + \rho(T)\frac{I^2(t)}{s^2} - \beta(T)\frac{I(t)}{s}\frac{\partial T}{\partial x}, \qquad (1)$$

with the initial condition in the form of $T(x,0) = T_a$, where $T(t, x)$ is the temperature, T_a is the ambient temperature, and $I(t)$ is the current, which in the general case is a function of time t, which varies on the interval $t \in [0, t_1]$. The coordinate x is directed along the leg from the cold junction to the hot one and varies on the segment $x \in [0, L]$, where L is the height, and s is the cross-section of the leg. The boundary conditions take into account the heat balance on the heat-absorbing surface and the temperature stabilization of the heat-releasing surface of the thermoelement, and they take the form:

$$g\frac{\partial T(0,t)}{\partial t} = \left[q_0 + Ks(T_a - T) - \alpha(T)I(t)T + I^2(t)\frac{r_0}{s} + \kappa(T)s\frac{\partial T}{\partial x} \right]_{x=0}, \qquad (2)$$

$$T(L,t) = T_h. \qquad (3)$$

The coefficients of the thermoelectric material, namely the heat capacity c, resistivity ρ, Seebeck coefficient α, thermal conductivity κ, and Thomson coefficient β, depend on temperature and are identical in

magnitude for thermoelectric legs of both conductivity types. In condition (2), g is the volumetric heat capacity of the cooling object, calculated per single leg. The first term on the right side of (2) accounts for the heat generation q_0 of the cooling object; the second term accounts for the heat exchange between the surface and the ambient environment, where K is the heat transfer coefficient; the third term is the Peltier heat; and the fourth term is the Joule heat generation due to contact resistance r_0.

The optimization problem consists in determining the optimal current function $I(t)$ such that, over a certain time interval t_1, it ensures a given cooling temperature $T(0,t_1)=T_c$ of the object under the condition of minimum power consumption J, which is determined by the functional:

$$J=\int_0^{t_1}\left[\alpha I(t)(T_h-T(0,t))+(\rho+\frac{r_0}{L})\frac{L}{s}I^2(t)\right]dt\,. \qquad (4)$$

The problem is solved by discretization along the coordinate using Pontryagin's maximum principle. An example of the solution to the problem is shown in Figs. 1, 2.

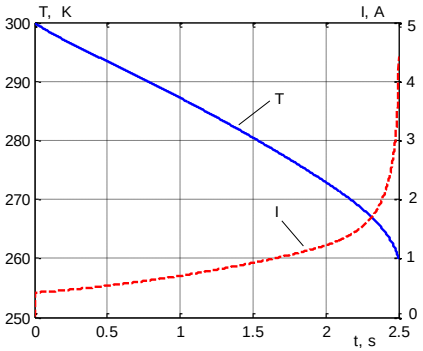

Fig. 1.

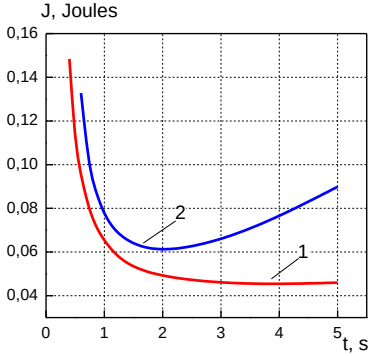

Fig. 2.

Fig. 1 shows the optimal current function $I_{opt}(t)$, calculated for lowering the temperature from 300 K to 260 K in 2.5 s. It also shows how the temperature decreases over time. Obviously, these functions are different for different time intervals. Fig. 2 shows the dependences of the electrical energy consumed by the thermoelectric element on the time required to reach a cooling temperature of 260 K under the conditions of applying $I_{opt}(t)$ (curve 1) and constant current (curve 2). When using $I_{opt}(t)$, the power consumption savings range from 25 to 50% compared to the option of powering the thermoelement with constant current.

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MODELING OF DYNAMIC SYSTEMS WITH FUZZY DISCRETE OBSERVATIONS

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A model of a dynamical system governed by the differential equation

$$L(\partial_s)y(s)=u(s),$$

is considered, where $L(\partial_s)$ is a linear differential operator defining the dynamics of the system, $u(s)$ is a known continuously differentiable function, and $y(s)$ is the unknown state function of the system defined on the closed and bounded domain $S_0^T=S_0\times[0,T]$, $S_0\in\mathbb{R}^m$, $s=(x,t)=(x_1,\ldots,x_n,t)\in S_0^T$, $\partial_s=(\partial_x,\partial_t)=(\partial_{x_1,\ldots},\partial_{x_n},\partial_t)$. Discrete fuzzy observations of the initial, boundary, and current states of the system are also assumed to be known:

$$L_r^0(\partial_t)y(s)_{t=0,\ x=x_l^0\in S_0}=\tilde{Y}_{rl}^0,\ (r=\overline{1,R_0},\ l=\overline{1,L_0}),$$

$$L_\rho^\Gamma(\partial_x)y(s)_{s=s_l^\Gamma\in\Gamma\times[0,T]}=\tilde{Y}_{\rho l}^\Gamma\ \ (\rho=\overline{1,R_\Gamma},\ l=\overline{1,L_\Gamma}),$$

$$y(s_k)=\tilde{Y}_{sk},\ \ s_k\in S_{cn},\ k=\overline{1,K},$$

where $\tilde{Y}_{rl}^0=tfn\left(Y_{rl}^0,a_{Lrl}^0,a_{Rrl}^0\right)$, $\tilde{Y}_{\rho l}^\Gamma=tfn\left(Y_{\rho l}^\Gamma,a_{L\rho l}^\Gamma,a_{R\rho l}^\Gamma\right)$, $\tilde{Y}_{sk}=tfn\left(Y_{sk},a_{Lsk},a_{Rsk}\right)$ are known triangular fuzzy numbers.

Following the methodology described in [1], we will seek the state function of the system in the form

$$y(s)=\sum_{m=1}^{M}G(s-s_m)u_m\Delta_m+\sum_{m=1}^{M_0}G(s-s_m^0)u_{0m}+\sum_{m=1}^{M_\Gamma}G(s-s_m^\Gamma)u_{\Gamma m},$$

where $G(s-s')$ is the transfer function of the differential operator $L(\partial_s)$, u_m are the values of the function $u(s)$, and $u_{0m},u_{\Gamma m}$ are unknown values of the modeling functions. Substituting $y(s)$ into the

discrete fuzzy observations, we obtain a system of linear algebraic equations with a fuzzy right-hand side. The solution [2] of this system is a set of triangular fuzzy numbers $u_{0m}, u_{\Gamma m}$. Using these values, the values of the state function $y(s)$ can be reconstructed as triangular fuzzy numbers.

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RESEARCH OF SIGNAL AND DATA CONVERSION PROCESSES IN MICROPROCESSOR-BASED AND SPECIALISED HARDWARE MODULES

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The report examines the processes of signal and data conversion in microprocessor-based and specialized hardware modules used in real-time digital information-processing systems. The processing chain under consideration comprises data acquisition, conversion, preprocessing and data transfer between heterogeneous computing units. The purpose of the study is to assess the impact of a digital system's architecture on its performance, reliability and energy efficiency. A comparative analysis is performed for general-purpose microprocessors, microcontrollers, digital signal processors, field-programmable gate arrays, and application-specific integrated circuits. Microprocessor-based architectures provide flexibility, programmability, and convenient implementation of complex control algorithms. However, their performance may depend on operating-system scheduling, interrupt processing, cache behaviour, and shared-memory access. Specialized hardware modules provide parallel data processing, predictable execution time, high throughput, and reduced latency, but require more complex design and verification procedures. The main architectural factors influencing real-time performance are identified: data-path

width, memory hierarchy, interrupt latency, direct memory access, pipelining, hardware parallelism, buffering strategy, bus arbitration, and synchronization between processing modules. Particular attention is paid to heterogeneous architectures in which the microprocessor performs supervisory control, configuration, decision-making, and adaptive algorithms, while FPGA- or ASIC-based modules execute time-critical filtering, coding, transformation, and interface operations. The study shows that the efficient distribution of computational tasks between programmable and specialized components makes it possible to combine software flexibility with deterministic hardware performance. The obtained results can be applied in embedded control systems, industrial automation, robotic platforms, telecommunications equipment, cyber-physical systems, and Hardware-in-the-Loop environments. Further research should focus on adaptive task distribution, fault-tolerant data paths, and energy-efficient implementation of real-time information-processing architectures.

PREDICTIVE ESTIMATES OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH NONLINEAR DIFFUSION UNDER UNCERTAINTY

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Problems of estimating solutions of stochastic equations - with unorthodox innovation/noise structures - present an interesting field of research within financial mathematics.

We investigate the problem of estimating solutions of equations of the form

$$dx(t) = \left[Ax(t) + f(t)\right]dt + \sigma(t,x)dW(t), \quad x(0) = x_0, \qquad (1)$$

where A is a given $n \times n$ matrix, $f(t)$ is a given function continuous on the interval $[0,T]$, and $\sigma(t,x)$ is a diagonal matrix with elements $\sigma_i(t,x) = b_i(t)\sqrt{(x(t), c_i(t)}$. Here $W(t) = (W_1(t),...,W_n(t))^T$ is a standard Wiener process, while $b_i(t)$ and $c_i(t)$ are given continuous functions.

The initial value x_0 is a random variable independent of $W_i(t)$, and

$$Ex_0 = m_0, \quad E[(x_0 - m_0)(x_0 - m_0)^T] = R_0.$$

In what follows, we assume that a solution $x(t)$ of equation (1) exists such that $\big(x(t), c_i(t)\big) > 0 \quad \forall t \in (0,T)$. Let the output function $y(t)$, with values in $\mathbb{R}^m$, have the form

$$y(t) = Hx(t) + \xi(t), \quad t \in (0,T), \tag{2}$$

where H is a known $m \times n$ matrix and $\xi(t)$ is a mean-square continuous vector random process such that $E\xi(t) = 0$, and

$$E[\xi_i(t)\xi_j(s)] = \begin{cases} 0, & i \neq j, \\ R_i(t,s), & i = j. \end{cases}$$

The correlation functions $R_i(t,s), i = \overline{1,m},$ are unknown and belong to a set V of a certain functional space.

Let L be a linear operator mapping $\mathbb{R}^n$ into $\mathbb{R}^s$, where $s \leq n.$

We investigate linear estimates of the vector $Lx(t_1),$ where $t_1 > T,$ of the form

$$Lx(t_1) = \int_0^T U(t)y(t)dt + c,$$

and their guaranteed mean-square errors for a particular choice of the set V. The set V has the form

$$V = \left\{ R : \int_0^T q_i^2(t) R_i(t,t)dt \leq 1, \quad i = \overline{1,m} \right\},$$

where $q_i^2(t)$ are known functions. Guaranteed mean-square errors are found, and it is shown that the best predictive estimates have the following form:

$$Lx(t_1) = L\hat{x}(t_1),$$

where the function $\hat{x}(t)$ is determined from the solution of the equation

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + f(t), \quad \hat{x}(T) = x_g(T).$$

The function $x_g(T)$ is obtained from the solution of the equation

$$\frac{dx_g(t)}{dt} = Ax_g(t) + f(t) + P(t)H^T Q(y(t) - Hx_g(t)),$$

with $x_g(0) = Ex_0,$ where $Q = \operatorname{diag}(q_1^2(t), ..., q_m^2(t)).$

The matrix-valued function $P(t)$ is determined from the solution of a nonlinear matrix differential equation.

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A VARIATIONAL METHOD FOR SOLVING THE INVERSE PROBLEM OF DETERMINING A COMPLEX POTENTIAL, DEPENDENT ONLY ON TIME, IN A NONLINEAR NONSTATIONARY SCHRÖDINGER EQUATION WITH A SPECIAL GRADIENT TERM

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In this paper, we consider the inverse problem of determining functions $\left(v_0(t), v_1(t), \psi(x,t)\right)$ from the following conditions:

$$i\frac{\partial \psi}{\partial t} + a_0\frac{\partial^2 \psi}{\partial x^2} + ia_1(x,t)\frac{\partial \psi}{\partial x} - a(x)\psi + v_0(t)\psi + \\ + iv_1(t)\psi + a_2\left|\psi\right|^2\psi = f(x,t),\ (x,t)\in\Omega, \quad (1)$$

$$\psi(x,0) = \varphi(x),\ x\in(0,l), \quad \frac{\partial \psi(0,t)}{\partial x} = \frac{\partial \psi(l,t)}{\partial x} = 0,\ t\in(0,T), \quad (2)$$

$$\psi(x,T) = y(x), \quad x\in\left(0,l\right), \quad (3)$$

where $i=\sqrt{-1}$; $a_0>0, l>0, T>0$ are given numbers; $\Omega=(0,l)\times(0,T)$; a_2 is complex number satisfying the conditions:

$$a_2 = \operatorname{Re}a_2 + i\operatorname{Im}a_2,\ \operatorname{Im}a_2>0,\ \operatorname{Re}a_2<0,\ \operatorname{Im}a_2 \ge 2\left|\operatorname{Re}a_2\right|; \quad (4)$$

$a(x), a_1(x,t)$ are real-valued measurable bounded functions satisfying the conditions:

$$\mu_0 \le a(x) \le \mu_1, \quad \left|\frac{da(x)}{dx}\right| \le \mu_2, \quad \left|\frac{d^2a(x)}{dx^2}\right| \le \mu_3, \quad \overset{0}{\forall} x \in (0,l),$$

$$\mu_0, \mu_1, \mu_2, \mu_3 = const > 0; \qquad (5)$$

$$\left|a_1(x,t)\right| \le \mu_4, \quad \left|\frac{\partial a_1(x,t)}{\partial x}\right| \le \mu_5, \quad \left|\frac{\partial^2 a_1(x,t)}{\partial x^2}\right| \le \mu_6, \quad \overset{0}{\forall}(x,t) \in \Omega,$$

$$a_1(0,t) = a_1(l,t) = 0, \mu_4, \mu_5, \mu_6 = const > 0; \qquad (6)$$

$\varphi(x), f(x,t), y(x)$ are complex-valued functions satisfying the conditions:

$$\varphi \in W_2^2(0,l), \frac{d\varphi(0)}{dx} = \frac{d\varphi(l)}{dx} = 0 \ ,$$

$$f \in W_2^{2,0}(\Omega), \ \frac{\partial f(0,t)}{\partial x} = \frac{\partial f(l,t)}{\partial x} = 0, \ t \in (0,T) \ , y \in L_2(0,l) \ . \qquad (7)$$

To solve the inverse problem (1)-(3), a variational method is applied and a variational problem of minimizing the functional is considered:

$$J_a(v) = \left\|\psi(.,T) - y\right\|^2_{L_2(0,l)} + \alpha \left\|v - \omega\right\|^2_H \qquad (8)$$

on the set:

$$V = \left\{ v = v(t) = \left(v_0(t), v_1(t)\right) : \ v_s \in L_2(0,T), \left|v_s(t)\right| \le b_s, s = 0,1, \overset{0}{\forall} t \in (0,T) \right\}$$

under conditions (1), (2), where $\alpha \ge 0$, $b_s > 0, s = 0,1$ are given numbers, $\omega \in H \equiv L_2(0,T) \times L_2(0,T)$ is a given element and the symbol $\overset{0}{\forall}$ means 'for almost all'.

For the variational problem (1), (2), (8), we first prove the existence and uniqueness theorems for the solution. Next, we investigate the differentiability of the functional under concideration and estabilish a formula for its gradient, which is used to prove a necessary condition for solving the variational problem in the form a variational inequality [1].

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THE MIXTURE OF SEMI-MARKOV PROCESSES IN NONLINEAR APROXIMATION

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Consider semi-Markov processes $x_1\left(\frac{t}{g(\varepsilon)}\right)$ and $x_2\left(\frac{t}{g(\varepsilon)}\right)$, $t>0$, $g(\varepsilon)\to 0$ when $\varepsilon\to 0$ on a time scale $\frac{t}{g(\varepsilon)}$ with a nonlinear normalization factor $g(\varepsilon)$.

Let the process $x_1\left(\frac{t}{g(\varepsilon)}\right)$ occurs with probability p_1, and the process $x_2\left(\frac{t}{g(\varepsilon)}\right)$ with probability p_2, when $p_1+p_2=1$. Then, let's construct a random process that will be a mixture of these semi-Markov processes

$$x\left(\frac{t}{g(\varepsilon)}\right)=p_1\cdot x_1\left(\frac{t}{g(\varepsilon)}\right)+p_2\cdot x_2\left(\frac{t}{g(\varepsilon)}\right).$$

The purpose of the work is the asymptotic analysis of this process in nonlinear approximation

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ECG R-PEAK DETECTION ON MICROCONTROLLERS USING PIECEWISE POLYNOMIAL SMOOTHING

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Standard Holter monitors provide accurate diagnostics but require expensive equipment and manual post-processing of the records by a specialist. Wearable devices based on affordable 8-bit microcontrollers can reduce costs and perform real-time signal analysis, but their implementation is hindered by the lack of a hardware floating-point unit and a small amount of SRAM (2 - 8 KB).

A novel approach to accurate R-peak detection based on piecewise Hermite polynomial smoothing was proposed in [1]. The fundamental advantage of this approach is the combination of accuracy and computational simplicity: it relies exclusively on integer arithmetic and requires only three samples per step. This makes it highly suitable for microcontrollers with strict resource constraints. The present work confirms this in practice.

The prototype was assembled using an Arduino Mega 2560 (ATmega2560, 8-bit, 8 KB SRAM) and an AD8232 analog front-end. The algorithm [1] runs in real time, occupying ~2300 bytes of memory. For validation purposes, signals were recorded directly from the hardware setup during both resting conditions and dynamic states involving motion artifacts. Statistical analysis using confidence intervals confirmed that the acquired signals correspond to true ECG waveforms. Furthermore, we demonstrate an energy-efficient transmission strategy. By sending only the R-peak indices instead of a continuous stream of raw ADC samples, the device significantly cuts down data traffic and prolongs its battery life.

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EXPERT-GUIDED GBDT IN TECHNICAL CONDITION ASSESSMENT OF DAMAGED OBJECTS

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The assessment of the technical condition of damaged objects relies on heterogeneous indicators, relationships between defects and expert knowledge of typical damage patterns. Gradient Boosted Decision Trees (GBDT) are well suited to such tabular data. However, high classification quality may require many weak learners, increasing model complexity and reducing interpretability. Previous studies have shown that clustering can enrich the original feature space with additional characteristics [1], which may significantly affect classification while reducing the number of GBDT estimators.

This study proposes a methodology for integrating building-technical expert knowledge into the construction of cluster-derived features for GBDT models. Experts possess domain knowledge of the systemic impact of explicit and implicit signs of object degradation on the deterioration of its overall technical condition, while not necessarily having expertise in machine learning or data analysis. Therefore, such knowledge should be collected in a simple domain-oriented form and automatically transformed into clustering parameters or constraints [2]. Possible mechanisms include expert-defined feature groups and weights, representative damage regimes and relationships, introducing constraint clustering [3].

Since access to real datasets of construction and technical expertise is currently significantly limited, it is proposed to validate the methodology on synthetic datasets with predefined latent damage modes and controlled levels of expert knowledge accuracy. Evaluation should consider classification quality, ensemble complexity, clustering stability and robustness to incomplete or inaccurate expert information. The expected result is a practical methodology enabling experts to incorporate domain knowledge into model training without advanced data-science skills, while preserving predictive quality and potentially reducing GBDT ensemble complexity.

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BAYESIAN INFERENCE OF THE AGGREGATED TRANSITION KERNEL IN A LATENT MIXTURE OF MARKOV CHAINS

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The study considers the problem of Bayesian inference for the aggregate transition matrix of a discrete Markov process generated by a latent mixture of component Markov kernels. Unlike most works, where the main attention is paid to the recovery of hidden regimes or component transition matrices, in this work the object of study is the aggregate transition kernel, which is an identified functional of the model even if the components are not identified. For a model with latent independent mode indicators, a Dirichlet prior on the mixture weights, and a Dirichlet prior on the rows of the component matrices, an algorithm based on the variational Bayes method in the mean-field approximation is derived, including a full ELBO (Evidence Lower Bound) derivation and coordinate updates. It is proved that the posterior distribution of the aggregated kernel is an induced distribution and does not belong to the Dirichlet family of distributions in general. Analytical formulas for the first two moments of the aggregated kernel under the variational posterior are obtained, and a compact Dirichlet projection based on moment agreement is proposed. The proposed approach provides an interpretable representation of the a posteriori uncertainty of the aggregated dynamics without the need to identify the mixture components.

KEY PERFORMANCE INDICATORS OF DIGITAL TWINS IN HIL SIMULATION

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The digital twin concept, pioneered by M. Grieves and advanced in the aerospace industry, has become a key technology of Industry 4.0 and Industry 5.0. In F. Tao's systematic review, the digital twin (DT) is defined as an integrated multi-physics product simulation based on physical models, sensor data, and optimization algorithms to reflect the current state and predict behavior Despite the significant number of publications, a unified methodology for assessing digital twin effectiveness remains absent: most approaches are confined to verifying individual aspects without a comprehensive evaluation of holistic behavior. For its assessment under full life-cycle conditions, a comprehensive set of parameters is proposed: physico-mathematical (geometric, mechanical, thermal, electromagnetic, and hydrodynamic indicators); functional-dynamic (time and frequency characteristics, transient processes, and nonlinear effects); operational (reliability, energy efficiency, degradation processes, and diagnostic features); and information-control parameters (exchange protocols, synchronization, and data integrity).

For an integrated DT/HIL environment, a four-level hierarchy of key performance indicators is proposed: physical-to-virtual correspondence (model accuracy, digital twin "age," and bidirectional synchronization coefficient); multi-physics integrity (multi-domain consistency, energy balance coefficient, and transient process index); computational efficiency and scalability (flexibility index, computational budget, and data integrity); as well as verification and validation.

Based on the key parameters, a system adequacy index of the digital twin is formed, which combines data freshness, multi-dimensional quality, and computational cost functions into a single scalar value with weighting coefficients. This metric provides a comprehensive assessment of the accuracy with which the real product is represented in the virtual environment.

Further development of the proposed approach involves unifying the assessment criteria, creating ready-made templates for building such systems in production, and developing algorithms for the automatic self-tuning of the digital twin to the state of real equipment in real time.

ASYMPTOTIC GAIN PROPERTY FOR THE PDE-ODE SYSTEM

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The investigation of the stability of dynamic systems is an extremely important task. The theory of input-to-state stability was developed in the works of Sontag [1] and was subsequently adapted to study the robust stability of systems [2], [3], [4]. Further research focused on extending this theory to the case of infinite-dimensional systems [5] and separately for systems known as PDE-ODE systems [6].

In bounded domain $\Omega \subset \mathbb{R}^d$ we consider the following PDE-ODE system

$$\begin{cases} u_t = A\Delta u - f(u) + B(x)v(t) + d_1(t,x), \\ u\big|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

$$\frac{dv}{dt} = -F(v) + \int_\Omega G(x)u(x,t)\,dx + d_2(t), \tag{2}$$

where A is a given $N \times N$ matrix such that $\frac{1}{2}\left(A + A^*\right) \geq \upsilon_1 I$, $u(x,t) = (u^1,\ldots,u^N), v(t) = (v^1,\ldots,v^M)$ are unknown vector functions, $B, G \in L^2(\Omega)$ are given matrix of corresponding dimensions, $d_1 \in L^\infty\left(0,+\infty;(L^2(\Omega))^N\right)$ and $d_2 \in L^\infty\left(0,+\infty;\mathbb{R}^M\right)$ are input signals and $\forall y, w \in \mathbb{R}^N$, $u \in \mathbb{R}^M$:

$$f \in \mathbb{C}^1\left(\mathbb{R}^N;\mathbb{R}^N\right), F \in \mathbb{C}^1\left(\mathbb{R}^M;\mathbb{R}^M\right), \sum_{i=1}^N f^i(y)\,y^i \geq \nu_2 \cdot \sum_{i=1}^N \left|y^i\right|^{p_i} - c_1,$$

$$\sum_{i=1}^N \left|f^i(y)\right|^{\frac{p_i}{p_i-1}} \leq c_2\left(\sum_{i=1}^N \left|y^i\right|^{p_i} + 1\right), \tag{3}$$

$$\left(Df(u)w, w\right)_{\mathbb{R}^N} \geq -c_3 \cdot \left|w\right|^2_{\mathbb{R}^N}, \sum_{i=1}^M F^i(u)u^i \geq \nu_3\left|u\right|^2_{\mathbb{R}^M} - c_4,$$

where $\upsilon_1, \upsilon_2, \upsilon_3, c_1, c_2, c_3, c_4$ are given constants, $p_i \geq 2, i = \overline{1,N}$.

Theorem. *The attractor Θ for the unperturbed case of the system (1)-(2) ($d \equiv 0$) has the AG property, i.e.*

$$\exists \gamma \in \mathcal{K} \forall z_0 \in X, \forall \|d\|_\infty \le R_0 : \overline{lim}_{t\to\infty} \|S_d(t, z_0)\|_\Theta \le \gamma\left(\|d\|_\infty\right),$$

where

$$S_d(t, z_0) := \{z(t),\ z(\cdot)\ is\ a\ solution\ of\ the\ system\ (1)\text{-}(2)$$
$$with\ disturbances\ d,\ z(0) = z_0\},$$

$\mathcal{K}$ *is the class of continuous strictly increasing functions* $\gamma : [0, +\infty) \mapsto [0, +\infty)$ *with* $\gamma(0) = 0$.

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ASYMPTOTIC ANALYSIS OF RELIABILITY CHARACTERISTICS IN SERIES REPAIRABLE SYSTEM WITH LARGE REDUNDANCY

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This paper investigates the asymptotic reliability behavior of series repairable systems with redundancy under an overloaded repair facility. The main focus is on the behavior of reliability characteristics as the number of redundant units tends to infinity in the case where the repair facility utilization factor exceeds unity ($\rho>1$).

For exponentially distributed failure and repair times, analytical expressions for the main reliability characteristics are derived using the results of [1]. It is shown that, for exponentially distributed failure and repair times, unlike the mean time to failure, which tends to infinity as $m\to\infty$, the mean time between failures (MTBF) converges to a finite limiting value determined by the system parameters. In addition, the stationary availability coefficient converges to the limiting value $1/\rho$, which depends only on the repair facility utilization factor.

For the case of a single repair channel and general repair time distributions, an asymptotic expression for the stationary availability coefficient is derived using the results of [2]. It is shown that, as the number of redundant units increases, the stationary availability converges to the same limiting value $1/\rho$.

The analytical results are complemented by simulation studies performed using GPSS World. The simulation results show that increasing the number of redundant units leads to an increase in the MTBF, with approaching limiting value. This limiting value increases as the coefficient of variation of the repair time distribution increases, whereas the convergence rate decreases. At the same time, the stationary availability asymptotically approaches $1/\rho$ independently of the particular forms of the failure and repair time distributions.

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ANALYSIS OF REGULAR AND CHAOTIC DYNAMICS IN A FRACTIONAL-ORDER NONLINEAR SYSTEM

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In this paper, we consider a mathematical model of a nonlinear fractional-order financial system that exhibits complex regular and chaotic dynamics. The corresponding system of fractional-order differential equations describing the chaotic Chen system is given by

$$d^{\alpha_1}X/dt^{\alpha_1} = Z + (Y-\alpha)X,$$
$$d^{\alpha_2}X/dt^{\alpha_2} = 1 - bY - X^2,$$
$$d^{\alpha_3}X/dt^{\alpha_3}\,\frac{d^{\alpha_3}Z}{dt^{\alpha_3}} = -X - cZ$$

where a≥0, b≥0, and c≥0 are model parameters representing the persistence of the interest rate, the investment cost, and the elasticity of demand in commercial markets, respectively. In this study, the Caputo-type fractional derivative is defined as $d^{\alpha}y/dx^{\alpha} = D_*^{\alpha}y(x) = I^{m-\alpha}y^{(m)}(x)$, where $0 \le m-1 \le \alpha \le m,\ m = [\alpha]$ ([.] denotes the ceiling function), $y^{(m)}$ is the ordinary m-th derivative of y, and $I^{\beta}z(x) = \frac{1}{\Gamma(\beta)}\int_0^x (x-t)^{\beta-1}z(t)dt$ is the Riemann–Liouville integral operator of order β, where Γ is the gamma function and $\beta > 0$ is an arbitrary but fixed base point.

In this study, the regular and chaotic regimes of the system are investigated depending on the bifurcation parameters. Furthermore, based on the Lyapunov function approach, theorems concerning the existence of a globally exponentially attractive set, a positively invariant set, and periodic solutions are formulated and proved. Conditions for the occurrence of a Poincaré–Andronov–Hopf bifurcation are also established. Theoretical results concerning attractor control are obtained.

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